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Statistical properties and applications of Poisson-Garima distribution

Abstract

The Poisson-Garima distribution, the Poisson compound of Garima distribution, was introduced by Shanker to model count data of over-dispersed nature and some of the properties along with applications were discussed. It has been observed that some useful statistical properties and reliability properties of the Poisson-Garima distribution was not studied by Shanker. The main purpose of this paper is to discuss the statistical and reliability properties of the Poisson-Garima distribution which was not studied by Shanker and also propose some new applications of the distribution. The proposed distribution is log-concave and a two-component mixture of negative binomial distributions. The hazard function of the distribution has been derived and is increasing and thus the distribution is unimodal. The simulation study has been presented to show the consistency of maximum likelihood estimator of the parameter of Poisson-Garima distribution. It has been shown that the maximum likelihood estimator of the parameter of the distribution is consistent and asymptotically normal. The Poisson-Garima distribution has been applied to four real datasets and found to provide quite satisfactory fit over other competing one parameter over-dispersed count distributions.

Keywords: Poisson-Garima distribution, statistical and reliability properties, simulation, maximum likelihood estimation, goodness of fit.

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Introduction

The Poisson distribution which is a limiting distribution of binomial distribution is the first discrete distribution for count data of equi-dispersed (mean equal to variance) nature. Count data arise in almost every fields of knowledge including insurance, agriculture, environmental science, biological sciences, clinical trials, engineering etc. It has been found that the count data are in general either over-dispersed (variance greater than mean) or under-dispersed (variance less than mean) and hence Poisson distribution has limited applications due to its limitation of mean equal to variance. During recent decades several over-dispersed count distributions have been introduced in statistical literature including negative binomial distribution which is the Poisson compound of gamma distribution, Poisson-Lindley distribution (PLD) of Sankaran,[1] which is the Poisson compound of Lindley distribution of Lindley,[2] Poisson-Shanker distribution (PShD) of Shanker,[3] which is the Poisson compound of Shanker distribution of Shanker,[4] Poisson-Garima distribution (PGD) of Shanker,[5] which is the Poisson compound of Garima distribution of Shanker,[6] Poisson-Komal distribution (PKD) of Shanker,[7] which is the Poisson compound of Komal distribution of Shanker.[8] The Garima distribution introduced by Shanker,[6] is defined by its probability density function (pdf)

$$f(\lambda;\theta)=\frac{\theta}{\theta+2}(1+\theta+\theta\lambda)e^{-\theta\lambda}\quad;\lambda>0,\theta>0\,.$$

During a short interval of time much works have been done relating to Garima distribution and its modifications. Shanker and Shukla,[5,9] proposed zero-truncated Poisson-Garima distribution and size-biased Poisson-Garima distribution to model count data excluding zero counts. Tesfalem and Shanker,[10] suggested weighted Garima distribution and discussed its properties and applications. A two-parameter power Garima distribution has been proposed by Berhane.[11] Shanker,[12] proposed quasi Garima distribution and discussed its properties and applications. A discrete Garima distribution has been proposed by Shukla.[13] Mohiuddin,[14] suggested Alpha power transformed Garima distribution and discussed its properties and applications. The length –biased power Garima distribution has been introduced by klinjan and Aryuyuen.[15] Aryuyuen,[16] suggested power-Garima generated family of distributions and discussed its properties and applications. Sottiwan,[17] proposed truncated length-biased power Garima distribution and discussed its properties and applications. Klinjan and Aryuyuen,[18] developed R package for the length-biased power Garima distribution for analyzing lifetime data. Thiamsorn,[19] suggested power-Garima-Rayleigh distribution and discussed its properties and applications. Ganaie,[20] proposed a new generalization of Garima distribution using weight function. The unit- Garima distribution has been proposed by Aryuyuen and Bodhisuwan,[21] for analyzing proportion data. Aryuyuen and Pudprommarat,[22] suggested the bivariate length-biased power Garima distribution derived from copula and discussed its properties and applications to environmental data.

Shanker,[23] derived the PGD assuming that the parameter of the Poisson distribution follows Garima distribution. A random variable X is said to be PGD if it follows the following stochastic representation $X\mid\lambda\sim\text{Poisson}(\lambda)$ and $\lambda\mid\theta\sim\text{Garima}(\theta)$ for $\lambda>0,\theta>0$ and the unconditional distribution of this stochastic representation is the PGD(θ), as suggested by Shanker.[23] That is, the mathematical way of deriving the probability mass function (pmf) of PGD used by Shanker,[23] is as follows:

$$P(x;\theta)=\int_0^\infty P(X=x|\lambda)f(\lambda;\theta)d\lambda$$

$$=\int_0^\infty \frac{e^{-\lambda}\lambda^x}{x!}\frac{\theta}{\theta+2}(1+\theta+\theta\lambda)e^{-\theta\lambda}d\lambda$$

$$=\frac{\theta}{\theta+2}\frac{\theta x+\left(\theta^2+3\theta+1\right)}{(\theta+1)^{x+2}}\quad;x=0,1,2,...,\theta>0\,.$$

The natures of pmf of PGD for varying values of parameter are shown in Figure 1 and it is clear that it is highly positively skewed for increasing values of parameter.

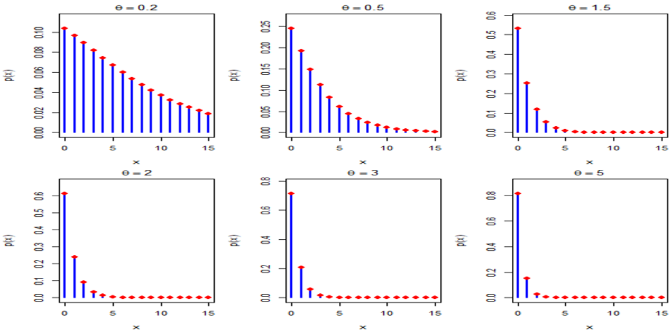

Figure 1 pmf of PGD for varying values of parameter.

The first four moments about origin and the variance of PGD obtained by Shanker,[23] are

$$\mu_1' = \frac{\theta+3}{\theta(\theta+2)},\quad \mu_2' = \frac{\theta^2+5\theta+8}{\theta^2(\theta+2)},\quad \mu_3' = \frac{\theta^3+9\theta^2+30\theta+30}{\theta^3(\theta+2)}$$

$$\mu_4' = \frac{\theta^4+17\theta^3+92\theta^2+204\theta+144}{\theta^4(\theta+2)},\ \text{and}\ \mu_2 = \frac{\theta^3+6\theta^2+12\theta+7}{\theta^2(\theta+2)^2}$$

The detailed discussion about its properties, estimation of parameter, and applications has been discussed by Shanker,[23] and it has been shown that it is better than Poisson distribution and PLD for modeling count data in various fields of knowledge. It seems that some interesting and useful statistical and reliability properties of PGD have not been discussed by Shanker.[23]

The main purpose of this paper is to derive some interesting and useful statistical and reliability properties of PGD, which has not been studied earlier by Shanker.[23] We have shown that the proposed distribution is log-concave and is a two-component mixture of negative binomial distributions. The hazard function of the distribution has been shown to be increasing and thus the distribution is unimodal. For showing the consistency of the maximum likelihood estimator of the distribution, simulation study has been presented. It has been shown that the maximum likelihood estimator of the parameter of the PGD is consistent and asymptotically normal. The goodness of fit of the PGD has been established with four real datasets and observed that it provides best fit over other competing one parameter over-dispersed distributions.

Statistical properties of Poisson-Garima distribution

In this section an attempt has been made to prove some important statistical results of PGD. The PGD is positively skewed, unimodal and decreasing which is supported by theorems 1 and 2. In theorem 3, it has been shown that PGD is also a two-component mixture of negative binomial distributions with different parameter (number of successes) and for the same probability of success.

The PGD has increasing hazard rate (IHR) and is unimodal. Since

$$Q(x,\theta) = \frac{P(x+1,\theta)}{P(x,\theta)} = \frac{1}{\theta+1}\left[1+\frac{\theta}{\theta x+\theta^2+3\theta+1}\right]$$ is a decreasing function of x for a given θ, $P(x,\theta)$ is log-concave. This implies that PGD has an increasing hazard rate and is unimodal. Grandell,[24] has detailed discussion about relationship between log-concavity, IHR and Unimodality of discrete distributions.

Theorem 1: The $Q(x,\theta)$ is decreasing function of x for given θ.

Proof: We have

$$Q(x,\theta) = \frac{P(x+1,\theta)}{P(x,\theta)} = \frac{1}{\theta+1}\left[1+\frac{\theta}{\theta x+\theta^2+3\theta+1}\right]$$

Differentiating partially with respect to x, we get

$$Q'(x,\theta) = -\frac{\theta^2}{(\theta+1)\left(\theta x+\theta^2+3\theta+1\right)^2}.$$ Since $Q'(x,\theta)<0$,

$Q(x,\theta)$ is decreasing function of x for given θ.

Theorem 2: The pmf $P(x,\theta)$ of PGD is log-concave

Proof: We have

$$P(x,\theta) = \frac{\theta\left(\theta x+\theta^2+3\theta+1\right)}{(\theta+2)(\theta+1)^{x+2}} = \frac{\theta^2 x+\theta^3+3\theta^2+\theta}{(\theta+2)(\theta+1)^{x+2}}$$

$$\log P(x,\theta) = \log\left(\theta^2 x+\theta^3+3\theta^2+\theta\right) - \log(\theta+2) - (x+2)\log(\theta+1).$$

Assuming $g(x,\theta) = \log P(x,\theta)$, and differentiating partially with respect to x, we get

$$g'(x,\theta) = \frac{\theta^2}{\theta^2 x+\theta^3+3\theta^2+\theta} - \log(\theta+1)$$ and

$$g''(x,\theta) = -\frac{\theta^4}{\left(\theta^2 x+\theta^3+3\theta^2+\theta\right)^2} < 0$$

This means that the pmf of PGD is log-concave.

Theorem 3: The PGD is a two-component mixture of negative binomial distributions and can be expressed as

$$P(x;\theta) = p_1 P_1(x;\theta) + p_2 P_2(x;\theta),$$

where $P_i(x;\theta)$ is the pmf of the negative binomial distribution (NBD) with parameters, the number of successes i and $p_1 = \frac{\theta+1}{\theta+2}$ with $P_1(x;\theta) = \frac{\theta}{(\theta+1)^{x+1}}$ as $NBD\left(1,\frac{\theta}{\theta+1}\right)$, $p_2 = \frac{1}{\theta+2}$ with

$P_2(x;\theta) = \frac{(x+1)\theta^2}{(\theta+1)^{x+2}}$ as the $NBD\left(2,\frac{\theta}{\theta+1}\right)$ .respectively.

Proof: We have

$$P(x;\theta) = \int_0^\infty \frac{e^{-\lambda}\lambda^x}{x!}\frac{\theta}{\theta+2}(1+\theta+\theta\lambda)e^{-\theta\lambda}d\lambda$$

$$= \int_0^\infty \frac{e^{-\lambda}\lambda^x}{x!}\left\{\frac{\theta+1}{\theta+2}\left(\theta e^{-\theta\lambda}\right)+\frac{1}{\theta+2}\left(\frac{\theta^2}{\Gamma(2)}e^{-\theta\lambda}\lambda\right)\right\}d\lambda$$

$$= \frac{\theta+1}{\theta+2}\left[\frac{\theta}{x!}\int_0^\infty e^{-(\theta+1)\lambda}\lambda^{x+1-1}d\lambda\right] + \frac{1}{\theta+2}\left[\frac{\theta^2}{x!\Gamma(2)}\int_0^\infty e^{-(\theta+1)\lambda}\lambda^{x+2-1}d\lambda\right]$$

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$$=\frac{\theta+1}{\theta+2}\left[\frac{\theta}{x!}\frac{\Gamma(x+1)}{(\theta+1)^{x+1}}\right]+\frac{1}{\theta+2}\left[\frac{\theta^2}{x!\Gamma(2)}\frac{\Gamma(x+2)}{(\theta+1)^{x+2}}\right]$$

$$=\frac{\theta+1}{\theta+2}\left[\frac{\theta}{(\theta+1)^{x+1}}\right]+\frac{1}{\theta+2}\left[\frac{(x+1)\theta^2}{(\theta+1)^{x+2}}\right]$$

$$=\frac{\theta+1}{\theta+2}\left[\binom{x+1-1}{x}\left(\frac{\theta}{\theta+1}\right)^1\left(\frac{1}{\theta+1}\right)^x\right]+\frac{1}{\theta+2}\left[\binom{x+2-1}{x}\left(\frac{\theta}{\theta+1}\right)^2\left(\frac{1}{\theta+1}\right)^x\right]$$

$$=\frac{\theta+1}{\theta+2}\left[NBD\left(1,\frac{\theta}{\theta+1}\right)\right]+\frac{1}{\theta+2}\left[NBD\left(2,\frac{\theta}{\theta+1}\right)\right]$$

Now, even though the PGD is a two-component mixture of NBD's, the presence of two modes is not perceptible in any of the plots of pmf of PGD in Figure 1 for the selected values of θ. This suggests that the two modes coming from the two sub-populations are very close to each other. Tajuddin,[25] observed that if the modes of the sub-populations are very close to each other, then the population will be unimodal. This means that the distribution in which the existence of the modes of sub-populations are very close, the distribution would be suitable for over-dispersed count data.

Reliability properties of Poisson-Garima distribution

Various interesting and useful reliability properties including reverse hazard rate function, second rate of failure, cumulative hazard function and Mills ratio of a distribution depends on cumulative distribution function, survival function and hazard function of the distribution. The following theorem 4 deals with the cumulative distribution function (cdf), survival function and the hazard function of PGD. The expression for reverse hazard rate function, second rate of failure, cumulative hazard function and Mills ratio of PGD have also been obtained.

Theorem 4: The cumulative distribution function (cdf), survival function and the hazard function of PGD are given by

$$F(x)=F(x;\theta)=1-\frac{\theta x+\left(\theta^2+4\theta+2\right)}{(\theta+2)(\theta+1)^{x+2}}$$

$$S(x)=S(x;\theta)=\frac{\theta x+\left(\theta^2+4\theta+2\right)}{(\theta+2)(\theta+1)^{x+2}}, \text{ and}$$

$$h(x)=h(x;\theta)=\frac{\theta\left\{\theta x+\left(\theta^2+3\theta+1\right)\right\}}{\theta x+\left(\theta^2+4\theta+2\right)}$$

Proof: We have

$$F(x)=F(x,\theta)=P(X\le x)=1-P(X\ge x+1)$$

$$=1-\sum_{t=x+1}^{\infty}\int_0^{\infty}P(X=t|\lambda)f(\lambda;\theta)\,d\lambda$$

$$=1-\sum_{t=x+1}^{\infty}\int_0^{\infty}\frac{e^{-\lambda}\lambda^t}{t!}\frac{\theta}{\theta+2}(1+\theta+\theta\lambda)e^{-\theta\lambda}\,d\lambda$$

$$=1-\sum_{t=x+1}^{\infty}\frac{\theta\left[\theta t+\left(\theta^2+3\theta+1\right)\right]}{(\theta+2)(\theta+1)^{t+2}}$$

$$=1-\frac{\theta}{(\theta+2)(\theta+1)^2}\sum_{t=x+1}^{\infty}\frac{\theta t+\left(\theta^2+3\theta+1\right)}{(\theta+1)^t}$$

$$=1-\frac{\theta x+\left(\theta^2+4\theta+2\right)}{(\theta+2)(\theta+1)^{x+2}}.$$

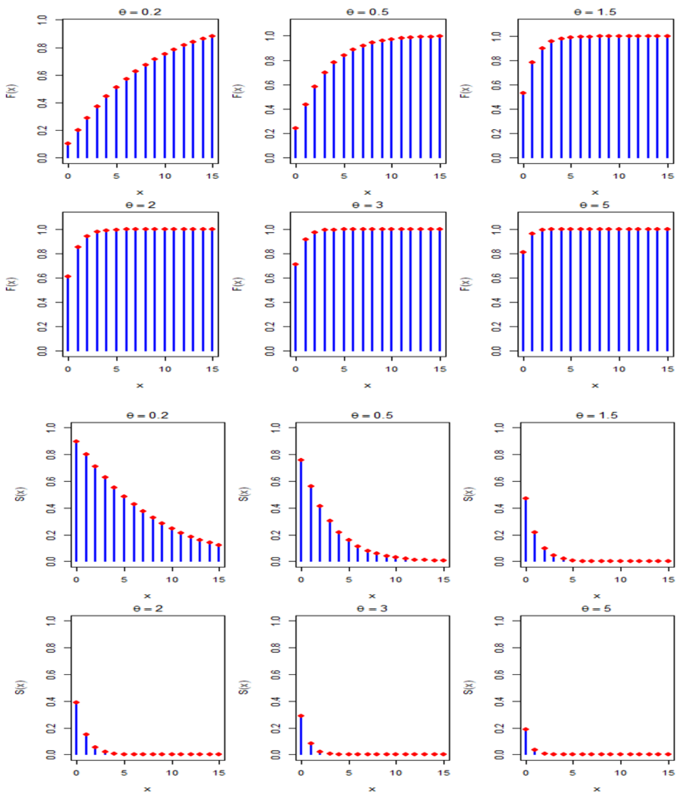
The survival function of PGD can thus be given by

$$S(x)=S(x,\theta)=1-F(x,\theta)=\frac{\theta x+\left(\theta^2+4\theta+2\right)}{(\theta+2)(\theta+1)^{x+2}}.$$

The hazard function of PGD can be expressed as

$$h(x)=h(x,\theta)=\frac{P(x,\theta)}{S(x,\theta)}=\frac{\theta\left\{\theta x+\left(\theta^2+3\theta+1\right)\right\}}{\theta x+\left(\theta^2+4\theta+2\right)}$$

The natures of cdf, survival function and hazard function of PGD for varying values of parameter are shown in the following Figure 2 and it is obvious from the figure that the PGD has a valid cdf since $F(x)\to 1$ as $x\to\infty$. Further, the hazard rate function shows an increasing pattern with a limiting value of θ, which means that $\lim_{x\to\infty} h(x)=\theta$.



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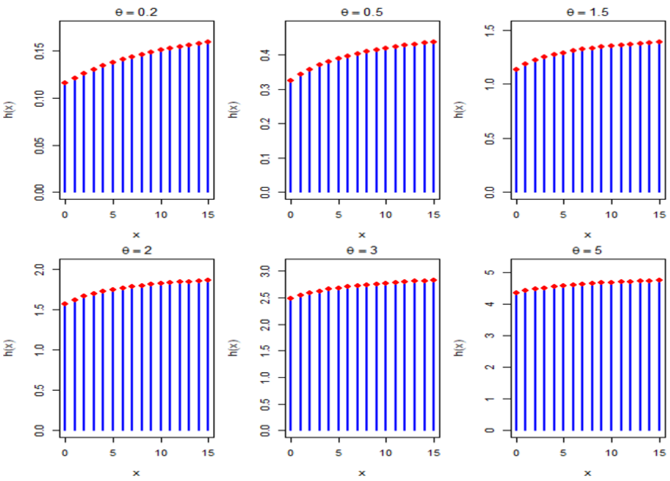

Figure 2 cdf, survival function and hazard function of PGD for varying values of parameter.

The reverse hazard rate function $Rh(x;\theta)$ and the second rate of failure $SRF(x;\theta)$ of the PGD can be obtained as

$$Rh(x;\theta)=\frac{P(x;\theta)}{F(x;\theta)}=\frac{\theta^2x+\theta^3+3\theta^2+\theta}{\left[(\theta+2)(\theta+1)^{x+2}-\left(\theta x+\theta^2+4\theta+2\right)\right]}$$

, and

$$SRF(x;\theta)=\ln\left[\frac{S(x;\theta)}{S(x+1;\theta)}\right]=\ln\left[\frac{(\theta+1)\left(\theta x+\theta^2+4\theta+2\right)}{\theta(x+1)+\theta^2+4\theta+2}\right]$$

The cumulative hazard function $H(x;\theta)$ and Mills ratio $M(x;\theta)$ of PGD are given by

$$H(x;\theta)=-\ln S(x;\theta)=-\ln\left[\frac{\theta x+\left(\theta^2+4\theta+2\right)}{(\theta+2)(\theta+1)^{x+2}}\right],$$

and $M(x;\theta)=\dfrac{S(x;\theta)}{P(x;\theta)}=\dfrac{\theta x+\theta^2+4\theta+2}{\theta^2x+\theta^3+3\theta^2+\theta}$.

Maximum likelihood estimation of parameter

Let $x_1, x_2, ..., x_n$ be a random sample of size n from the PGD and let f_x be the observed frequency in the sample corresponding to $X = x\ (x = 1, 2, 3, ..., k)$ such that $\sum_{x=1}^{k} f_x = n$, where k is the largest observed value having non-zero frequency.

The log likelihood function of PGD is obtained as

$$\log L=n\log\left(\frac{\theta}{\theta+2}\right)-\sum_{x=1}^{k}f_x(x+2)\log(\theta+1)+\sum_{x=1}^{k}f_x\log\left[\theta x+\left(\theta^2+3\theta+1\right)\right]$$

The first derivative of the log likelihood function is given by

$$\frac{\partial\log L}{\partial\theta}=\frac{n}{\theta}-\frac{n}{\theta+2}-\frac{n(\bar{x}+2)}{\theta+1}+\sum_{x=1}^{k}\frac{(x+2\theta+3)f_x}{\theta x+\left(\theta^2+3\theta+1\right)}$$

where $\bar{x}$ is the sample mean.

The maximum likelihood estimate (MLE), $\hat{\theta}$ of θ is the solution of the equation $\frac{\partial\log L}{\partial\theta}=0$ and is given by the solution of the non-linear equation

$$\frac{2n}{\theta(\theta+2)}-\frac{n(\bar{x}+2)}{\theta+1}+\sum_{x=1}^{k}\frac{(x+2\theta+3)f_x}{\theta x+\left(\theta^2+3\theta+1\right)}=0$$

This non-linear equation can be solved by any numerical iteration methods such as Newton- Raphson, Bisection method or Regula – Falsi method etc. In the following theorem 5, the consistency and asymptotic normality of the PGD has been established.

Theorem 5: The ML estimator $\hat{\theta}$ of θ of the PGD is consistent and asymptotically normal. That is $\sqrt{n}\left(\hat{\theta}-\theta\right)\xrightarrow{d}N\left[0,\ I^{-1}(\theta)\right]$, where

$$I(\theta)=\frac{-2\left(2\theta^3+3\theta^2-2\theta-2\right)}{\theta^2(\theta+2)^2(\theta+1)}+\frac{\theta-1}{\theta+2}\int_0^1\frac{t^{\theta^2+3\theta}}{\theta+1-t^{\theta}}dt$$ is the Fisher's

information about θ.

Proof: The PGD satisfies the regularity conditions under which the ML estimator $\hat{\theta}$ of θ is consistent and asymptotically normal. We have

$$I(\theta)=E\left[-\frac{\partial^2}{\partial\theta^2}\ln f(X;\theta)\right]$$

$$=\frac{1}{\theta^2}-\frac{1}{(\theta+2)^2}-\frac{\mu+2}{(\theta+1)^2}-2E\left[\frac{1}{\theta X+\left(\theta^2+3\theta+1\right)}\right]+\frac{1}{\theta^2}E\left[\frac{1}{\theta X+\left(\theta^2+3\theta+1\right)}\right]$$

$$+\frac{\theta^2-1}{\theta^2}E\left[\frac{1}{\left\{\theta X+\left(\theta^2+3\theta+1\right)\right\}^2}\right]$$

where $\mu=E(X)=\dfrac{\theta+3}{\theta(\theta+2)}$.

Now , we have

$$E\left[\frac{1}{\theta X+\left(\theta^2+3\theta+1\right)}\right]=\sum_{x=0}^{\infty}\frac{1}{\theta x+\left(\theta^2+3\theta+1\right)}\cdot\frac{\theta}{\theta+2}\cdot\frac{\theta x+\left(\theta^2+3\theta+1\right)}{(\theta+1)^{x+2}}$$

$$=\frac{\theta}{(\theta+2)(\theta+1)^2}\sum_{x=0}^{\infty}\left(\frac{1}{\theta+1}\right)^x=\frac{1}{(\theta+1)(\theta+2)}$$

Again, we have

$$E\left[\frac{1}{\left\{\theta X+\left(\theta^2+3\theta+1\right)\right\}^2}\right]=\sum_{x=0}^{\infty}\frac{1}{\left\{\theta x+\left(\theta^2+3\theta+1\right)\right\}^2}\cdot\frac{\theta}{\theta+2}\cdot\frac{\theta x+\left(\theta^2+3\theta+1\right)}{(\theta+1)^{x+2}}$$

$$=\frac{\theta}{(\theta+2)(\theta+1)^2}\sum_{x=0}^{\infty}\frac{1}{\left\{\theta x+\left(\theta^2+3\theta+1\right)\right\}(\theta+1)^x}$$

$$=\frac{\theta}{(\theta+1)(\theta+2)}\int_0^1\frac{t^{\theta^2+3\theta}}{\theta+1-t^{\theta}}dt\ .$$

Using these values in the expressions for $I\left(\theta\right)$ we get after a little algebraic simplification

$$I\left(\theta\right)=\frac{-4\theta^{2}-3\theta+6}{\theta^{2}\left(\theta+2\right)^{2}}+\frac{\theta-1}{\theta\left(\theta+2\right)}\int_{0}^{1}\frac{t^{\theta^{2}+3\theta}}{\theta+1-t^{\theta}}dt.$$

Simulation study

To better understand how well the maximum likelihood estimator (MLE) works for the PGD, we carried out a detailed simulation study. We used inverse transform sampling to generate random samples from the distribution, taking advantage of a closed-form expression for the cumulative distribution function (CDF). For each sample size we tested 20, 50,100, 200, 300,400 and 500, and we repeated the process 10,000 times to get a reliable picture of the estimator's behavior. We then calculated the bias and mean squared error (MSE) of these estimates to evaluate accuracy and consistency. The simulation results for different parameter values are present in Table 1 & 2. As expected, both bias and MSE decreases as the sample size an increase which shows that the MLE becomes more reliable with more data. The formula for bias and MSE are as follows:

$$bias(\hat{\theta})=E\left[\hat{\theta}\right]-\theta=\frac{1}{N}\sum_{i=1}^{N}\hat{\theta}_{i}-\theta$$

$$MSE\left(\hat{\theta}\right)=E\left[\left(\hat{\theta}-\theta\right)^{2}\right]=\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\theta}_{i}-\theta\right)^{2}$$

Table 1 Bias and MSE for $\theta=1.5$

n	$\theta=1.5$	
	Bias	**MSE**
50	0.05537	0.96833
100	0.03093	0.04345
200	0.01361	0.02030
300	0.00805	0.01352
400	0.00800	0.01004
500	0.00580	0.00832

Table 2 Bias and MSE for $\theta=2$

n	$\theta=2$	
	Bias	**MSE**
50	0.09035	0.22266
100	0.05146	0.09642
200	0.02183	0.04393
300	0.01253	0.02916
400	0.01305	0.02181
500	0.00928	0.01804

Applications

In this section four examples of observed count datasets, for which the Poisson distribution (PD), Poisson-Lindley distribution (PLD), Poisson-Komal distribution (PKD), Poisson-Shanker distribution (PShD) and Poisson-Garima distribution (PGD) has been fitted, are presented. The first and second dataset in table 3 and 4 are related to number of days that experienced the number of thunderstorms events at Cape Kennedy, Florida for the month of June and August ,11-year period of record January 1957 to December 1967 available in Falls.[26] The third dataset in table 5 is related to the number of European red mites on apple leaves available in Bliss,[27] and the fourth dataset in table 6 is related to the number of households according to the number of male migrants used 15 years and above, available in Shukla and Yadav.[28] Based on the values of chi-square, PLD provides best fit in Table 3, PGD provides best fit in Table 4 and PKD provides the best fit in Table 5 & 6 among all considered over-dispersed count distributions.

Table 3 Number of thunderstorm events in the month of June

Number of observations	Observed frequency	Expected frequency				
		PD	**PLD**	**PKD**	**PShD**	**PGD**
0	187	155.6	185.3	187.2	186.4	186.5
1	77	117.0	83.5	81.7	82.4	82.4
2	40	44.0	36.0	35.2	35.5	35.5
3	17	11.0	15.0	15.0	15.0	15.0
4	6	2.1	6.1	6.3	6.3	6.3
5	2	0.3	2.5	2.7	2.6	2.6
6	1	0.0	1.6	1.8	1.7	1.7
Total	330	330	330	330	330	330
$\hat{\theta}(SE)$		0.7515 (0.0477)	1.8042 (0.1257)	1.5886 (0.1088)	1.6766 (0.1057)	1.6906 (0.1268)
$-2\log L$		824.506	788.8836	789.2348	789.0612	789.0402
χ^{2}		32.493	1.3739	1.4913	1.4339	1.4338
d.f		2	3	3	3	3
p-value		0.0000	0.8487	0.8282	0.8383	0.8383

Table 4 Number of thunderstorm events in the month of August

Number of observations	Observed frequency	Expected frequency				
		PD	PLD	PKD	PShD	PGD
0	185	151.8	184.8	187.0	186.0	186.2
1	89	122.9	87.2	85.3	86.1	86.0
2	30	49.7	39.3	38.4	38.8	38.7
3	24	13.4	17.1	17.0	17.1	17.1
4	10	2.7	7.3	7.5	7.4	7.4
5	3	0.5	5.3	5.8	5.6	5.6
Total	341	341	341	341	341	341
$\hat{\theta}(SE)$		0.8094 (0.0487)	1.6934 (0.1132)	1.4944 (0.0977)	1.5844 (0.0951)	1.5800 (0.1141)
$-2\log L$		888.6172	847.6051	847.9943	847.779	847.7928
χ^2		49.49	7.0191	7.0868	7.0038	6.9731
d.f		2	4	4	4	4
p-value		0.0000	0.2192	0.2143	0.2204	0.2227

Table 5 Number of European red mites[27]

Number of European red mites	Observed frequency	Expected frequency				
		PD	PLD	PKD	PShD	PGD
0	70	47.7	67.3	68.6	67.7	68.4
1	38	54.6	38.9	38.0	38.6	38.1
2	17	31.3	21.2	20.6	21.0	20.7
3	10	12.0	11.2	11.0	11.1	11.0
4	9	3.4	5.7	5.8	5.7	5.8
5	3	0.8	2.9	3.0	2.9	3.0
6	2	0.2	1.4	1.5	1.5	1.5
7	1	0.0	0.7	0.8	0.7	0.8
8	0	0.0	0.7	0.7	0.8	0.7
Total	150	150	150	150	150	150
$\hat{\theta}(SE)$		1.1467 (0.0874)	1.2601 (0.1139)	1.1289 (0.0983)	1.2247 (0.0970)	1.1491 (0.1142)
$-2\log L$		485.6199	445.0218	444.7778	444.9169	444.7943
χ^2		23.295	3.0161	2.5141	2.8706	2.5555
d.f		3	4	4	4	4
p-value		0.0001	0.6975	0.7744	0.7199	0.7681

Table 6 Number of male migrants per households

Number of male migrants	Observed frequency	Expected frequency				
		PD	PLD	PKD	PShD	PGD
0	242	209.0	240.2	242.1	241.5	241.1
1	97	136.7	98.8	96.9	97.4	97.8
2	35	44.7	39.1	38.4	38.6	38.7
3	19	9.8	15.0	15.1	15.1	15.1
4	6	1.6	5.7	5.9	5.8	5.8
5	3	0.2	2.1	2.3	2.2	2.2
6	0	0.0	0.8	0.9	0.8	0.8
7	0	0.0	0.3	0.3	0.3	0.3
8	0	0.0	0.0	0.1	0.3	0.2
Total	402	402	402	402	402	402
$\hat{\theta}(SE)$		0.6542 (0.0403)	2.0329 (0.1348)	1.7837 (0.1172)	1.8679 (0.1138)	1.9184 (0.1360)
$-2\log L$		932.6495	892.3796	892.458	892.4004	892.3785
χ^2		42.031	1.544	1.3348	1.3627	1.3806
d.f		2	3	3	3	3
p-value		0.0000	0.8188	0.8554	0.8506	0.8476

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Conclusion

In this paper some interesting and useful statistical and reliability properties of the Poisson-Garima distribution (PGD) of Shanker,[23] the Poisson compound of Garima distribution, has been derived and discussed. The proposed distribution has been observed to be log-concave and is a two-component mixture of negative binomial distributions. The hazard function of the distribution is increasing and thus the distribution is unimodal. The consistency of maximum likelihood estimator of the parameter of the PGD has been shown using simulation study. It has been shown that the maximum likelihood estimator of the parameter of the distribution is consistent and asymptotically normal. The applications and the goodness of fit of the PGD has been established with four real datasets and found to compete well with other one parameter over-dispersed count distributions.

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Conflict of interest

The authors declare that there are no conflict of interest.

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