MedCrave Step into the World of Research

Biometrics & Biostatistics International Journal

Research Article

Open Access

On some statistical properties and applications of Uma distribution

Abstract

In this paper some of the important statistical properties such as Bonferroni and Lorenz curves with their indices, stress-strength reliability, Renyi entropy and order statistics of Uma distribution have been obtained. The applications and goodness of fit of the distribution have been discussed with four real lifetime data sets and the fit has been compared with one-parameter lifetime distributions including Sujatha, Akash, Shanker, Lindley and exponential distributions. The goodness of fit of Uma distribution reveals that it is the best distribution among the considered distributions.

Keywords: Uma distribution, statistical properties, parameter estimation, simulation study, goodness of fit

Volume 14 Issue 1 - 2025

Jyotirmoyee Baishya, Rama Shanker
Department of Statistics, Assam University, India

Correspondence: Rama Shanker, Department of Statistics, Assam University, Silchar, Assam, India

Received: December 26, 2024 | **Published:** March 6, 2025

Introduction

In all applied sciences, including engineering, medicine, finance, and insurance, lifetime data analysis and modeling are essential. While several lifetime distributions have been developed in statistical literature, each has pros and cons of its own. The first one parameter lifetime distribution is exponential distribution(ED) and it has a constant hazard rate but in real life situations we have to deal with the situations where we get a decreasing, increasing or sometimes irregular hazard rate. In the year 1958, Lindley[1] has developed a one parameter distribution known as Lindley distribution (LD) to overcome the difficulty of constant hazard rate. Later, Shanker et al.[2] have detailed comparative study on applications of LD and ED regarding goodness of fit on several datasets from different fields of knowledge and observed that there are several datasets where these two distributions do not provide good fit because of the level of over-dispersion observed in the datasets. Shanker[3,4] proposed other two lifetime distributions named Shanker distribution(ShD) and Akash distribution(AD) for modelling data from biological sciences and engineering and shows that these two lifetime distributions provide much better fit than ED and LD. Shanker et al.[5,6] have detailed comparative study on the goodness of fit of ED, LD, ShD, and AD for data relating to biological sciences and engineering and observed that there are some datasets where these lifetime distributions failed to provide good fit due to theoretical or applied point of view. To overcome this difficulty, Shanker[7] suggested Sujatha distribution (SD) and discussed its several statistical properties and detailed applications for modelling lifetime data from engineering and biological sciences. During testing goodness of fit , it was also observed that although SD provide better fit over ED, LD, ShD, AD but it failed to provide better fit over some datasets having less over-dispersed or less under-dispersed and to overcome this Shanker[8] proposed Uma distribution(UD) which provides much better fit than SD and other lifetime distributions. UD is defined by its probability density function (pdf) and cumulative distribution function (cdf)

$$f(x;\theta)=\frac{\theta^4}{\theta^3+\theta^2+6}\left(1+x+x^3\right)e^{-\theta x};x>0,\theta>0 \quad (1)$$

$$F(x;\theta)=1-\left[1+\frac{\theta x\left(\theta^2x^2+3\theta x+\theta^2+6\right)}{\theta^3+\theta^2+6}\right]e^{-\theta x};x>0,\theta>0 \quad (2)$$

Like other lifetime distributions including LD, ShD, AD and SD, UD is also a convex combination of an exponential (θ) distribution , gamma $(2,\theta)$ distribution, gamma $(4,\theta)$ distribution, with mixing proportions gamma $\frac{\theta^3}{\theta^3+\theta^2+6}$, $\frac{\theta^2}{\theta^3+\theta^2+6}$ and $\frac{6}{\theta^3+\theta^2+6}$ respectively.

UD, being one parameter lifetime distribution, is not very suitable for several datasets because it has only scale parameter and to overcome this difficulty Shanker et al.[9,10] introduced power Uma distribution (PUD) and weighted Uma distribution (WUD) by power transformation and weighted transformation in the UD. Recently, Ganaie et al.[11] derived length biased Uma distribution (LBUD) and studied some of its properties and applications. It should be worth to note that LBUD is nothing but the particular case of WUD and all results of LBUD are the particular cases of WUD.

It appears that there are some important statistical properties of UD including Bonferroni and Lorenz curves and their indices, stress-strength reliability, distribution of order statistics and Renyi entropy measure have not been studied. The main purpose of this paper is to discuss these statistical properties and present the simulation study of the maximum likelihood estimates. Finally the applications of the UD have been explained with four real lifetime datasets from different fields of knowledge. The goodness of fit of UD shows that UD is the best distribution among the considered distributions and can be considered an important lifetime distribution for statisticians working in applied fields of knowledge.

Some statistical properties of Uma distribution

In this section, an attempt has been made to discuss Bonferroni and Lorenz curves and their indices, stress-strength reliability, distribution of order statistics, and Renyi entropy measure of UD.

Bonferroni and lorenz curves and indices

Although Bonferroni and Lorenz curves by Bonferroni[12] and Bonferroni & Gini indices have been proposed in economics to study income and poverty, but it has several applications in other fields of knowledge including reliability, demography, insurance and medicine. The Bonferroni and Lorenz curves are defined as

$$B(p)=\frac{1}{p\mu}\int_0^q x\,f(x)dx=\frac{1}{p\mu}\left[\int_0^\infty x\,f(x)dx-\int_q^\infty x\,f(x)dx\right]=\frac{1}{p\mu}\left[\mu-\int_q^\infty x\,f(x)dx\right] \quad (3)$$

$$L(p)=\frac{1}{\mu}\int_0^q x\,f(x)dx=\frac{1}{\mu}\left[\int_0^\infty x\,f(x)dx-\int_q^\infty x\,f(x)dx\right]=\frac{1}{\mu}\left[\mu-\int_q^\infty x\,f(x)dx\right] \quad (4)$$

The Bonferroni and Gini indices are thus defined as $B=1-\int_0^1 B(p)dp$ and $G=1-2\int_0^1 L(p)dp$ respectively.

Using pdf of UD, we get

$$\int_q^\infty x\,f(x)dx=\frac{\left\{\theta^4\left(q+q^2+q^4\right)+\theta^3\left(1+2q+4q^3\right)+2\theta^2\left(1+6q^2\right)+24\left(1+\theta q\right)\right\}e^{-\theta q}}{\theta\left(\theta^3+\theta^2+6\right)} \quad (5)$$

After some simple algebraic simplifications, we obtain the expressions for Bonferroni and Lorenz curves of UD as

$$B(p)=\frac{1}{p}\left[1-\frac{\left\{\theta^4\left(q+q^2+q^4\right)+\theta^3\left(1+2q+4q^3\right)+2\theta^2\left(1+6q^2\right)+24\left(1+\theta q\right)\right\}e^{-\theta q}}{\left(\theta^3+2\theta^2+24\right)}\right] \quad (6)$$

$$L(p)=1-\frac{\left\{\theta^4\left(q+q^2+q^4\right)+\theta^3\left(1+2q+4q^3\right)+2\theta^2\left(1+6q^2\right)+24\left(1+\theta q\right)\right\}e^{-\theta q}}{\left(\theta^3+2\theta^2+24\right)} \quad (7)$$

Again, the expressions for Bonferroni and Lorenz indices of UD are thus obtained as

$$B=1-\frac{\left\{\theta^4\left(q+q^2+q^4\right)+\theta^3\left(1+2q+4q^3\right)+2\theta^2\left(1+6q^2\right)+24\left(1+\theta q\right)\right\}e^{-\theta q}}{\left(\theta^3+2\theta^2+24\right)} \quad (8)$$

$$G=-1+\frac{2\left\{\theta^4\left(q+q^2+q^4\right)+\theta^3\left(1+2q+4q^3\right)+2\theta^2\left(1+6q^2\right)+24\left(1+\theta q\right)\right\}e^{-\theta q}}{\left(\theta^3+2\theta^2+24\right)} \quad (9)$$

Stress-strength reliability

Let X and Y be independent strength and stress random variables following UD with parameter θ_1 and θ_2 respectively. Then the stress-strength reliability R of UD can be obtained

$$R=P\left(Y<X\right)=\int_0^\infty P\left(Y<X\mid X=x\right)f_X\left(x\right)dx \quad (10)$$

$$=\int_0^\infty f\left(x;\theta_1\right)F\left(x;\theta_2\right)dx \quad (11)$$

$$=1-\frac{\theta_1^4\left[\begin{array}{l}720\theta_2^3+360\theta_2^2\left(\theta_1+\theta_2\right)+48\left(\theta_2^3+\theta_2\right)\left(\theta_1+\theta_2\right)^2+12\left(\theta_2^3+2\theta_2^2+3\right)\left(\theta_1+\theta_2\right)^3+\\ 2\left(3\theta_2^2+\theta_2^3+6\theta_2\right)\left(\theta_1+\theta_2\right)^4+\left(2\theta_2^3+\theta_2^2+6\theta_2+6\right)\left(\theta_1+\theta_2\right)^5+\left(\theta_2^3+\theta_2^2+6\right)\left(\theta_1+\theta_2\right)\end{array}\right]}{\left(\theta_1^3+\theta_1^2+6\right)\left(\theta_2^3+\theta_2^2+6\right)\left(\theta_1+\theta_2\right)^7} \quad (12)$$

Renyi entropy

Entropy of a random variable X is a measure of variation of uncertainty. A popular entropy measure is Renyi entropy, proposed by Renyi[13]. If X is a continuous random variable having pdf $f(.)$, then Renyi entropy is defined as

$$T_R\left(\beta\right)=\frac{1}{1-\beta}\log\left\{\int f^\beta\left(x\right)dx\right\},\text{ where }\beta>0\text{ and }\beta\neq 0 \quad (13)$$

Thus, the Renyi entropy for the UD is obtained as

$$T_R\left(\beta\right)=\frac{1}{1-\beta}\log\left\{\int_0^\infty \frac{\theta^{4\beta}}{\left(\theta^3+\theta^2+6\right)^\beta}\left(1+x+x^3\right)^\beta e^{-\theta\beta x}\right\}dx \quad (14)$$

Order statistics

Let $\left(X_1,X_2,\ldots,X_n\right)$ be a random sample of size n from UD. Let $X_{(1)}<X_{(2)}<\ldots<X_{(n)}$ denote the corresponding order statistics. The pdf and the cdf of the k^{th}, $\left(k=1,2,3,\ldots,n\right)$ order statistic, say $Y=X_{(k)}$ are given by

$$f_Y(y)=\frac{n!}{(k-1)!(n-k)!}F^{k-1}\left(y\right)\left\{1-F\left(y\right)\right\}^{n-k}f\left(y\right) \quad (15)$$

$$=\frac{n!}{(k-1)!(n-k)!}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l F^{k+l-1}\left(y\right)f\left(y\right) \quad (16)$$

$$F_Y(y)=\sum_{j=k}^{n}\binom{n}{j}F^j\left(y\right)\left\{1-F\left(y\right)\right\}^{n-j} \quad (17)$$

$$=\sum_{j=k}^{n}\sum_{l=0}^{n-j}\binom{n}{j}\binom{n-j}{l}(-1)^l F^{j+l}\left(y\right), \quad (18)$$

Thus, the pdf and the cdf of the k^{th} order statistics of UD are given by

$$f_Y\left(y\right)=\frac{n!\theta^4\left(1+x+x^3\right)e^{-\theta x}}{\left(\theta^3+\theta^2+6\right)(k-1)!(n-k)!}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\left[1-\left\{\frac{\theta^3+\theta^2+6+\theta x\left(\theta^2x^2+3\theta x+\theta^2+6\right)}{\left(\theta^3+\theta^2+6\right)}\right\}e^{-\theta x}\right]^{k+l-1} \quad (19)$$

$$F_Y(y)=\sum_{j=k}^{n}\sum_{l=0}^{n-j}\binom{n}{j}\binom{n-j}{l}(-1)^l\left[1-\left\{\frac{\theta^3+\theta^2+6+\theta x\left(\theta^2x^2+3\theta x+\theta^2+6\right)}{\left(\theta^3+\theta^2+6\right)}\right\}e^{-\theta x}\right]^{j+l} \quad (20)$$

Parameter estimation using maximum likelihood estimation

Suppose $\left(x_1,x_2,x_3,\ldots,x_n\right)$ be a random sample of size n from UD. The log-likelihood function, L of UD is given by

$$\log L=\sum_{i=1}^{n}\log f\left(x_i;\theta\right)=n\left\{4\log\theta-\log\left(\theta^3+\theta^2+6\right)\right\}+\sum_{i=1}^{n}\log\left(1+x_i+x_i^3\right)-\theta\sum_{i=1}^{n}x_i \quad (21)$$

The maximum likelihood estimate (MLE) $\left(\hat{\theta}\right)$ of $\left(\theta\right)$ of Uma distribution is the solution of the following log likelihood equations

$$\frac{\partial \ln L}{\partial\theta}=\frac{4n}{\theta}-\frac{\left(3\theta^2+2\theta\right)n}{\left(\theta^3+\theta^2+6\right)}-\sum_{i=1}^{n}x_i=0 \quad (22)$$

This gives

$$\bar{x}\theta^4+\left(\bar{x}-1\right)\theta^3-2\theta^2+6\bar{x}\theta-24=0 \quad (23)$$

This is a fourth-degree polynomial equation in θ. It should be noted that the method of moment estimate is also same as that of the MLE. The above equation can easily be solved using Newton-Raphson method, taking the initial value of the parameter $\theta = 0.5$.

The simulation study

To assess the effectiveness of maximum likelihood estimators for UD, a simulation study has been conducted. The investigation involved examining mean estimates, biases (B), mean square errors (MSEs), and variances of the maximum likelihood estimates (MLEs) for UD, using the specified formulas

$$\text{Mean}=\frac{1}{n}\sum_{i=1}^{n}\hat{H}_i,\ \text{B}=\frac{1}{n}\sum_{i=1}^{n}\left(\hat{H}_i-H\right),\ \text{MSE}=\frac{1}{n}\sum_{i=1}^{n}\left(\hat{H}_i-H\right)^2,$$

$$\text{Variance}=\text{MSE-B}^2 \quad (24)$$

Where $H=\left(\theta\right)$ and $\hat{H}_i=\left(\hat{\theta}\right)$

Simulation results for various parameter values of UD are outlined in Table 1 and Table 2. The simulation study consists of following steps:

1) The quantile function has been used to generate the data.

The sample sizes taken are $n = 25, 50,100,200,300$.

The parameter values are set as $\theta=0.6$ and $\theta=0.2$

Each sample size has been replicated 10000 times.

The biases, MSE's and variances of the MLEs of the parameter are decreasing for increasing sample size as evident in Table 1 and Table 2. This supports the first-order asymptotic theory of MLEs. It should be noted that there is no close form for quantile function for Uma distribution and hence approximate quantile function using R software has been used to generate data.

Table 1 The mean values, Biases, MSE's and Variances of Uma distribution for $\theta=0.6$

Parameter	Sample size	Mean	Bias	MSE	Variance
	25	0.60067	0.00067	0.0000088	0.0000084
	50	0.60063	0.00063	0.0000076	0.0000073
$\hat{\theta}$	100	0.60057	0.00058	0.0000075	0.0000072
	200	0.60044	0.00044	0.0000073	0.0000071
	300	0.60033	0.00033	0.0000071	0.000007

Table 2 The mean values, Biases, MSE's and Variances of Uma distribution for $\theta=0.2$

Parameter	Sample size	Mean	Bias	MSE	Variance
	25	0.20106	0.001061	0.00000153	0.00000411
	50	0.20028	0.000277	0.00000132	0.00000124
$\hat{\theta}$	100	0.20013	0.000127	1.248E-06	0.00000123
	200	0.20004	0.000041	9.962E-06	0.00000099
	300	0.20001	0.000015	9.629E-06	0.00000096

Data analysis

To examine the applications of UD, the goodness of fit of UD has been tested for four real lifetime datasets from four different fields. The goodness of fit of UD is compared with the goodness of fit given by ED, LD, ShD, AD, and SD. The goodness of fit of all distributions is based on their maximum likelihood estimate. The pdf and the cdf of ED, LD, ShD, AD, and SD are presented in Table 3 along with their introducer and year

Table 3 Descriptive data summary

Datasets	Minimum value	1st quartile	Median	Mean	3rd quartile	Maximum	Variance
1	19.89	30.34	40.4	51.5	61.34	185.56	1048.259
2	17.88	47.2	67.8	73.85	101.88	173.4	1452.811
3	1.6	5.075	6.5	6.253	7.825	9	3.824096
4	4.1	8.45	10.6	13.49	16.85	39.2	64.82658

Citation: Baishya J, Shanker R. On some statistical properties and applications of Uma distribution. *Biom Biostat Int J.* 2025;14(1):21‒27.
DOI: 10.15406/bbij.2025.14.00430

The datasets are as follows:

1) This following right-skewed data set discussed by Van Montfort,[14] presents the maximum annual flood discharges of the North Saskatchewan in units' of 1000 cubic feet per second, of the north Saskatchewan River at Edmonton, over a period of 47 years.

19.885, 20.940, 21.820, 23.700, 24.888, 25.460, 25.760, 26.720, 27.500, 28.100, 28.600, 30.200, 30.380, 31.500, 32.600, 32.680, 34.400, 35.347, 35.700, 38.100, 39.020, 39.200, 40.000, 40.400, 40.400, 42.250, 44.020, 44.730, 44.900, 46.300, 50.330, 51.442, 57.220, 58.700, 58.800, 61.200, 61.740, 65.440, 65.597, 66.000, 74.100, 75.800, 84.100, 106.600, 109.700, 121.970, 121.970, 185.560

2) The following skewed to right data developed by Lawless,[15] is the number of million revolutions before failure for each of the 23 ball bearings in the life tests.

17.88, 28.92, 33.0, 41.52, 42.12, 45.6, 48.8, 51.84, 51.96, 54.12, 55.56, 67.8, 68.44, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 105.84, 127.92, 128.04, 173.4

3) The following skewed to left life time data developed by Maguire & Pearson,[16] represents the time to failure (103h) of turbocharger of one type of engine.

1.6, 2.0, 2.6, 3.0, 3.5, 3.9, 4.5, 4.6, 4.8, 5.0, 5.1, 5.3, 5.4, 5.6, 5.8, 6.0, 6.0, 6.1, 6.3, 6.5, 6.5, 6.7, 7.0, 7.1, 7.3, 7.3, 7.3, 7.7, 7.7, 7.8, 7.9, 8.0, 8.1, 8.3, 8.4, 8.4, 8.5, 8.7, 8.8, 9.0

4) The following skewed to right a set of data, discussed by Nassar & Nada,[17] represents the monthly actual taxes revenue (in 1000 million Egyptian pounds) in Egypt between January 2006 and November 2010.

5.9, 20.4, 14.9, 16.2, 17.2, 7.8, 6.1, 9.2, 10.2, 9.6, 13.3, 8.5, 21.6, 18.5, 5.1, 6.7, 17, 8.6, 9.7, 39.2, 35.7, 15.7, 9.7, 10, 4.1, 36, 8.5, 8, 9.2, 26.2, 21.9, 16.7, 21.3, 35.4, 14.3, 8.5, 10.6, 19.1, 20.5, 7.1, 7.7, 18.1, 16.5, 11.9, 7, 8.6, 12.5, 10.3, 11.2, 6.1, 8.4, 11, 11.6, 11.9, 5.2, 6.8, 8.9, 7.1, 10.8

The descriptive summary of datasets 1, 2, 3 and 4 are presented in Table 3.

In order to compare lifetime distributions, values of $-2\log L$, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Kolmogorov-Smirnov Statistics (K-S) and the corresponding probability value (p-value) for the four datasets has been computed and presented in the Tables 5–8. The confidence intervals for the parameter of UD for four datasets are given in Table 8. The formula for computing AIC, BIC and K-S are as follows

Table 4 pdf and cdf of the considered distributions

Distributions	Pdf/cdf	Author (Year)
SD	$f(x;\theta)=\frac{\theta^3}{\theta^2+\theta+2}\left(1+x+x^2\right)e^{-\theta x};x>0,\theta>0$ $F(x;\theta)=1-\left[1+\frac{\theta x(\theta x+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x};x>0,\theta>0$	Shanker[7]
AD	$f(x;\theta)=\frac{\theta^3}{\theta^2+2}\left(1+x^2\right)e^{-\theta x};x>0,\theta>0$ $F(x;\theta)=1-\left[1+\frac{\theta x(\theta x+2)}{\theta^2+2}\right]e^{-\theta x};x>0,\theta>0$	Shanker[4]
ShD	$f(x;\theta)=\frac{\theta^2}{\theta^2+1}(\theta+x)e^{-\theta x};x>0,\theta>0$ $F(x;\theta)=1-\left[1+\frac{\theta x}{\theta^2+1}\right]e^{-\theta x};x>0,\theta>0$	Shanker[3]
LD	$f(x;\theta)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x};x>0,\theta>0$ $F(x;\theta)=1-\left[1+\frac{\theta x}{\theta+1}\right]e^{-\theta x};x>0,\theta>0$	Lindley[1]
ED	$f(x;\theta)=\theta e^{-\theta x};x>0,\theta>0$ $F(x;\theta)=1-e^{-\theta x};x>0,\theta>0$	

Table 5 Goodness of fit of dataset 1

Distributions	ML estimate $\hat{\theta}$ (Standard error)	$-2\log L$	AIC	BIC	K-S	p-value
UD	0.0776(0.0061)	443.28	445.28	447.15	0.1	0.43
SD	0.0572(0.0052)	447.08	449.08	450.95	0.18	0.12
AD	0.0582(0.0048)	444.08	446.08	447.95	0.13	0.44
ShD	0.0388(0.0041)	451.08	453.08	454.95	0.16	0.19
LD	0.0381(0.0039)	452.28	454.28	456.15	0.19	0.07
ED	0.0194(0.0028)	474.38	476.38	478.25	0.33	0.00

Citation: Baishya J, Shanker R. On some statistical properties and applications of Uma distribution. *Biom Biostat Int J.* 2025;14(1):21–27. DOI: 10.15406/bbij.2025.14.00430

Table 6 Goodness of fit of dataset 2

Distributions	ML estimate $\hat{\theta}$ (Standard error)	$-2\log L$	AIC	BIC	K-S	p-value
UD	0.0541(0.0056)	227.53	229.53	230.67	0.11	0.93
SD	0.0402(0.0048)	229.39	331.39	232.52	0.21	0.25
AD	0.0406 (0.0049)	228.38	300.38	231.52	0.14	0.9
ShD	0.0271 (0.0039)	232.23	234.23	235.37	0.17	0.58
LD	0.0267 (0.0039)	232.63	234.63	235.76	0.2	0.32
ED	0.0135 (0.0028)	243.89	245.89	247.03	0.29	0.05

Table 7 Goodness of fit of dataset 3

Distributions	ML estimate $\hat{\theta}$ (Standard error)	$-2\log L$	AIC	BIC	K-S	p-value
UD	0.6057 (0.0463)	185.7	187.7	189.39	0.2	0.08
SD	0.4170 (0.0383)	207.65	209.65	211.34	0.24	0.02
AD	0.4503(0.0401)	193.72	195.72	197.4	0.22	0.05
ShD	0.3016 (0.0325)	203.87	205.87	207.56	0.27	0.01
LD	0.2844 (0.0321)	208.57	210.57	212.25	0.32	0.00
ED	0.1599 (0.0252)	226.64	228.64	230.32	0.34	0.00

Table 8 Goodness of fit of dataset 4

Distributions	ML estimate $\hat{\theta}$ (Standard error)	$-2\log L$	AIC	BIC	K-S	p-value
UD	0.2935 (0.0189)	387.08	389.08	391.16	0.11	0.54
SD	0.2073 (0.0156)	400.64	402.64	404.72	0.15	0.21
AD	0.2189 (0.0163)	388.93	400.93	393.01	0.14	0.27
ShD	0.1462 (0.0133)	397.06	399.06	401.14	0.16	0.12
LD	0.1392 (0.0129)	401.26	403.26	405.34	0.16	0.14
ED	0.0741 (0.0097)	425.01	427.01	429.09	0.28	0.00

$$AIC = -2\log L + 2k\ ,\ BIC = -2\log L - k\log(n)\ ,$$

$$K-S = \underset{x}{Sup}\left|F_n\left(x\right) - F_0\left(x\right)\right| \tag{25}$$

Where, $k=$ number of parameters, $n=$ sample size, $F_n\left(x\right)=$ empirical CDF of considered distribution and $F_0\left(x\right)=$ CDF of considered distribution.

From the Tables 4–7 we observed that UD has the least AIC, BIC and K-S values as compared to SD, ShD, AD, LD and ED and hence we can conclude that UD is the best distribution among the considered distributions for these datasets. The P-P plots and the Q-Q plots of UD for datasets 1, 2, 3, and 4 are shown in Figure 1 and Figure 2 and it establish the same conclusion as given by the goodness of fit by UD (Table 9).

Table 9 Confidence Interval (CI) of the estimated parameter of UD for datasets

Datasets	Parameter	90% CI (Lower, Upper)	95%CI (Lower, Upper)	99%CI (Lower, Upper)
1	Θ	(0.0762, 0.0790)	(0.0759, 0.0793)	(0.0753, 0.0799)
2	Θ	(0.0519, 0.0562)	(0.5146, 0.0567)	(0.0540,0.0541)
3	Θ	(0.5937, 0.6177)	(0.5914, 0.6200)	(0.5868, 0.6246)
4	Θ	(0.2895,0.2975)	(0.2887, 0.2983)	(0.2872, 0.2998)

Citation: Baishya J, Shanker R. On some statistical properties and applications of Uma distribution. *Biom Biostat Int J.* 2025;14(1):21‒27.
DOI: 10.15406/bbij.2025.14.00430

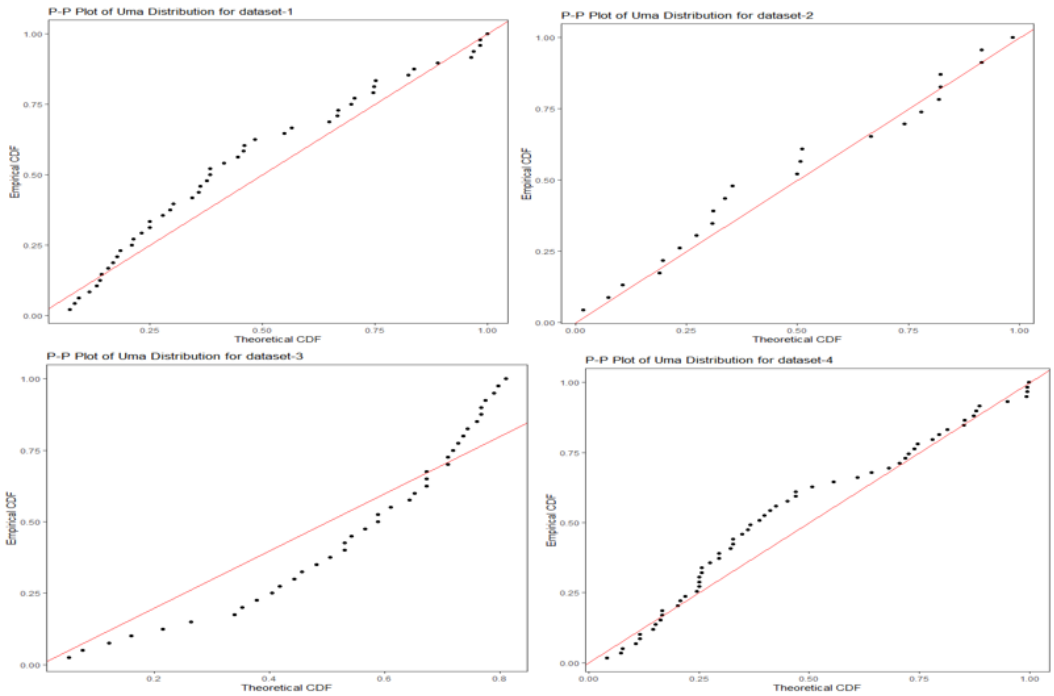

Figure 1 P-P Plot of UD for the dataset 1, 2, 3 and 4.

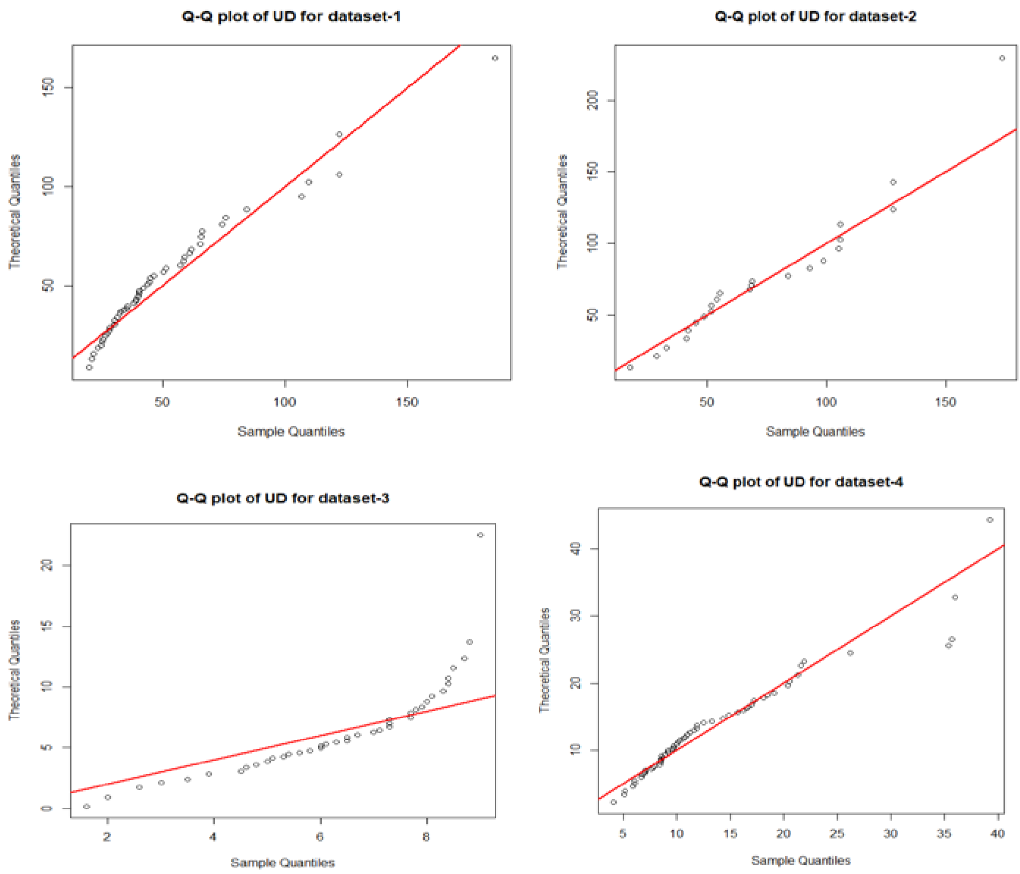

Figure 2 Q-Q Plot of UD for the dataset 1, 2, 3 and 4.

Citation: Baishya J, Shanker R. On some statistical properties and applications of Uma distribution. *Biom Biostat Int J.* 2025;14(1):21‒27. DOI: 10.15406/bbij.2025.14.00430

Conclusion

In this paper some of the important statistical properties such as Bonferroni and Lorenz curves and their indices, stress-strength reliability, Renyi entropy measure and order Statistics of Uma distribution (UD) have been obtained. To know the performance of maximum likelihood estimates of the parameter of the distribution, a simulation study has been presented using quantile function. To demonstrate the applications and goodness of fit of UD, four real lifetime datasets are considered from different fields of knowledge and the goodness of fit of UD shows that the UD provides a better fit as compared to Sujatha distribution, Akash distribution, Shanker distribution, Lindley distribution and exponential distribution.

Acknowledgments

Authors are grateful to the editor in chief of the journal and the anonymous reviewer for few minor corrections which improved the quality of the paper.

Funding

None.

Conflicts of interest

The authors declare that there are no conflicts of interest.

References

1. Lindley DV. Fiducial distributions and Bayes' theorem. *Journal of the Royal Statistical Society: Series B*. 1958;20(1):102–107.
2. Shanker R, Hagos F, Sujatha S. On Modelling of Lifetimes data using Exponential and Lindley distributions. *Biometrics & Biostatistics International Journal.* 2015;2(5):140–147.
3. Shanker R. Shanker distribution and its applications. *International Journal of Statistics and Applications.* 2015;5(6):338–348.
4. Shanker R. Akash Distribution and its applications. *International Journal of Probability and Statistics.* 2015;4(3):65–75.
5. Shanker R, Hagos F, Sujatha S. On modelling of lifetime data using one parameter akash, lindley and exponential distributions. *Biometrics & Biostatistics International Journal.* 2016;3(2):54–62.
6. Shanker R, Hagos F. On modelling of lifetime data using Akash, Shanker, Lindley and exponential distributions**.** *Biometrics & Biostatistics International Journal.* 2016;3(6):214–224.
7. Shanker R. Sujatha distribution and its applications. *Statistics in Transition New series.* 2016;17(3):391–410.
8. Shanker R. Uma distribution with properties and applications. *Biometrics & Biostatistics International journal*. 2022;11(5):165–169.
9. Shanker R, Baishya j, Prodhani HR, Ray M. Power Uma distribution with properties and applications in survival analysis. Journal of XIDIAN University. 2023;17(12):849–860.
10. Shanker R, Zakaria AFM, Prodhani H R, Ray M. Weighted Uma distribution with properties and applications. Journal of XIDIAN University. 2023;17(10):830–846.
11. Ganaie RA, Subramanian C, Soumya VP, et al. Enhancing engineering sciences with Uma distribution- A perfect fit and valuable contributions. *Reliability Theory& Applications*. 2023;4(76):99–111.
12. Bonferroni CE. Elementi di Statistica Generale. Seeber, Firenze. 1930.
13. Renyi A. On measures of entropy and information. *Proceedings of the fourth Berkeley symposium on mathematical statistics and probability.* 1961;4:547–562.
14. Van Montfort M A J. On testing that the distribution of extremes is of type I when type II is the alternative. *Journal of Hydrology*, 1970;11(4):421–427.
15. Lawless J. F. Statistical models and methods for lifetime data. *John Wiley & Sons*. 2002.
16. Maguire BA, Pearson ES, Wynn AHA. The time intervals between industrial accidents. *Biometrika*. 1952;39(1-2):168–180.
17. Nassar MM, Nada NK. The beta generalized Pareto distribution. Journal of Statistics: Advances in Theory and Applications. 2011;6(1-2):1–17.