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A compound of exponential and Komal distributions with properties and application

Abstract

In this paper exponential-Komal distribution, the compound of exponential distribution with Komal distribution has been proposed. The key feature of the proposed compound distribution is that it doesn't have a moment generating function or moments which might seem like a limitation, but this distribution can be very much useful for modelling data from biomedical sciences and engineering of heavy tailed behaviour. Important statistical properties of the distribution have been studied. The estimation of its parameter has been discussed using maximum likelihood method. Goodness of fit of the proposed distribution has been explained with an example of real life data having decreasing failure rate. The fit has been found quite satisfactory over exponential, Lindley, Shanker, Komal, exponential-Lindley and exponential-Shanker distributions.

Keywords: lifetime distribution, statistical properties, stress-strength reliability, maximum likelihood estimation, goodness of fit.

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Introduction

In the realm of distribution theory one parameter exponential distribution and Lindley distribution given by Lindley[1] are popular. Both exponential and Lindley distributions are useful for modelling different types of random events, particularly those that involve waiting times, failure times, or life spans, but Lindley provides more flexibility when the assumptions of the exponential model are too strict. A comparative study of exponential and Lindley distribution done by Shanker et al[2] found that in certain datasets exponential outshines while in others, the Lindley offers a more precise fit and also there were some datasets where both the distributions aren't provides optimal fit. Highlighting the need for more flexible models, Shanker[3] proposed a new one parameter lifetime distribution named Shanker distribution which provides much better fit than both exponential and Lindley distributions. Shanker[4] proposed another one parameter lifetime distribution called Komal distribution which provides much better fit than Shanker, exponential and Lindley distributions. The Komal distribution is defined by its probability density function (pdf) and cumulative distribution function (cdf)

$$f(x,a)=\frac{a^2}{a^2+a+1}(1+a+x)e^{-ax};x>0,a>0 \qquad (1)$$

$$F(x,a)=1-\left[1+\frac{ax}{a^2+a+1}\right]e^{-ax};x>0,a>0 \qquad (2)$$

Komal distribution is a two- component mixture of exponential (a) distribution and a gamma $(2,a)$ distribution with mixing proportion $\frac{a(a+1)}{a^2+a+1}$ and $\frac{1}{a^2+a+1}$ respectively. Recently, Shanker et al[5,6] derived the weighted version and the power version of Komal distribution respectively.

Belhamra et al[7] proposed the compound of exponential and Lindley distribution and named exponential-Lindley distribution (E-LD) having pdf and cdf

$$f(x,a)=\frac{a^2(2+a+x)}{(a+1)(a+x)^3};x>0,a>0 \qquad (3)$$

$$F(x,a)=\frac{x}{a+x}+\frac{ax}{(a+1)(a+x)^2};x>0,a>0 \qquad (4)$$

Recently, Ray and Shanker[8] proposed a compound distribution namely exponential-Shanker distribution (E-SD) and discussed its various statistical properties, estimation of parameter and application for engineering data and showed that it provides much closer fit than other compound distributions. The E-SD is defined by its pdf and cdf

$$f(x,a)=\frac{a^2(a^2+ax+2)}{(a^2+1)(a+x)^3};x>0,a>0 \qquad (5)$$

$$F(x,a)=\frac{x}{a+x}+\frac{ax}{(a^2+1)(a+x)^2};x>0,a>0 \qquad (6)$$

Both the E-LD and the E-SD is a particular cases of gamma-Lindley distribution (G-LD) of Abdi et al[9] and gamma-Shanker distribution (G-SD) by Ray and Shanker[10] respectively.

The main purpose for introducing the compound of exponential and Komal distribution are that the compound distributions are much useful for the study of heterogeneous population which is the reality of present real life situations and to examine its fit over other compound distributions. Further, as we know that Komal distribution provides much closer fit than exponential, Lindley and Shanker distributions, it is expected that the compound of exponential and Komal would provide much closer fit over the compound of exponential and Lindley distributions and the compound of exponential and Shanker distributions. Statistical properties, estimation of parameter and application of the proposed distribution have been discussed. One of the most important advantages of compounding exponential and Komal distribution is that the hazard rate for exponential distribution is constant but the hazard rate for the compound of exponential and Komal distributions is not constant but it is decreasing. Further, although the moment generating function and the moments of the proposed distribution do not exist, but the distribution is very much useful to model data of heavy tailed behaviour.

Exponential - Komal distribution

The pdf and the cdf of the compound of exponential and Komal distribution named exponential-Komal distribution (E-KD) are obtained as

$$f(x,a)=\frac{a^2\left[(a+x)(1+a)+2\right]}{\left(a^2+a+1\right)(a+x)^3};x>0,a>0 \qquad (7)$$

$$F(x,a)=\frac{x\left[a(a+x)(a+1)+(x+2a)\right]}{(a+x)^2\left(a^2+a+1\right)};x>0,a>0 \qquad (8)$$

The shapes of the pdf and the cdf of E-KD for varying values of parameter are shown in the following figures 1and 2 respectively. For better visualization of the plots of cdf we used 3D plots of the same in the figure 3.

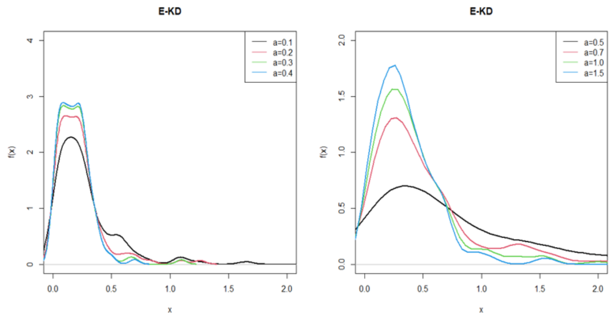

Figure 1 Pdf of E-KD for some selected values of parameter.

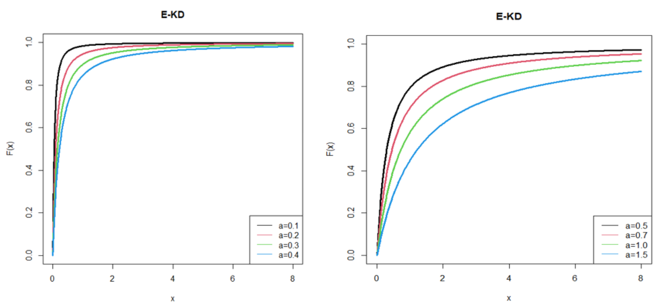

Figure 2 Cdf of E-KD for some selected values of parameter.

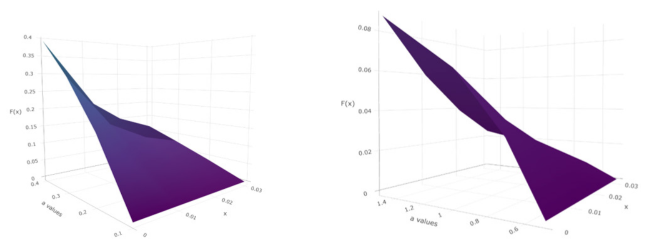

Figure 3 3D plots of cdf of E-KD for some selected values of parameter.

Theorem 1: The pdf of E-KD distribution is decreasing for $a \geq 0$.

Proof: We have,

$$f(x,a)=\frac{a^2\left[(a+x)(1+a)+2\right]}{\left(a^2+a+1\right)(a+x)^3};x>0,a>0$$

$$\log f(x,a)=C+\log\left[(a+x)(1+a)+2\right]-3\log(a+x),$$

Where, C is a constant. We have

$$\frac{d}{dx}\log f(x,a)=\frac{-2\left[(a+x)(1+a)+3\right]}{(a+x)\left[(a+x)(1+a)+2\right]}<0$$

For $a \geq 0$, $\frac{d}{dx}\log f(x,a)<0$ and this means that $f(x,a)$ is decreasing for all x.

Hazard function and reversed hazard function

The hazard function and the reversed hazard function are two important functions of a distribution. The reliability (survival) function of E-KD can be obtained by

$$R(x)=\frac{a\left[(a+x)\left(a^2+a+1\right)-x\right]}{(a+x)^2\left(a^2+a+1\right)} \qquad (9)$$

The corresponding hazard function and reversed hazard function of E-KD are obtained as

$$h(x)=\frac{a\left[(a+x)(1+a)+2\right]}{(a+x)\left[(a+x)\left(a^2+a+1\right)-x\right]} \qquad (10)$$

$$r(x)=\frac{a^2\left[(a+x)(1+a)+2\right]}{x(a+x)\left[a(a+x)(1+a)+(x+2a)\right]} \qquad (11)$$

The behavior of $h(x)$ when $x \to 0$ and $x \to \infty$, respectively are $= H$

$$\lim_{x\to 0} h(x)=\frac{a(1+a)+2}{a\left(a^2+a+1\right)} \text{ and } \lim_{x\to\infty} h(x)=0$$

$$\lim_{x\to 0} r(x)=\infty \text{ and } \lim_{x\to\infty} r(x)=0.$$

The shapes of the hazard function and the reversed hazard function of E-KD for varying values of parameter are shown in the following figures 4 and 6 respectively. Also, for better visualization of the plots of the hazard function and the reversed hazard function, we used 3D plots of the same in the figures 5 and 7 respectively.

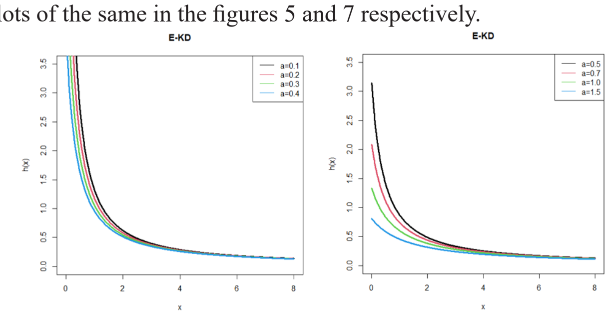

Figure 4 Hazard function of E-KD for some parameter values.

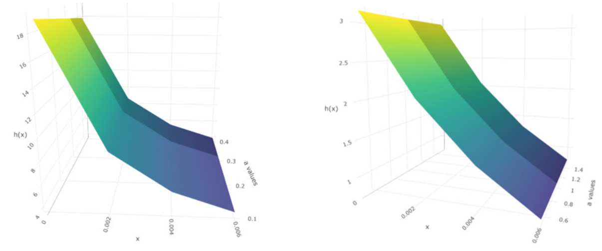

Figure 5 3D Plots of hazard function of E-KD for some parameter values.

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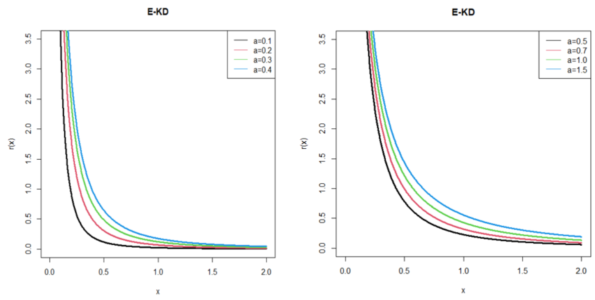

Figure 6 Reversed hazard function of E-KD for some parameter values.

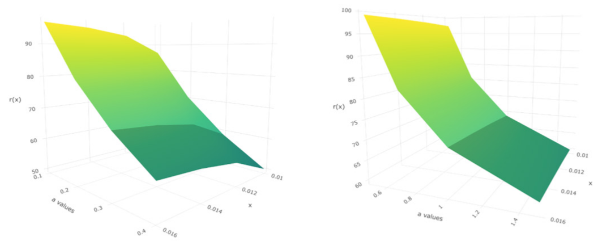

Figure 7 3D Plots of reversed hazard function of E-KD for some parameter values.

Theorem 2: The hazard function of the E-KD is decreasing

Proof: We have

$$f(x,a)=\frac{a^2\left[(a+x)(1+a)+2\right]}{\left(a^2+a+1\right)(a+x)^3};x>0,a>0$$

$$f'(x,a)=\frac{-2a\left[(1+a)(a+x)+3\right]}{(a+x)^4\left(a^2+a+1\right)}$$

Now, suppose that

$$\xi(x)=-\frac{f'(x,a)}{f(x,a)}=\frac{2\left[(a+x)(1+a)+3\right]}{a(a+x)\left[(a+x)(1+a)+2\right]}.$$

This gives

$$\phi'(x)=\frac{-2\left[(a+x)(1+a)\left\{(a+x)(1+a)+6\right\}+6\right]}{a(a+x)^2\left[(a+x)(1+a)+2\right]}<0$$

Theorem 3: The reversed hazard function of the E-KD is decreasing

Proof: We have,

$$r(x)=\frac{a^2\left[(a+x)(1+a)+2\right]}{x(a+x)\left[a(a+x)(1+a)+(x+2a)\right]}$$

This gives

$$\frac{d}{dx}\log r(x)=-\frac{\left[3a(1+a)+2\right]}{\left\{(a+x)(1+a)+2\right\}\left\{a(a+x)(1+a)+x+2a\right\}}-\frac{1}{(a+x)}-\frac{1}{x}<0$$ for all a

Quantiles and moments

The pth quantiles x_p of E-KD defined by $F\left(x_p\right)=p$, is the root of the equation

$$\frac{x_p\left[a\left(a+x_p\right)(a+1)+\left(x_p+2a\right)\right]}{\left(a+x_p\right)^2\left(a^2+a+1\right)}=p$$

This gives

$$x_p=\frac{2a}{\left[\left(1+\frac{a}{x_p}\right)\left\{p\left(1+\frac{a}{x_p}\right)\left(a^2+a+1\right)-a(a+1)\right\}-1\right]} \qquad (12)$$

It should be noted that this x_p may be used to generate E-KD random variates. Further, the median of E-KD can be obtained from above equation by taking $p=\frac{1}{2}$.

The moments and the moment generating function of E-KD do not exit and it has been shown mathematically in the following theorems 4 and 5 respectively.

Theorem 4: The moments of the E-KD does not exist.

Proof: The r^{th} moment of E-KD is given by

$$E\left(X^r\right)=\int_0^\infty x^r f(x,a)dx=\frac{a^2}{a^2+a+1}\int_0^\infty x^r\frac{\left\{(a+x)(1+a)+2\right\}}{(a+x)^3}dx$$

$$=\frac{1+a}{a^2+a+1}\int_0^\infty\frac{x^r}{\left(1+\frac{x}{a}\right)^2}dx+\frac{2}{a\left(a^2+a+1\right)}\int_0^\infty\frac{x^r}{\left(1+\frac{x}{a}\right)^3}dx$$

Let, $\frac{x}{a}=z$. As $x\to 0,\ z\to 0$ and *as* $x\to\infty,\ z\to\infty$ we have

$$E\left(X^r\right)=\frac{(1+a)a^{r+1}}{\left(a^2+a+1\right)}\int_0^\infty\frac{z^r}{(1+z)^2}dz+\frac{2a^r}{\left(a^2+a+1\right)}\int_0^\infty\frac{z^r}{(1+z)^3}dz$$

Using beta integral of second kind $\int_0^\infty\frac{x^{a-1}}{(1+x)^{a+b}}dx=B(a,b);\ a>0,b>0$, we get

$$E\left(X^r\right)=\frac{a^{r+1}}{\left(a^2+a+1\right)}\left[(1+a)\left\{B(r+1,1-r)\right\}+\frac{2}{a}\left\{B(r+1,2-r)\right\}\right]$$

Here the range is $-1<r<1$ but $r\geq 1$. Hence $E\left(X^r\right)$ does not exist.

Theorem 5: The moment generating function of the E-KD does not exist.

Proof:

$$=\frac{a^2}{a^2+a+1}\left[(1+a)\int_0^\infty\frac{e^{tx}}{(a+x)^2}dx+2\int_0^\infty\frac{e^{tx}}{(a+x)^3}dx\right]$$

Now, we have

$$\int_0^\infty\frac{e^{tx}}{(a+x)^2}dx=\left[\frac{e^{tx}}{-(a+x)}\right]_0^\infty+t\int_0^\infty\frac{e^{tx}}{a+x}dx$$

$$=\lim_{x\to\infty}\left[-\frac{e^{tx}}{(a+x)}\right]+\frac{1}{a}+t\int_0^a\frac{e^{tx}}{a+x}dx$$

$$=-\infty+t\int_0^a\frac{e^{tx}}{a+x}dx$$

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At $\lim_{x\to\infty}\frac{e^{tx}}{a+x}=\infty$, integral function is unbounded in the neighborhood of ∞, so $\int_0^\infty \frac{e^{tx}}{a+x}dx$ is divergent . This means that moment generating function does not exist

Entropies

Renyi entropy

Renyi entropy, proposed by Renyi[11] measures the variation of uncertainty in the distribution. The Renyi entropy is defined as

$$e(\eta)=\frac{1}{1-\eta}\log\left[\int_0^\infty f^\eta(x)dx\right] \text{ Where } 0<\eta<1$$

$$=\frac{1}{1-\eta}\log\left[\int_0^\infty\left(\frac{a^2((a+x)(1+a)+2)}{(a^2+a+1)(a+x)^3}\right)^\eta dx\right]$$

$$=\frac{1}{1-\eta}\log\left[\left(\frac{a^2}{a^2+a+1}\right)^\eta\int_0^\infty\left(\frac{1+a}{(a+x)^2}+\frac{2}{(a+x)^3}\right)^\eta dx\right] \quad (13)$$

Applying binomial expansion $(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}$, we get

$$e(\eta)=\frac{1}{1-\eta}\log\left[\left(\frac{a^2}{a^2+a+1}\right)^\eta\int_0^\infty\sum_{m=0}^{\eta}\binom{\eta}{m}\left(\frac{(1+a)}{(a+x)^2}\right)^m\left(\frac{2}{(a+x)^3}\right)^{\eta-m}dx\right] \text{ Where, } \frac{2}{\beta+x}<1$$

$$=\frac{\eta}{1-\eta}\log\left(\frac{a^2}{a^2+a+1}\right)+\frac{1}{1-\eta}\left[\sum_{m=0}^{\eta}\binom{\eta}{m}\frac{2^{\eta-m}(1+a)^m}{(3\eta-m-1)a^{(3\eta-2m-1)}}\right] \quad (14)$$

Tsallis entropy

Tsallis[12] introduced entropy called Tsallis entropy for generalizing the standard statistical mechanics which is defined as

$$S_\lambda=\frac{1}{1-\lambda}\log\left[1-\int_0^\infty f^\lambda(x)dx\right]$$

$$=\frac{1}{1-\lambda}\log\left[1-\left(\frac{a^2}{a^2+a+1}\right)^\lambda\int_0^\infty\left(\frac{1+a}{(a+x)^2}+\frac{2}{(a+x)^3}\right)^\lambda dx\right] \quad (15)$$

Applying binomial expansion $(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}$, we get

$$S_\lambda=\frac{1}{1-\lambda}\left[1-\left(\frac{a^2}{a^2+a+1}\right)^\lambda\sum_{m=0}^{\lambda}\binom{\lambda}{m}\frac{2^{\lambda-m}(1+a)^m}{(3\lambda-m-1)a^{(3\lambda-2m-1)}}\right] \quad (16)$$

Extreme order statistics

Let, $X_{1:n},\ldots,X_{n:n}$ be the order statistics of a random sample of size n from the E-KD(a) distribution with distribution function $F(x)$.

The cdf of the minimum order statistic $X_{1:n}$ is given by

$$F_{X_{1:n}}(x)=1-[1-F(x)]^n=1-\left[\frac{a\left[(a+x)(a^2+a+1)-x\right]}{(a+x)^2(a^2+a+1)}\right]^n$$

The cdf of the maximum order statistic $X_{n:n}$ is given by

$$F_{X_{n:n}}(x)=[F(x)]^n=\left[\frac{x\left[a(a+x)(a+1)+(x+2a)\right]}{(a+x)^2(a^2+a+1)}\right]^n$$

Stochastic orderings

Stochastic ordering is used to compare two lifetime distributions to examine how one variable is greater than the other.

A random variable X is said to be smaller than a random variable Y in the

i. Stochastic order $(X\prec_{st}Y)$ if $F_X(x)\geq F_Y(y)$ for all x

ii. Hazard rate order $(X\prec_{hr}Y)$ if $h_X(x)\geq h_Y(y)$ for all x

iii. Mean residual life order $(X\prec_{mrl}Y)$ if $m_X(x)\geq m_Y(y)$ for all x

iv. Likelihood ratio order $(X\prec_{lr}Y)$ if $\frac{f_X(x)}{f_Y(Y)}$ decrease in x

The following results due to Shaked and Shanthikumar[13] are well known for establishing stochastic ordering of distributions:

$$X\prec_{lr}Y\Rightarrow X\prec_{hr}Y\Rightarrow X\prec_{mrl}Y$$
$$\Downarrow$$
$$X\prec_{st}Y$$

Theorem 6: Let $X_1\sim$ E-KD(a_1) and $X_2\sim$ E-KD(a_2) .If $a_1\leq a_2$ then $X_1\prec_{lr}X_2\Rightarrow X_1\prec_{hr}X_2\Rightarrow X_1\prec_{st}X_2$.

Proof: We have

$$\frac{f_{X_1}(x)}{f_{X_2}(x)}=\frac{a_1^2(a_2+x)^3(a_2^2+a_2+1)\left[(a_1+x)(1+a_1)+2\right]}{a_2^2(a_1+x)^3(a_1^2+a_1+1)\left[(a_2+x)(1+a_2)+2\right]}$$

Let,

$$\varphi(x)=\frac{f_{X_1}(x)}{f_{X_2}(x)}=\frac{a_1^2(a_2+x)^3(a_2^2+a_2+1)\left[(a_1+x)(1+a_1)+2\right]}{a_2^2(a_1+x)^3(a_1^2+a_1+1)\left[(a_2+x)(1+a_2)+2\right]}$$

$$\frac{d\log\varphi(x)}{dx}=\left(\frac{3}{a_2+x}-\frac{1+a_2}{(a_2+x)(1+a_2)+2}\right)-\left(\frac{3}{a_1+x}-\frac{1+a_1}{(a_1+x)(1+a_1)+2}\right)$$

$$=\psi(a_2)-\psi(a_1),$$

Where,

$$\psi(a)=\left(\frac{3}{a+x}-\frac{1+a}{(a+x)(1+a)+2}\right)$$

$$\frac{d}{da}\psi(a)=\frac{-3}{(a+x)^2}-\frac{1+2a+a^2}{\{(a+x)(1+a)+2\}^2}<0$$

For $a_1\leq a_2$, $\frac{d}{dx}\log\left(\frac{f_{X_1}(x)}{f_{X_2}(x)}\right)<0$ .This means that $X_1\prec_{lr}X_2$ and hence $X_1\prec_{hr}X_2$ and $X_1\prec_{st}X_2$.

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Estimation of parameters

Let $(x_1, x_2, ..., x_n)$ be the observed values of a random sample $(X_1, X_2, ..., X_n)$ from the E-KD. Then the Likelihood function is given by

$$L(a) = \left(\frac{a^2}{a^2+a+1}\right)^n \frac{\prod_{i=1}^{n}\left[(a+x_i)(1+a)+2\right]}{\prod_{i=1}^{n}(a+x_i)^3} \quad (17)$$

The log-likelihood function of E-KD is thus obtained as

$$\log L(a) = 2n\log a - n\log\left(a^2+a+1\right) + \sum_{i=1}^{n}\log\left[(a+x_i)(1+a)+2\right] - 3\sum_{i=1}^{n}\log(a+x_i) \quad (18)$$

The maximum likelihood estimate of the parameter a is the solution of the following log-likelihood equation

$$\frac{\partial \log L(a)}{\partial a} = \frac{2n}{a} - \frac{n(2a+1)}{\left(a^2+a+1\right)} + \sum_{i=1}^{n}\frac{(1+a)}{(a+x_i)(1+a)} - 3\sum_{i=1}^{n}\frac{1}{(a+x_i)} = 0 \quad (19)$$

It can be easily shown that the maximum likelihood estimate $\hat{a}$ will satisfy the second order sufficient condition of maximum likelihood estimator. For, we have

$$\frac{\partial^2 \log L(a)}{\partial a^2} = \frac{-2n}{a^2} + \frac{n\left(2a^2+2a-1\right)}{\left(a^2+a+1\right)} + \sum_{i=1}^{n}\frac{(1+a)(x-1)}{\left[(a+x)(1+a)+2\right]^2} + 3\sum_{i=1}^{n}\frac{1}{(a+x_i)^2} < 0 \quad (20)$$

Estimation of the stress-strength parameter $R = P(X > Y)$

In reliability, the stress-strength model describes the life of a component which has a random strength X subjected to a random stress Y. The component fails at the instant if the stress applied to it exceeds its strength, and the component will function satisfactory whenever, $X > Y$. In this section our objective is to estimate $R = P(X > Y)$ when $X \sim$ E-KD(a_1) and $Y \sim$ E-KD(a_2) and X and Y are independently distributed. Thus, the stress- strength parameter is given by

$$R = P(X > Y) = \int_0^\infty P(X > Y \mid Y = y) f_Y(y)\,dy$$

$$= \int_0^\infty \left[1 - F_X(y)\right] f_Y(y)\,dy$$

$$= 1 - \int_0^\infty \frac{a_1 a_2^2}{\left(a_1^2+a_1+1\right)\left(a_2^2+a_2+1\right)} \frac{\left[(a_1+y)\left(a_1^2+a_1+1\right) - y\right]\left\{(a_2+y)(1+a_2)+2\right\}}{(a_1+y)^2(a_2+y)^3}\,dy$$

$$= \mathrm{H}\,(a_1, a_2)$$

Let, $(x_1, x_2, ..., x_n)$ be the observed value of a random sample of size n from E-KD(a_1) and $(y_1, y_2, ..., y_m)$ be the observed value of a random sample of size m from E-KD(a_2).

The log-likelihood functions of a_1 and a_2 is given by

$$\log L(a_1, a_2) = 2n\log(a_1) - n\log\left(a_1^2+a_1+1\right) + \sum_{i=1}^{n}\log\left[(a_1+x_i)(1+a_1)+2\right] - 3\sum_{i=1}^{n}\log(a_1+x_i)$$
$$+ 2m\log(a_2) - m\log\left(a_2^2+a_2+1\right) + \sum_{i=1}^{m}\log\left[(a_2+y_i)(1+a_2)+2\right] - 3\sum_{i=1}^{m}\log(a_2+y_i)$$

The maximum likelihood estimates of a_1 and a_2 are the solutions of following log-likelihood equations

$$\frac{\partial}{\partial a_1}\left(\log L(a_1,a_2)\right) = \frac{2n}{a_1} - \frac{n(2a_1+1)}{\left(a_1^2+a_1+1\right)} + \sum_{i=1}^{n}\frac{(1+2a_1+x_i)}{(a_1+x)(1+a_1)+2} - 3\sum_{i=1}^{n}\frac{1}{(a_1+x_i)} = 0$$

$$\frac{\partial}{\partial a_2}\left(\log L(a_1,a_2)\right) = \frac{2m}{a_2} - \frac{n(2a_2+1)}{\left(a_2^2+a_2+1\right)} + \sum_{i=1}^{m}\frac{1+2a_2+y_i}{(a_2+y)(1+a_2)+2} - 3\sum_{i=1}^{m}\frac{1}{(a_2+y_i)} = 0$$

Solving these non-linear equations using any iterative methods available in R packages we can obtain the MLEs of the parameters as $(\hat{a}_1, \hat{a}_2)$ and hence the MLE of R can thus be obtained as

$\hat{R} = \mathrm{H}(\hat{a}_1, \hat{a}_2)$.

A simulation study

This section contains a simulation study to examine the consistency of maximum likelihood estimator of the parameter of the E-KD. The mean, bias (B), MSE and variance of the MLE's are computed using the formulae

$$Mean = \frac{1}{n}\sum_{i=1}^{n}\hat{H}_i, \quad B = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{H}_i - H\right), \quad MSE = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{H}_i - H\right)^2,$$

$$Variance = MSE - B^2$$

Where, $H = (a)$ and $\hat{H} = (\hat{a})$.

The simulation results of E-KD have been presented in table 1 using acceptance-rejection method of simulation.

Table 1 The mean, biases, MSE and variances of E-KD for $a = 1.5$

Parameters	Sample Size	Mean	Bias	MSE	Variance
	20	1.482652	-0.0173482	0.000865	0.000564
	40	1.486166	-0.0138335	0.000719	0.0005279
$\hat{a}$	60	1.484523	-0.0154774	0.000537	0.0002981
	80	1.482652	-0.0114015	0.000468	0.0003381
	100	1.482957	-0.0100429	0.000444	0.000344

Application

This section deals with the goodness of fit of E-KD over E-LD, E-SD, Shanker, Komal, Lindley and exponential distributions. The applications and the goodness of fit have been presented with one real dataset relating to failure times of 50 components. The summary of the dataset is presented in table 2. The total time to test (TTT) plots and Violin plot of the dataset related to failure times and simulated dataset are given in figures 8 and 9 respectively. The goodness of fit of the considered distributions for the dataset is provided in table3. The fitted plots of the considered distributions for the dataset are given in figure 10. The dataset is as follows:

Dataset 1: The following extreme skewed to right data, discussed by Murthy et al[14] presents the failure times of 50 components and the observations are:

0.036, 0.058, 0.061, 0.074, 0.078, 0.086, 0.102, 0.103, 0.114, 0.116, 0.148, 0.183, 0.192, 0.254, 0.262, 0.379, 0.381, 0.538, 0.570, 0.574, 0.590, 0.618, 0.645, 0.961, 1.228, 1.600, 2.006, 2.054, 2.804, 3.058, 3.076, 3.147, 3.625, 3.704, 3.931, 4.073, 4.393, 4.534, 4.893, 6.274, 6.816, 7.896, 7.904, 8.022, 9.337, 10.940, 11.020, 13.880, 14.730, 15.080

Table 2 Summary of the dataset 1

Min	1st Qu.	Median	Mean	Variance	3rd Qu.	Max
0.036	0.2075	1.414	3.343	17.48477	4.4988	15.08

Table 3 Goodness of fit of E-KD along with other distributions for dataset 1

Distributions	ML estimates and standard error $\hat{a}(\text{S.E})$	$-2\log L$	AIC	K-S	P-value
E-KD	1.4566 (0.2932)	**211.9573**	**213.9573**	**0.1423**	**0.3202**
E-SD	1.5819 (0.2822)	212.1365	214.1365	0.1614	0.1887
E-LD	1.7208 (0.3529)	212.3744	214.3744	0.2769	0.0009
KD	0.4847 (0.0483)	234.9971	236.9971	0.3093	0.0003
SD	0.5713 (0.0509)	249.9203	251.9203	0.3454	0
LD	0.4987 (0.0513)	240.3559	242.3559	0.3469	0
ED	0.2991(0.0423)	220.6857	222.6857	0.28	0.0097

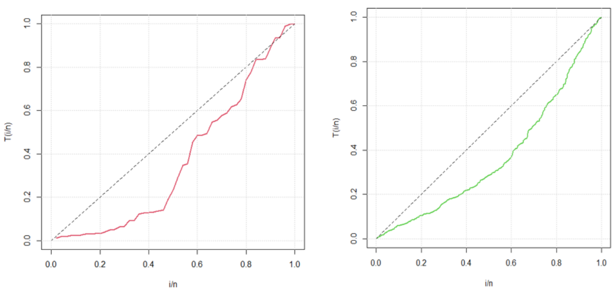

Figure 8 TTT-plot of dataset and simulated dataset related to failure times of E-KD.

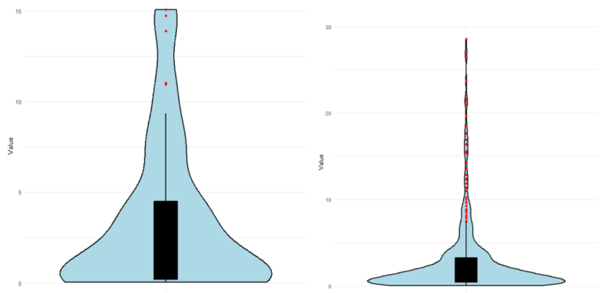

Figure 9 Violin-plot of the dataset related to failure times and simulated data of E-KD respectively.

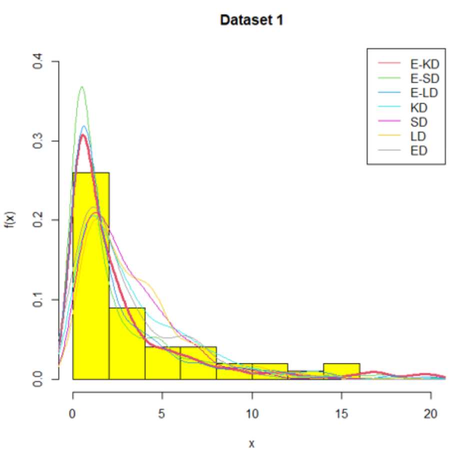

Figure 10 Fitted plots of distributions for the dataset 1.

Concluding remarks

In this paper, we introduced exponential-Komal distribution (E-KD) by compounding exponential distribution with Komal distribution. Several key characteristics of this distribution have been thoroughly explored, including its shape, hazard and reversed hazard functions, quantile function, Rényi entropy, Tsallis entropy and stress-strength reliability. Maximum Likelihood estimation has been discussed for estimating its parameter. The goodness of fit of E-KD over E-LD, E-SD, Komal distribution, Shanker distribution, Lindley distribution and exponential distribution shows that E-KD gives much closer fit than these distributions for the dataset related to failure times of 50 components.The proposed distribution is very much useful for modelling data from biomedical sciences and engineering having heavy tailed behaviour.

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Conflict of interests

The authors declare that there are no conflicts of interest.

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