Research Article

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Adya distribution with properties and application

Abstract

In the present paper, a new one parameter lifetime distribution named, “Adya distribution’ has been proposed for modeling lifetime data from engineering. Its various statistical properties including moments and moments based measures, hazard rate function, mean residual life function, stochastic ordering, deviations from the mean and the median, Bonferroni and Lorenz curves, and stress-strength reliability have been studied. Both the method of moment and the maximum likelihood estimation have been discussed for estimating the parameter of the proposed distribution. A numerical example has been presented to test the goodness of fit of the proposed distribution over other one parameter lifetime distributions available in statistical literature.

Keywords: lifetime distributions, statistical and mathematical properties, parameter estimation, goodness of fit

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Rama Shanker,[1] Kamlesh Kumar Shukla,[2] Amaresh Ranjan,[3] Ravi Shanker[4]

[1]Department of Statistics, Assam University, Silchar, Assam, India
[2]Department of Community Medicine, Noida International Institute of Medical Science, India
[3]Department of Mathematics, Nalanda Open University, India
[4]Department of Mathematics, G.L.A. College, N.P University, India

Correspondence: Rama Shanker, Department of Statistics, Assam University, Silchar, Assam, India, Email shankrrama2009@gmail.com

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Introduction

The classical one parameter exponential distribution and Lindley distribution proposed by Lindley[1] were useful for modeling lifetime data from engineering and biomedical. It has been observed by Shanker et al.[2] that exponential and Lindley distributions are not suitable for several lifetime data. In search for better one parameter lifetime distributions, Shanker has introduced several one parameter lifetime distributions including Shanker,[3] Aradhana,[4] Sujatha,[5] Devya.[6] The probability density function (pdf) and the cumulative distribution function (cdf) of these distributions are presented in Table 1.

Table 1 pdf and cdf of Shanker, Aradhana, Sujatha, Devya, and Lindley distributions for $x>0,\theta>0$

Distributions		Probability density functions and Cumulative distribution functions
Shanker	pdf	$f(x)=\dfrac{\theta^2}{\theta^2+1}(\theta+x)\,e^{-\theta x}$
	cdf	$F(x)=1-\left[1+\dfrac{\theta x}{\theta^2+1}\right]e^{-\theta x}$
Aradhana	pdf	$f(x)=\dfrac{\theta^3}{\theta^2+2\theta+2}(1+x)^2\,e^{-\theta x}$
	cdf	$F(x)=1-\left[1+\dfrac{\theta x(\theta x+2\theta+2)}{\theta^2+2\theta+2}\right]e^{-\theta x}$
Sujatha	pdf	$f(x)=\dfrac{\theta^3}{\theta^2+\theta+2}(1+x+x^2)e^{-\theta x}$
	cdf	$F(x)=1-\left[1+\dfrac{\theta x(\theta x+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x}$
Devya	pdf	$f(x)=\dfrac{\theta^5}{\theta^4+\theta^3+2\theta^2+6\theta+24}(1+x+x^2+x^3+x^4)e^{-\theta x}$
	cdf	$F(x)=1-\left[1+\dfrac{\left\{\begin{array}{l}\theta^4(x^4+x^3+x^2+x)+\theta^3(4x^3+3x^2+2x)\\+6\theta^2(2x^2+x)+24\theta x\end{array}\right\}}{\theta^4+\theta^3+2\theta^2+6\theta+24}\right]e^{-\theta x}$

Table Continued...

Distributions		Probability density functions and Cumulative distribution functions
Lindley	pdf	$f(x)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x}$
	cdf	$F(x)=1-\left[1+\frac{\theta x}{\theta+1}\right]e^{-\theta x}$

The reasons for introducing such lifetime distributions with their advantages and disadvantages, statistical properties, parameter estimation and applications are available in the respective papers.

In this paper, a new lifetime distribution which gives better fit over several one parameter lifetime distributions are introduced. The new one parameter lifetime distribution is defined by its cdf and pdf, respectively

$$F(x,\theta)=1-\left[1+\frac{\theta x\left(\theta x+2\theta^2+2\right)}{\theta^4+2\theta^2+2}\right]e^{-\theta x}\ ;x>0,\theta>0 \qquad (1.1)$$

$$f(x;\theta)=\frac{\theta^3}{\theta^4+2\theta^2+2}(\theta+x)^2 e^{-\theta x}\quad ;x>0,\ \theta>0 \qquad (1.2)$$

We name this distribution, "Adya distribution". This is a convex combination of exponential (θ), gamma $(2,\theta)$ and gamma $(3,\theta)$ distributions. We have

$$f(x;\theta)=p_1\, g_1(x;\theta)+p_2\, g_2(x;2,\theta)+(1-p_1-p_2)g_3(x;3,\theta)$$

Where $p_1=\frac{\theta^4}{\theta^4+2\theta^2+2}$, $p_2=\frac{2\theta^4}{\theta^4+2\theta^2+2}$, $g_1(x;\theta)=\theta e^{-\theta x}$, $g_2(x;2,\theta)=\frac{\theta^2}{\Gamma(2)}x^{2-1}e^{-\theta x}$, and $g_3(x;3,\theta)=\frac{\theta^3}{\Gamma(3)}x^{3-1}e^{-\theta x}$;$x>0,\theta>0$.

The pdf and the cdf of Adya distribution for values of the parameter θ are shown in Figures 1 and 2, respectively.

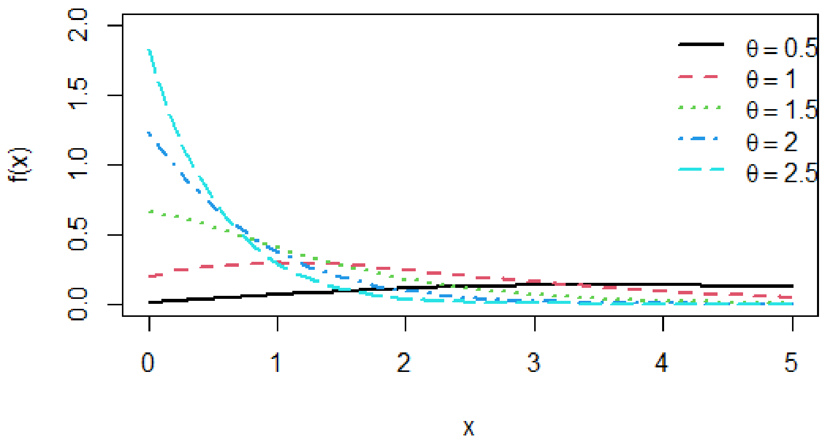

Figure 1 The pdf of Adya distribution.

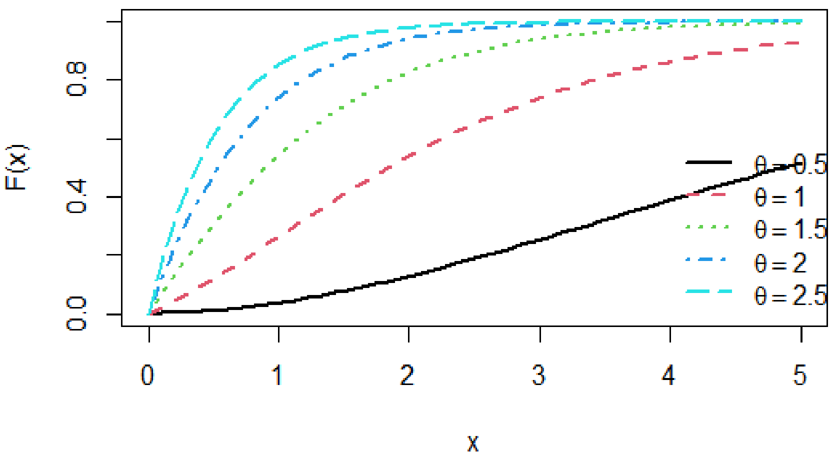

Figure 2 The cdf of Adya distribution.

Citation: Shanker R, Shukla KK, Ranjan A, et al. Adya distribution with properties and application. *Biom Biostat Int J.* 2021;10(3):81‒88. DOI: 10.15406/bbij.2021.10.00334

Moments and moments based measures

The r th moment about origin μ_r' of (1.2) can be obtained as

$$\mu_r' = \frac{r!\left\{\theta^4 + 2(r+1)\theta^2 + (r+1)(r+2)\right\}}{\theta^r\left(\theta^4 + 2\theta^2 + 2\right)} \quad ; r = 1,2,3,... \tag{2.1}$$

Substituting r=1,2,3, and 4 in (2.1), the first four moments about origin of (1.2) are obtained as

$$\mu_1' = \frac{\theta^4 + 4\theta^2 + 6}{\theta\left(\theta^4 + 2\theta^2 + 2\right)}, \quad \mu_2' = \frac{2\left(\theta^4 + 6\theta^2 + 12\right)}{\theta^2\left(\theta^4 + 2\theta^2 + 2\right)},$$

$$\mu_3' = \frac{6\left(\theta^4 + 8\theta^2 + 20\right)}{\theta^3\left(\theta^4 + 2\theta^2 + 2\right)}, \quad \mu_4' = \frac{24\left(\theta^4 + 10\theta^2 + 30\right)}{\theta^4\left(\theta^4 + 2\theta^2 + 2\right)}$$

Thus, the central moments of (1.2) are obtained as

$$\mu_2 = \frac{\theta^8 + 8\theta^6 + 24\theta^4 + 24\theta^2 + 12}{\theta^2\left(\theta^4 + 2\theta^2 + 2\right)^2}$$

$$\mu_3 = \frac{2\left(\theta^{12} + 12\theta^{10} + 54\theta^8 + 100\theta^6 + 108\theta^4 + 72\theta^2 + 24\right)}{\theta^3\left(\theta^4 + 2\theta^2 + 2\right)^3}$$

$$\mu_4 = \frac{3\begin{pmatrix} 3\theta^{16} + 48\theta^{14} + 304\theta^{12} + 944\theta^{10} + 1816\theta^8 + 2304\theta^6 \\ +1920\theta^4 + 960\theta^2 + 240 \end{pmatrix}}{\theta^4\left(\theta^4 + 2\theta^2 + 2\right)^4}$$

Descriptive measures including coefficient of variation $(C.V)$, coefficient of skweness $\left(\sqrt{\beta_1}\right)$, coefficient of kurtosis (β_2) and index of dispersion (γ) of (1.2) are thus obtained as

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\theta^8 + 8\theta^6 + 24\theta^4 + 24\theta^2 + 12}}{\theta^4 + 4\theta^2 + 6}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{2\left(\theta^{12} + 12\theta^{10} + 54\theta^8 + 100\theta^6 + 108\theta^4 + 72\theta^2 + 24\right)}{\left(\theta^8 + 8\theta^6 + 24\theta^4 + 24\theta^2 + 12\right)^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3\begin{pmatrix} 3\theta^{16} + 48\theta^{14} + 304\theta^{12} + 944\theta^{10} + 1816\theta^8 + 2304\theta^6 \\ +1920\theta^4 + 960\theta^2 + 240 \end{pmatrix}}{\left(\theta^8 + 8\theta^6 + 24\theta^4 + 24\theta^2 + 12\right)^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\theta^8 + 8\theta^6 + 24\theta^4 + 24\theta^2 + 12}{\theta\left(\theta^4 + 2\theta^2 + 2\right)\left(\theta^4 + 4\theta^2 + 6\right)}$$

The natures of these descriptive measures for values of parameter θ are shown in Figure 3.

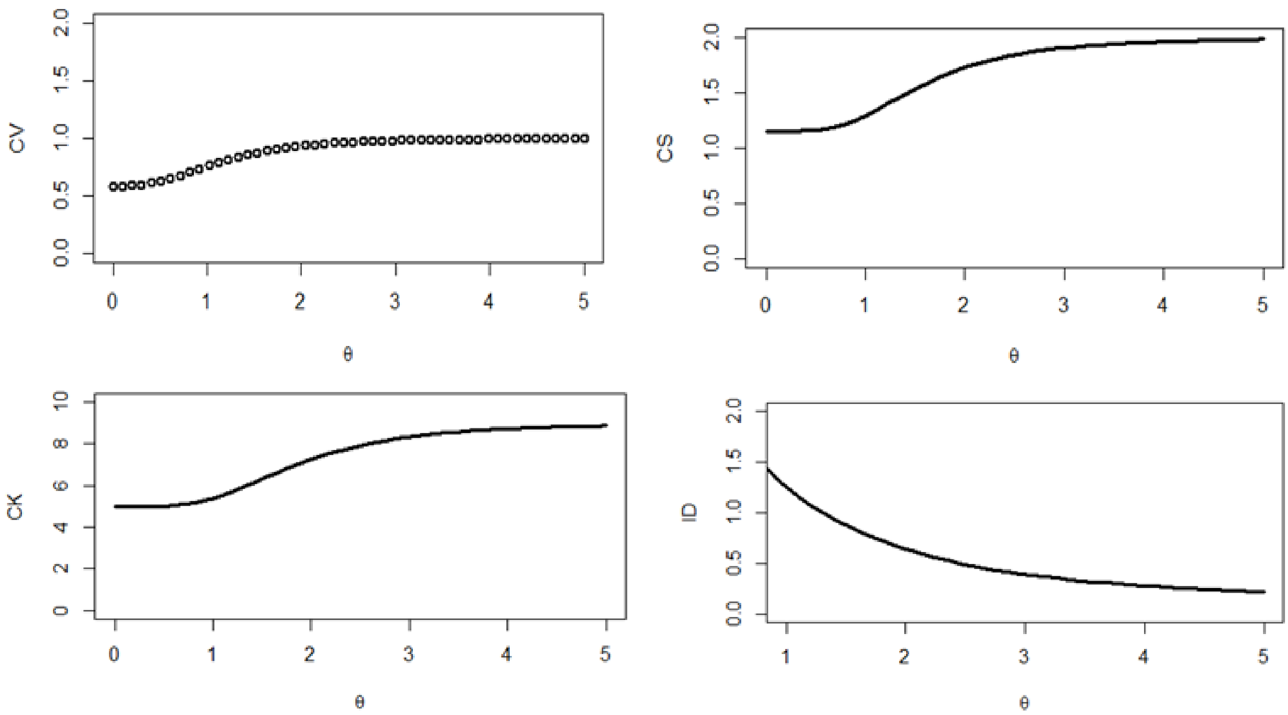

Figure 3 Coefficients of variation, skewness, kurtosis and index of dispersion of Adya distribution.

The condition under which Adya distribution is over-dispersed, equi-dispersed, and under-dispersed along with condition under which Shanker, Aradhana, Sujatha, Devya, Lindley and exponential distributions are over-dispersed, equi-dispersed, and under-dispersed are presented in Table 2.

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Table 2 Over-dispersion, equi-dispersion and under-dispersion of Adya, Shanker, Aradhana, Sujatha, Devya, Lindley and exponential distributions for parameter θ

Distribution	Over-dispersion $(\mu<\sigma^2)$	Equi-dispersion $(\mu=\sigma^2)$	Under-dispersion $(\mu>\sigma^2)$
Adya	$\theta<1.305719841$	$\theta=1.305719841$	$\theta>1.305719841$
Shanker	$\theta<1.171535555$	$\theta=1.171535555$	$\theta>1.171535555$
Aradhana	$\theta<1.283826505$	$\theta=1.283826505$	$\theta>1.283826505$
Sujatha	$\theta<1.364271174$	$\theta=1.364271174$	$\theta>1.364271174$
Devya	$\theta<1.451669994$	$\theta=1.451669994$	$\theta>1.451669994$
Lindley	$\theta<1.170086487$	$\theta=1.170086487$	$\theta>1.170086487$
Exponential	$\theta<1$	$\theta=1$	$\theta>1$

Hazard rate function and mean residual life function

The hazard rate function and the mean residual life function of a continuous random variable X having pdf and cdf $f(x)$ and $F(x)$ are, respectively, defined as

$$h(x)=\lim_{\Delta x\to 0}\frac{P(X<x+\Delta x|X>x)}{\Delta x}=\frac{f(x)}{1-F(x)} \tag{3.1}$$

and $m(x)=E[X-x|X>x]=\frac{1}{1-F(x)}\int_x^\infty[1-F(t)]\,dt$ (3.2)

Thus, $h(x)$ and $m(x)$ of (1.2) are obtained as

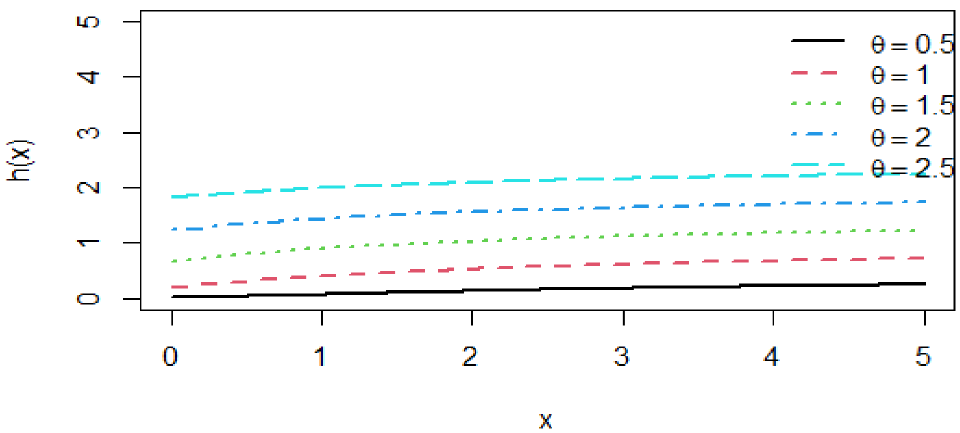
$$h(x)=\frac{\theta^3(\theta+x)^2}{\theta^2x^2+2\theta(\theta^2+1)x+(\theta^4+2\theta^2+2)} \tag{3.3}$$

and

$$m(x)=\frac{1}{\left[\theta^2x^2+2\theta(\theta^2+1)x+(\theta^4+2\theta^2+2)\right]e^{-\theta x}}\times\int_x^\infty\left[\theta^2t^2+2\theta(\theta^2+1)t+(\theta^4+2\theta^2+2)\right]e^{-\theta t}dt$$

$$=\frac{\theta^2x^2+2\theta(\theta^2+2)x+(\theta^4+4\theta^2+6)}{\theta\left[\theta^2x^2+2\theta(\theta^2+1)x+(\theta^4+2\theta^2+2)\right]} \tag{3.4}$$

This gives $h(0)=\frac{\theta^5}{\theta^4+2\theta^2+2}=f(0)$ and $m(0)=\frac{\theta^4+4\theta^2+6}{\theta(\theta^4+2\theta^2+2)}=\mu_1'$. The hazard rate function and mean residual life function of Adya distribution are shown in Figure 4.



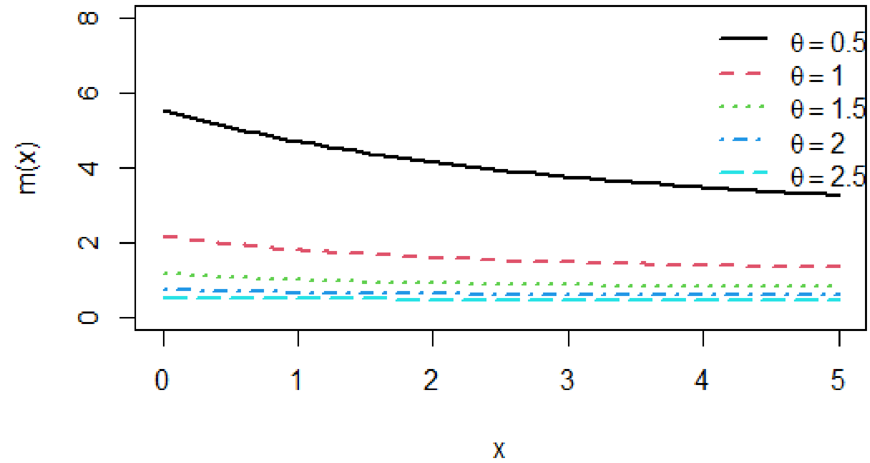


Figure 4 Graphs of $h(x)$ and $m(x)$ of Adya distribution.

Stochastic orderings

A random variable X is said to be smaller than a random variable Y in the

i. stochastic order $(X \le_{st} Y)$ if $F_X(x) \ge F_Y(x)$ for all x

ii. hazard rate order $(X \le_{hr} Y)$ if $h_X(x) \ge h_Y(x)$ for all x

iii. mean residual life order $m_X(x) \le m_Y(x)$ if $m_X(x) \le m_Y(x)$ for all x

iv. likelihood ratio order $(X \le_{lr} Y)$ if $\frac{f_X(x)}{f_Y(x)}$ decreases in x.

Shaked and Shanthikumar[7] proposed following results for establishing stochastic ordering of distributions

$$X \le_{lr} Y \Rightarrow X \le_{hr} Y \Rightarrow X \le_{mrl} Y \qquad (4.1)$$
$$\Downarrow$$
$$X \le_{st} Y$$

The distribution (1.2) is ordered with respect to the strongest 'likelihood ratio' ordering.

Theorem: Suppose X ~ Adya distributon(θ_1) and Y ~ Adya distribution(θ_2). If $\theta_1 > \theta_2$, then $X \le_{lr} Y$ and hence $X \le_{hr} Y$, $X \le_{st} Y$ and $X \le_{st} Y$.

Proof: We have

$$\frac{f_X(x)}{f_Y(x)} = \frac{\theta_1^3\left(\theta_2^4 + 2\theta_2^2 + 2\right)}{\theta_2^3\left(\theta_1^4 + 2\theta_1^2 + 2\right)}\left(\frac{(\theta_1 + x)^2}{(\theta_2 + x)^2}\right)e^{-(\theta_1 - \theta_2)x} \quad ; x > 0$$

Now

$$\ln\frac{f_X(x)}{f_Y(x)} = \ln\left[\frac{\theta_1^3\left(\theta_2^4 + 2\theta_2^2 + 2\right)}{\theta_2^3\left(\theta_1^4 + 2\theta_1^2 + 2\right)}\right] + 2\ln\left(\frac{\theta_1 + x}{\theta_2 + x}\right) - (\theta_1 - \theta_2)x$$

.

This gives $\frac{d}{dx}\left\{\ln\frac{f_X(x)}{f_Y(x)}\right\} = \frac{-2(\theta_1 - \theta_2)}{(\theta_1 + x)(\theta_2 + x)} - (\theta_1 - \theta_2)$

Thus for $\theta_1 > \theta_2$, $\frac{d}{dx}\left\{\ln\frac{f_X(x)}{f_Y(x)}\right\} < 0$. This means that $X \le_{lr} Y$ and hence $X \le_{hr} Y$, $X \le_{mrl} Y$ and $X \le_{st} Y$.

Mean deviations

The mean deviation about the mean and the mean deviation about the median are used to measure the amount of scatter in the population from the mean and the median and defined by

$$\delta_1(X) = \int_0^\infty |x - \mu| f(x)dx \quad \text{and} \quad \delta_2(X) = \int_0^\infty |x - M| f(x)dx$$

, respectively, where $\mu = E(X)$ and M = Median (X). The computation of these measures are simplified as

$$\delta_1(X) = \int_0^\mu (\mu - x)f(x)dx + \int_\mu^\infty (x - \mu)f(x)dx = 2\mu F(\mu) - 2\int_0^\mu x f(x)dx \qquad (5.1)$$

and

$$\delta_2(X) = \int_0^M (M - x)f(x)dx + \int_M^\infty (x - M)f(x)dx = \mu - 2\int_0^M x f(x)dx \qquad (5.2)$$

Using pdf (1.2) and expression for the mean of Adya distribution ,we get

$$\int_0^\mu x f(x;\theta)dx = \mu - \frac{\left\{\theta^5\mu + \theta^4\left(2\mu^2 + 1\right) + \theta^3(\mu^3 + 4\mu) + \theta^2(3\mu^2 + \mu) + 6\theta\mu + 6\right\}e^{-\theta\mu}}{\theta\left(\theta^4 + 2\theta^2 + 2\right)} \qquad (5.3)$$

$$\int_0^M x f(x;\theta)dx = \mu - \frac{\left\{\theta^5 M + \theta^4(2M^2 + 1) + \theta^3(M^3 + 4M) + \theta^2(3M^2 + M) + 6\theta M + 6\right\}e^{-\theta M}}{\theta\left(\theta^4 + 2\theta^2 + 2\right)} \qquad (5.4)$$

Using expressions from (5.1), (5.2), (5.3), and (5.4), the mean deviation about mean, $\delta_1(X)$ and the mean deviation about median, $\delta_2(X)$ of Adya distribution are obtained as

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$$\delta_1(X) = \frac{2\left\{2\theta^3\mu + \theta^2(\mu^2+\mu) + 4\theta\mu + \left(\theta^4+6\right)\right\}e^{-\theta\mu}}{\theta\left(\theta^4+2\theta^2+2\right)} \tag{5.5}$$

$$\delta_2(X) = \frac{2\left\{\theta^5 M + \theta^4\left(2M^2+1\right) + \theta^3(M^3+4M) + \theta^2(3M^2+M) + 6\theta M + 6\right\}e^{-\theta M}}{\theta\left(\theta^4+2\theta^2+2\right)} - \mu \tag{5.6}$$

Bonferroni and lorenz curves

The Bonferroni[8] and Lorenz curves are defined as

$$B(p) = \frac{1}{p\mu}\int_0^q x\,f(x)\,dx = \frac{1}{p\mu}\left[\int_0^\infty x\,f(x)\,dx - \int_q^\infty x\,f(x)\,dx\right] = \frac{1}{p\mu}\left[\mu - \int_q^\infty x\,f(x)\,dx\right] \tag{6.1}$$

and

$$L(p) = \frac{1}{\mu}\int_0^q x\,f(x)\,dx = \frac{1}{\mu}\left[\int_0^\infty x\,f(x)\,dx - \int_q^\infty x\,f(x)\,dx\right] = \frac{1}{\mu}\left[\mu - \int_q^\infty x\,f(x)\,dx\right] \tag{6.2}$$

respectively or equivalently

$$B(p) = \frac{1}{p\mu}\int_0^p F^{-1}(x)\,dx \tag{6.3}$$

and $$L(p) = \frac{1}{\mu}\int_0^p F^{-1}(x)\,dx \tag{6.4}$$

respectively, where $\mu = E(X)$ and $q = F^{-1}(p)$.

The Bonferroni and Gini indices are thus defined as

$$B = 1 - \int_0^1 B(p)\,dp \tag{6.5}$$

and $$G = 1 - 2\int_0^1 L(p)\,dp \tag{6.6}$$

respectively.

Using pdf (1.2), we have

$$\int_q^\infty x\,f(x)\,dx = \frac{\left\{\theta^5 q + \theta^4(2q^2+1) + \theta^3(q^3+4q) + \theta^2(3q^2+1) + 6\theta q + 6\right\}e^{-\theta q}}{\theta\left(\theta^4+2\theta^2+2\right)} \tag{6.7}$$

Now using equation (6.7) in (6.1) and (6.2), we have

$$B(p) = \frac{1}{p}\left[1 - \frac{\left\{\theta^5 q + \theta^4(2q^2+1) + \theta^3(q^3+4q) + \theta^2(3q^2+4) + 6\theta q + 6\right\}e^{-\theta q}}{\theta^4+4\theta^2+6}\right] \tag{6.8}$$

and

$$L(p) = 1 - \frac{\left\{\theta^5 q + \theta^4(2q^2+1) + \theta^3(q^3+4q) + \theta^2(3q^2+4) + 6\theta q + 6\right\}e^{-\theta q}}{\theta^4+4\theta^2+6} \tag{6.9}$$

Now using equations (6.8) and (6.9) in (6.5) and (6.6), the Bonferroni and Gini indices are obtained as

$$B = 1 - \frac{\left\{\theta^5 q + \theta^4(2q^2+1) + \theta^3(q^3+4q) + \theta^2(3q^2+4) + 6\theta q + 6\right\}e^{-\theta q}}{\theta^4+4\theta^2+6} \tag{6.10}$$

$$G = \frac{2\left\{\theta^5 q + \theta^4(2q^2+1) + \theta^3(q^3+4q) + \theta^2(3q^2+4) + 6\theta q + 6\right\}e^{-\theta q}}{\theta^4+4\theta^2+6} - 1 \tag{6.11}$$

Stress-strength reliability

Suppose X and Y be independent strength and stress random variables having Adya distribution with parameter θ_1 and θ_2 respectively. Then $R = P(Y < X)$ is known as stress-strength parameter and is a measure of the component reliability.

Thus,

$$R = P(Y<X) = \int_0^\infty P(Y<X \mid X=x) f_X(x)\,dx = \int_0^\infty f(x;\theta_1)\,F(x;\theta_2)\,dx$$

$$= 1 - \frac{\theta_1^3\left[\begin{array}{l} 24\theta_2^2 + 12\theta_2\left(\theta_2^2+\theta_1\theta_2+1\right)(\theta_1+\theta_2) + 2\left(\theta_2^4+4\theta_1\theta_2^3+\theta_1^2\theta_2^2+2\theta_2^2+4\theta_1\theta_2+2\right)(\theta_1+\theta_2)^2 \\ +2\theta_1\left(\theta_2^4+\theta_1\theta_2^3+2\theta_2+\theta_1\theta_2+2\right)(\theta_1+\theta_2)^3 + \theta_1^2(\theta_2^4+2\theta_2^2+2)(\theta_1+\theta_2)^4 \end{array}\right]}{\left(\theta_1^4+2\theta_1^2+2\right)\left(\theta_2^4+2\theta_2^2+2\right)(\theta_1+\theta_2)^5}$$

Estimation of parameter

Estimation using method of moment

Since Adya distribution has one parameter, equating the population mean to the corresponding sample mean, the moment estimate $\tilde{\theta}$ of θ is the solution of the following fifth degree polynomial equation

$\bar{x}\theta^5 - \theta^4 + 2\bar{x}\theta^3 - 4\theta^2 + 2\theta\bar{x} - 6 = 0$, where $\bar{x}$ is the sample mean.

Estimation using maximum likelihood estimation

Taking $(x_1, x_2, x_3, \ldots, x_n)$ a random sample from (1.2), the natural log likelihood function of Adya distribution is

$$\ln L = n\ln\left(\frac{\theta^3}{\theta^4+2\theta^2+2}\right) + 2\sum_{i=1}^n \ln(\theta+x_i) - n\theta\bar{x}.$$

This gives, $\dfrac{d\ln L}{d\theta} = \dfrac{3n}{\theta} - \dfrac{4n\theta\left(\theta^2+1\right)}{\theta^4+2\theta^2+2} + 2\sum_{i=1}^n \dfrac{1}{\theta+x_i} - n\bar{x}$,

The MLE $\tilde{\theta}$ of θ is the solution of $\dfrac{d\ln L}{d\theta} = 0$ which is given by

$$\frac{3n}{\theta} - \frac{4n\theta\left(\theta^2+1\right)}{\theta^4+2\theta^2+2} + 2\sum_{i=1}^n \frac{1}{\theta+x_i} - n\bar{x} = 0.$$

This non-linear equation is not in compact form and its solution can be obtained analytically. We have to use non-linear optimization technique such as quasi-Newton algorithm available in R software to maximize the log-likelihood function.

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A simulation study

A simulation study has been conducted to examine the performance of the maximum likelihood estimate (MLE) of the parameter of Adya distribution. Random number $n = 20, 40, 60$ and 80 generated corresponding to the parameter $\theta = 0.1, 0.2, 0.3, 0.4$ and 0.5 using Acceptance-Rejection method. Its Bias and Mean Square Error have been calculated and presented in the Table 3.

Table 3 Average Bias and Average Mean Square Error of simulated MLE (θ) for fixed values θ =0.1,0.2,0.3 &0.4

n	θ	Bias (θ)	MSE(θ)
20	0.1	0.01200	0.00288
	0.2	0.021244	0.009028
	0.3	0.018753	0.007033
	0.4	0.023497	0.0110371
40	0.1	0.004884	0.000954
	0.2	0.007587	0.002302
	0.3	0.008900	0.003168
	0.4	0.010251	0.0042034
60	0.1	0.003484	0.000728
	0.2	0.005428	0.001768
	0.3	0.005854	0.002056
	0.4	0.006242	0.002338
80	0.1	0.0023048	0.000424
	0.2	0.0038444	0.001182
	0.3	0.004266	0.001456
	0.4	0.004735	0.001794

It is obvious from above table that as the sample size increases, the bias and the mean square decreases. Further for increasing sample size and increasing value of parameter the bias and the mean square decreases. This verifies the asymptotic property of the ML estimate of the parameter of Adya distribution

Data analysis

The data set considered for the goodness of fit of Adya distribution is the strength data of glass of the aircraft window reported by Fuller et al.[9] and are given as

18.83, 20.80, 21.657, 23.03, 23.23, 24.05, 24.321, 25.50, 25.52, 25.80, 26.69, 26.77, 26.78, 27.05, 27.67, 29.90, 31.11, 33.20, 33.73, 33.76, 33.89, 34.76, 35.75, 35.91, 36.98, 37.08, 37.09, 39.58, 44.045, 45.29, 45.381

The goodness of fit of the distributions are based on the values of $-2\ln L$, AIC (Akaike Information Criterion) and K-S (Kolmogorov-Smirnov) statistic. AIC and K-S are computed using $AIC = -2\ln L + 2k$, $K\text{-}S = \sup_x \left| F_n(x) - F_0(x) \right|$, where k = the number of parameters, n = the sample size and $F_n(x)$ is the empirical distribution function. The distribution having lower $-2\ln L$, AIC, and K-S are said to be best distribution. The MLE $(\hat{\theta})$ and standard error, S.E $(\hat{\theta})$ of θ, $-2\ln L$, AIC, K-S and p-value of the fitted distributions are presented in the Table 4.

Table 4 MLE's, S.E $(\hat{\theta})$ $-2\ln L$, AIC and K-S Statistics of the fitted distributions of the given data set

Distributions	MLE $(\hat{\theta})$	S.E $(\hat{\theta})$	$-2\ln L$	AIC	K-S	p-value
Adya	0.096970	0.01000	240.63	242.63	0.298	0.006
Shanker	0.647164	0.008200	252.35	254.35	0.358	0.0004
Aradhana	0.094319	0.009780	242.22	244.22	0.306	0.0044
Sujatha	0.095613	0.009904	241.50	243.50	0.303	0.0051
Devya	0.160873	0.012916	227.68	229.68	0.422	0.0000
Lindley	0.062992	0.008001	253.98	255.98	0.365	0.0003
Exponential	0.032449	0.005822	274.52	276.53	0.458	0.0000

Clearly Adya distribution gives better fit than Shanker, Aradhana, Sujatha, Devya, Lindley and exponential distributions.

Concluding remarks

Adya distribution, a one parameter lifetime distribution, for modeling lifetime data has been presented and studied. The statistical properties including coefficient of variation, skewness, kurtosis, index of dispersion, hazard rate function, mean residual life function, stochastic ordering, mean deviations, Bonferroni and Lorenz curves and stress-strength reliability have been discussed. Over-dispersed, equi-dispersed, and under-dispersed of Adya distribution are presented. Method of moment and method of maximum likelihood are explained for estimating parameter. The asymptotic property of the ML estimate of the parameter has been discussed with simulation study. Finally, the goodness of fit test has been presented with a real lifetime data.

NOTE: The paper is named Adya distribution in the name of my loving niece Adya Vedanshi, the daughter of my younger brother Dr. Ravi Shanker**.**

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None.

Conflicts of interest

All authors declare that there is no conflict of interest.

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