Research Article

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AHM as a measure of central tendency of sex ratio

Abstract

In some recent studies, four formulations of average namely Arithmetic-Geometric Mean (abbreviated as AGM), Arithmetic-Harmonic Mean (abbreviated as *AHM*), Geometric-Harmonic Mean (abbreviated as *GHM*) and Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) have recently been derived from the three Pythagorean means namely Arithmetic Mean (*AM*), Geometric Mean (*GM*) and Harmonic Mean (*HM*). Each of these four formulations has been found to be a measure of central tendency of data. in addition to the existing measures of central tendency namely *AM*, *GM* & *HM*. This paper focuses on the suitability of *AHM* as a measure of central tendency of numerical data of ratio type along with the evaluation of central tendency of sex ratio namely male-female ratio and female-male ratio of the states in India.

Keywords: *AHM*, sex ratio, central tendency, measure

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Introduction

Several research had already been done on developing definitions/ formulations of average,[1,2] a basic concept used in developing most of the measures used in analysis of data. Pythagoras[3] is the first mathematician to introduce the concept of average and develop its formulation. He had developed three formulations/definitions of average which were later named as Pythagorean means[4,5] as a mark of honor to him. The three Pythagorean means are Arithmetic Mean (*AM*), Geometric Mean (*GM*) & Harmonic Mean (*HM*). A number of definitions/formulations of average have already been developed in continuation to the three Pythagorean means.[6-19] The next attempt had been initiated towards the development of generalized formulation/ definition of average. Kolmogorov[20] formulated one generalized definition of average namely Generalized f - Mean.[7,8] It has been shown that the definitions/formulations of the existing means and also of some new means can be derived from this Generalized f - Mean.[9,10] In an study, Chakrabarty formulated one generalized definition of average namely Generalized f_H – Mean.[11] In another study, Chakrabarty formulated another generalized definition of average namely Generalized f_G – Mean[12,13] and developed one general method of defining average[15-17] as well as the different formulations of average from the first principles.[19]

In many real situations, observed numerical data

$$x_1, x_2, \dots\dots\dots, x_N$$

are found to be composed of some parameter μ and respective errors

$$\varepsilon_1, \varepsilon_2, \dots\dots, \varepsilon_Z$$

usually of random in nature i.e

$$xi = \mu + \varepsilon i \;\; , (i = 1\ ,\ 2\ ,\ \dots\dots\dots\dots\ , N) \qquad (1.1)^{21\text{-}29}$$

The statistical methods of estimation of the parameter developed so far namely least squares estimation, maximum likelihood estimation, minimum variance unbiased estimation, method of moment estimation and minimum chi-square estimation,[31-52] cannot provide appropriate value of the parameter μ.[21-23] Therefore, some methods have recently been developed for determining the value of parameter μ in the situation mentioned above.[21-30,53-60] These methods, however, involve huge computational tasks. Moreover, these methods may not be able to yield the appropriate value of the parameter if observed data used are of relatively small size (and/or of moderately large size too) In reality, of course, the appropriate value of the parameter is not perfectly attainable in practical situation. What one can expect is to obtain that value which is more and more close to the appropriate value of the parameter. Four methods have therefore been developed for determining such value of parameter. These four methods involve lighter load of computational work than respective load involved in the earlier methods and can be applied even if the observed data used are of small size.[61-64] The methods developed are based on the concepts of Arithmetic-Geometric Mean (abbreviated as *AGM*),[61,62,67,68,69] Arithmetic-Harmonic Mean (abbreviated as *AHM*),[63] Geometric-Harmonic Mean (abbreviated as *GHM*)[64] and Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*)[65,66] respectively. Each of these four formulations namely *AGM*, *AHM*, *GHM* & *AGHM* has been found to be a measure of parameter μ of the model described by equation (1.1). In other words, each of these four formulations can be regarded as a measure of the central tendency, in addition to the usual measures of central tendency namely *AM*, *GM* & *HM* of the observed values $x_1, x_2, \dots\dots\dots, x_N$ since the values can be expressed by the model (1.1) if μ is the central tendency of them and vice versa. However, for different types of data different measures are suitable. This paper focuses on the suitability of *AHM* as a measure of central tendency of numerical data of ratio type along with the evaluation of central tendency of sex ratio namely male-female ratio and female-male ratio of the states in India.

Four formulations of average from pythagorean means

Let

$$x_1, x_2, \dots\dots\dots, x_N$$

N positive numbers or values or observations (not all equal or identical)

and

$$a_0 = AM\left(x_1, x_2, \dots\dots\dots, x_N\right) = \frac{1}{N}\sum_{i=1}^{N} x_i, \qquad (2.1)$$

$$g_0 = GM(x_1, x_2, \ldots\ldots\ldots, x_N) = \left(\prod_{i=1}^{N} x_i\right)^{1/N} \tag{2.2}$$

$$\& \ h_0 = HM(x_1, x_2, \ldots\ldots\ldots, x_N) = \left(\frac{1}{N}\sum_{i=1}^{N} x_i^{-1}\right)^{-1} \tag{2.3}$$

i.e. a_0, g_0 & h_0 are respectively the Arithmetic Mean (AM), the Geometric Mean (GM) & the Harmonic Mean (HM) of $x_1, x_2, \ldots\ldots\ldots, x_N$

which satisfy the inequality [4 , 5] namely

$$AM > GM > HM \text{ i.e. } h_0 < g_0 < a_0 \tag{2.4}$$

Arithmetic-geometric mean (*AGM*)

The two sequences $\{a_n\}$ & $\{g_n\}$ respectively defined by

$$a_{n+1} = \tfrac{1}{2}(a_n + g_n) \tag{2.5}$$

$$\& \ g_{n+1} = (a_n g_n)^{1/2} \tag{2.6}$$

where the square root assumes the principal value,

converge to a common point M_{AG} which can be termed as the Arithmetic-Geometric Mean (abbreviated as AGM) of $x_1, x_2, \ldots\ldots\ldots, x_N$ [61 , 62, 66 , 67 , 68].

Thus,

$$AGM(x_1, x_2, \ldots\ldots\ldots, x_N) = \text{common converging point of } \{a_n\} \,\&\, \{g_n\} \tag{2.7}$$

Arithmetic-harmonic mean (*AHM*)

The two sequences $a'_{n+1} = ½(a'_n + h'_n)$ respectively defined by

$$a'_{n+1} = ½(a'_n + h'_n) \tag{2.8}$$

$$\& \ h'_{n+1} = ½(a'^{-1}_n + h'^{-1}_n)\}^{-1} \tag{2.9}$$

converge to common point M_{AH} which can be termed as the Arithmetic-Harmonic Mean (abbreviated as AHM) of $x_1, x_2, \ldots\ldots\ldots, x_N$. [63,66]

Thus,

$$AHM\ (x_1, x_2, \ldots\ldots\ldots, x_N) = \text{common converging point of } \{a'_n\} \ \&\ \{h'_n\} \tag{2.10}$$

Geometric-harmonic mean (*GHM*)

The two sequences $\{g''n\}$ & $\{h''_n\}$ defined respectively by

$$g''_{n+1} = \left(g''_n . h''_n\right)^{1/2} \tag{2.11}$$

$$\& \ h''_{n+1} = \left\{\tfrac{1}{2}\left(g''^{-1}_n + h''^{-1}_n\right)\right\}^{-1} \tag{2.12}$$

where the square root takes the principal value,

converge to common point M_{GH} which can be termed as the Geometric-Harmonic Mean (abbreviated as GHM) of $x_1, x_2, \ldots\ldots\ldots, x_N$. [64,66]

Thus,

$$GHM(x_1, x_2, \ldots\ldots\ldots, x_N) = \text{common converging point of } \{g''_n\} \ \&\ \{h''_n\} \tag{2.13}$$

Arithmetic-geometric-harmonic mean (*AGHM*)

The three sequences $a'''_n = 1/3\left(a'''_{n-1} + g'''_{n-1} + h'''_{n-1}\right)$, defined respectively by

$$a'''_n = 1/3\left(a'''_{n-1} + g'''_{n-1} + h'''_{n-1}\right), \tag{2.14}$$

$$g'''n = \left(a'''_{n-1}\ g'''_{n-1}\ h'''_{n-1}\right)^{1/3} \tag{2.15}$$

$$\& \ h'''n = \left\{1/3\left(a'''^{-1}_{n-1} + g'''^{-1}_{n-1} + h'''^{-1}_{n-1}\right)\right\}^{-1} \tag{2.16}$$

converges to a common limit M_{AGH} which can be termed as the Arithmetic-Geometric-Harmonic Mean (abbreviated as AGHM) of $x_1, x_2, \ldots\ldots\ldots, x_N$. [65,66]

Thus,

$$AGHM(x_1, x_2, \ldots\ldots\ldots, x_N) = \text{common converging point of } \{a'''_n\}, \{g'''_n\} \,\&\, \{h'''_n\}, \tag{2.17}$$

AHM as measure of central tendency of sex ratio

Let

$$x_1, x_2, \ldots\ldots\ldots, x_N$$

be observed values (which are strictly positive and not all identical) on the Ratio Male/Female.

Also let μ be the central tendency of the observed values.

Then x_i can be expressed as

$$x_i = \mu + \varepsilon_i \tag{3.1}$$

where ε_i is the error associated to x_i for ($i = 1, 2, \ldots\ldots\ldots\ldots, N$) which is random in nature

i.e. each ε_i assumes either positive real value or negative real value with equal probability.

Again since μ is the central tendency of the observed values

$$x_1, x_2, \ldots\ldots\ldots, x_N$$

therefore, μ^{-1} will be the central tendency of reciprocals

$$x_1^{-1}, x_2^{-1}, \ldots\ldots\ldots, x_N^{-1}$$

of the observed values.

Accordingly, the reciprocals can be expressed as

$$x_i^{-1} = \mu^{-1} + \varepsilon'_i \quad , \left(I = 1, 2, \ldots\ldots\ldots\ldots, N\right) \tag{3.2}$$

where

$x_1^{-1}, x_2^{-1}, \ldots\ldots\ldots, x_N^{-1}$ are the random errors, which assume positive and negative values in random order, associated to are the random errors associated to

$x_1^{-1}, x_2^{-1}, \ldots\ldots\ldots, x_N^{-1}$ respectively.

Let us now write

$$AM(x_1, x_2, \ldots\ldots\ldots, x_N) = a_0 \tag{3.3}$$

$$\& \ HM(x_1, x_2, \ldots\ldots\ldots, x_N) = h_0 \tag{3.4}$$

and then define the two sequences $\{a'_n\}$ & $\{h'_n\}$ respectively by

$$a'_{n+1} = ½\,(a'_n + h'_n) \quad (3.5)$$

$$\& \; h'_{n+1} = ½(a'^{-1}_n + h'^{-1}_n)\}^{-1} \quad (3.6)$$

Then, both of $\{a'_n\}$ & $\{h'_n\}$ converges to some common real number C.

Let us now search the relation between C and μ.

Equation (3.1) together with (3.3) & (3.4) implies that $a_0 = \mu + \delta_0$ & $h_0 = \mu + e_0$

By inequality (2.4), $h_0 < a_0$ i.e. $e_0 < \delta_0$

Therefore, $a'_1 = \mu + \delta_1$ where $\delta_1 = ½(\delta_0 +)e_0$

$$\text{Since } ½(\delta_0 + e_0) < \delta_0$$

Therefore, $\delta_1 < \delta_0$

At the n^{th} step, one can obtain that

$$\delta_{n+1} = ½(\delta_n + e_n) < \delta_n$$

which implies, $\delta_{n+1} < \delta_n$ since $½(\delta_n + e_n) < \delta_n$

This implies, h'_n becomes more and more smaller as n becomes more and more larger.

This means. a'_n becomes more and more closer to μ as n becomes more and more larger.

Since $\{h'_n\}$ converges to the same point to which $\{a'_n\}$ converges,

Therefore, h'_n also becomes more and more closer to μ as n becomes more and more larger.

Accordingly, the $AHM(x_1, x_2, \ldots\ldots\ldots, x_N)$ can be regarded as the value of μ i.e. the value of the central tendency of $x_1, x_2, \ldots\ldots\ldots, x_N$.

Example

Data on the population of India (state-wise) in 2011, published in "Census Report" by Register General of India, have been shown in the following table (Table 1):

From the data in the observed values on the two ratios

Male/Female & Female/Male have been computed which have been shown in Table 2.

Table 1 Population of India in 2011 (State-wise)

State	Number of Persons	Number of Males	Number of Females
Jammu & Kashmir	1,25,41,302	66,40,662	59,00,640
Himachal Pradesh	68,64,602	34,81,873	33,82,729
Punjab	2,77,43,338	1,46,39,465	1,31,03,873
Chandigarh	10,55,450	5,80,663	4,74,787
Uttarakhand	1,00,86,292	51,37,773	49,48,519
Haryana	2,53,51,462	1,34,94,734	1,18,56,728
Delhi	1,67,87,941	89,87,326	78,00,615
Rajasthan	6,85,48,437	3,55,50,997	3,29,97,440
Uttar Pradesh	19,98,12,341	10,44,80,510	9,53,31,831
Bihar	10,40,99,452	5,42,78,157	4,98,21,295
Sikkim	6,10,577	3,23,070	2,87,507
Arunachal Pradesh	13,83,727	7,13,912	6,69,815
Nagaland	19,78,502	10,24,649	9,53,853
Manipur	28,55,794	14,38,586	14,17,208
Mizoram	10,97,206	5,55,339	5,41,867
Tripura	36,73,917	18,74,376	17,99,541
Meghalaya	29,66,889	14,91,832	14,75,057
Assam	3,12,05,576	1,59,39,443	1,52,66,133
West Bengal	9,12,76,115	4,68,09,027	4,44,67,088
Jharkhand	3,29,88,134	1,69,30,315	1,60,57,819
Odisha	4,19,74,218	2,12,12,136	2,07,62,082
Chhattisgarh	2,55,45,198	1,28,32,895	1,27,12,303
Madhya Pradesh	7,26,26,809	3,76,12,306	3,50,14,503
Gujarat	6,04,39,692	3,14,91,260	2,89,48,432
Daman & Diu	2,43,247	1,50,301	92,946

Table Continued...

State	Number of Persons	Number of Males	Number of Females
Dadra & Nagar Haveli	3,43,709	1,93,760	1,49,949
Maharashtra	11,23,74,333	5,82,43,056	5,41,31,277
Andhra Pradesh	8,45,80,777	4,24,42,146	4,21,38,631
Karnataka	6,10,95,297	3,09,66,657	3,01,28,640
Goa	14,58,545	7,39,140	7,19,405
Lakshadweep	64,473	33,123	31,350
Kerala	3,34,06,061	1,60,27,412	1,73,78,649
Tamil Nadu	7,21,47,030	3,61,37,975	3,60,09,055
Pondicherry	12,47,953	6,12,511	6,35,442
Andaman & Nicobar	3,80,581	2,02,871	1,77,710
India	**1,21,08,54,977**	**62,32,70,258**	**58,75,84,719**

Table 2 Central tendency of the ratio Male/Female

State	Value of the Ratio Male/Female	Value of the Ratio Female/Male
Jammu & Kashmir	1.1254138534125111852273651671683	0.88856201384741461016988968870875
Himachal Pradesh	1.0293088804926436613751796256809	0.97152567023553127871119940330966
Punjab	1.11718611741734676457868601138	0.89510600284914783429585712319405
Chandigarh	1.2229968385823537712700642603947	0.81766360177934533455722165869015
Uttarakhand	1.0382445737805593956494862402266	0.96316419584905755859591305415792
Haryana	1.1381499179200197558719403869263	0.878618874592118673847146598073
Delhi	1.1521304409972803426396508480421	0.86795727672502366109786158864161
Rajasthan	1.077386518469311558714857879884	0.92817200035205763708961523638845
Uttar Pradesh	1.0959666766496911194331303675474	0.91243650131493423988837726768373
Bihar	1.0894569681498644304609103396449	0.91788847952225054362107394324387
Sikkim	1.1236943796151050235298618816238	0.88992168879809329247531494722506
Arunachal Pradesh	1.0658345961198241305435082821376	0.93823188292114434272011116216004
Nagaland	1.0742210801874083323111632505218	0.93090707159232088256563955071444
Manipur	1.0150845888535768920299631387912	0.98513957455445833617176866728857
Mizoram	1.0248621894302476437945104610541	0.97574094381990099740878994632108
Tripura	1.0415856043291039214999824955364	0.96007471286444128606000076825568
Meghalaya	1.0113724418785172369610123540989	0.98875543626896326127874988604615
Assam	1.0441048168517855831597956077024	0.95775824788858682201128358123932
West Bengal	1.0526667948213744061675457587868	0.94996821873695430584361430969287
Jharkhand	1.0543346515488809532602154750904	0.9484654597389357492757813425208
Odisha	1.0216767277963741786589610810708	0.97878318336258074151514020087369
Chhattisgarh	1.0094862433659738915914763831542	0.99060289981333128651017560729672
Madhya Pradesh	1.0741921997293521487367677330733	0.93093209972289388478334723747063
Gujarat	1.0878399216924771607664276945985	0.9192528974705997791133158851059
Daman & Diu	1.6170787338884943945947109074086	0.61839907918110990612171575704752
Dadra & Nagar Haveli	1.29217267204182755470193199021	0.77389037985136251032204789430223
Maharashtra	1.0759593940486569345112623151307	0.92940310343605596519523288750508

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Table Continued...

State	Value of the Ratio Male/Female	Value of the Ratio Female/Male
Andhra Pradesh	1.0072027731513157131279371653056	0.99284873578258743089946488568226
Karnataka	1.02781463086286005607953097l1955	0.97293808627776643762353811714322
Goa	1.0274323920462048498411882041409	0.97330005141109938577265470682144
Lakshadweep	1.0565550239234449760765550239234	0.94647223983334842858436735802916
Kerala	0.92224729321594561234305382426448	1.0843078720382305015931455433978
Tamil Nadu	1.0035802105886977594941050244168	0.99643256159206485698216349975338
Pondicherry	0.96391330758747454527714567183158	1.0374376949964980220763382208646
Andaman & Nicobar	1.1415846041303246862866467840864	0.87597537351321775906857066806002
India	1.0607325851848778252519531570732	0.94274467850509882664736426425148

Central tendency of the ratio Male/Female

From the observed values on the ratio Male/Female in Table 3 it has been obtained that

AM of Male/Female = 1.0835068016450523020161865887443 & *HM* of Male/Female = 1.0740468088974845410059550737324

The following table (Table 3) shows the values of a'_n $\&h'_n$, in this case, for $n = 1, 2, 3,$ ………… :

Table 3 Values of $\{a'_n\}$ & $\{h'_n\}$ of the Ratio Male / Female

n	Value of a'_n	Value of h'_n
0	1.0835068016450523020161865887443	1.0740468088974845410059550737324
1	1.0787768052712684215110708312384	1.0787560661660274789282541031017
2	1.0787664357186479502196624671701	1.0787664356189714883012948072843
3	1.0787664356688097192604786372272	1.078766435668809719258176146917
4	1.0787664356688097192593273920721	1.0787664356688097192593273920721

The digits in a'_n $\&h'_n$, , which are agreed, have been underlined in the above table

It is seen that the values of a'_n $\&h'_n$ become identical at $n = 4$ which is

1.0787664356688097192593273920721

Therefore, this value can be regarded as the *AHM* and consequently the central tendency of the Ratio Male/Female.

Central tendency of the ratio female/male

From the observed values on Female/Male in Table 3 it has been obtained that *AM* of Female/Male = 0.931058117500955072681326 5197974 & *HM* of Female/Male = 0.9229291394218599224261917 9784686.

The computed values of $\{a'_n\}$ & $\{h'_n\}$, in this case, have been shown in the following table Table 4:

Table 4 Values of $\{a'_n\}$ & $\{h'_n\}$ of the Ratio Female/Male

n	Value of a'_n	Value of h'_n
0	0.9310581175009550726813265197974	0.92292913942185992242619179784686
1	0.926993628461407497553759158822l3	0.9269758073344381333499624625797l
2	0.92698471789792281545186081070092	0.92698471781227076522756233102558
3	0.92698471785509679033971157086325	0.92698471785509679033773303940364
4	0.92698471785509679033872230513345	0.92698471785509679033872230513345

The digits in a'_n & h'_n , which are agreed, have been underlined in the above table

It is seen that the values of a'_n & h'_n become identical at $n = 4$ which is

0.92698471785509679033872230513345

Therefore, this value can be regarded as the *AHM* and consequently the central tendency of the Ratio Female/Male.

Results and discussions

If μ is the central tendency of $x_1, x_2, \ldots\ldots\ldots, x_N$ then the central tendency of $x_1^{-1}, x_2^{-1}, \ldots\ldots\ldots, x_N^{-1}$ should logically be μ^{-1}. Similarly, the central tendency of $-x_1^{-1}, -x_2^{-1}, \ldots\ldots\ldots, -x_N^{-1}$ should logically be $-\mu$.

It is seen in the in the above example that the AHM of the ratio Male/Female is 1.0787664356688097192593273920721 and of the ratio Female/Male is 0.92698471785509679033872230513345.

These two values are reciprocals each other i.e.

$(1.0787664356688097192593273920721)^{-1} = 0.92698471785509679033872230513345$

& $(0.92698471785509679033872230513345)^{-1} = 1.0787664356688097192593273920721$

Moreover, it is found that the AHM of the additive inverses of the observed values of the ratio Male/Female, is

−1.0787664356688097192593273920721 and of the ratio Female/Male is

−0.92698471785509679033872230513345

Thus, *AHM* can logically be regarded as an acceptable measure of central tendency of data of ratio type.

It is to be noted that each of *AM* & *HM* does not satisfy these two properties of central tendency and therefore cannot logically be regarded as acceptable measure of central tendency of data of ratio type.

Of course, *GM* satisfies the first property but not the second property of central tendency. Thus, is to be studied further on the acceptability of *GM* as a measure of central tendency of data of ratio type.

Regarding accuracy, it is to be noted that $a_0 = \mu + \delta_0$ & $\delta_{n+1} < \delta_n$

This means, δ_n becomes more and more smaller as *n* becomes more and more larger which means, a'_n becomes more and more closer to μ as *n* becomes more and more larger which further means, A*HM* ($x_1, x_2, \ldots\ldots\ldots, x_N$) becomes more and more closer to μ as *n* becomes more and more larger.

Since $\delta_n < \delta_0$ for all $n > 1$ therefore, the deviation of A*HM* ($x_1, x_2, \ldots\ldots\ldots, x_N$) from μ is more than that the deviation of a_0. But, a_0 = AM ($x_1, x_2, \ldots\ldots\ldots, x_N$)

Hence, A*HM* ($x_1, x_2, \ldots\ldots\ldots, x_N$) is more accurate measure of central tendency than AM ($x_1, x_2, \ldots\ldots\ldots, x_N$) in the case of data of ratio type.

Similarly, A*HM* ($x_1, x_2, \ldots\ldots\ldots, x_N$) can be shown to be more accurate measure of central tendency than *HM* ($x_1, x_2, \ldots\ldots\ldots, x_N$) in the case of data of ratio type.

Therefore, *AHM* can be regarded as a measure of central tendency of data of ratio type which is more accurate than each of *AM* and *HM*. However, it is yet to be studied on the comparison of accuracy of A*HM* with that of *GM* as measure of central tendency of data of ratio type.

It is to be noted that the GM of *AM* of the Ratio Male/Female & *HM* of the Ratio Male/Female is found to be 1.0787664356688097192593273920721 which is nothing but the AHM of the observed values of the Ratio Male/Female.

Similarly, the GM of *AM* of the Ratio Female/Male & *HM* of the Ratio Female/Male is found to be 0.92698471785509679033872230513345 which is nothing but the *AHM* of the observed values of the Ratio Female/Male.

Thus, *AHM* of the observed values can be regarded as the *GM* of *AM* of the observed values and *HM* of observed values. In general, *AHM*($x_1, x_2, \ldots\ldots\ldots, x_N$) can be defined as the *GM* of *AM*($x_1, x_2, \ldots\ldots\ldots, x_N$) and *HM*($x_1, x_2, \ldots\ldots\ldots, x_N$) in the instant case. However, it is to be established for general case.

On the whole, the two values 1.0787664356688097192593273920721 and 0.92698471785509679033872230513345 can be regarded as the respective values of central tendency of the Ratio Male/Female and the Ratio Female/Male of the states in India which are very close to the respective actual values while the overall values of these two ratios in India (combing the states) are 1.06073258518487782525195315707 32 and 0.94274467850509882664736426425148 respectively.

However, it is yet to be determined the size of errors or discrepancies in values obtained by AHM. It is also to be assessed the performance of AHM by applying it in the data with various sample sizes.

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Conflicts of interest

None.

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