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A review on mathematically transformed Lindley random variables

Abstract

Lindley distribution has been used quite successfully to analyze lifetime data. Recent advances in the theory of the distribution have built numerous specialized applications. This article reviews recent developments of Lindley distribution, along with a brief review of sum and difference of two Lindley random variables. An extensive set of references to the distribution is given.

Keywords: Lindley distribution, probability density function, random variable

AMS subject classification: 60E05, 62E15, 62F10

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Introduction

In statistical theory, modelling lifetime data utilizing lifetime distributions has gained the attention of many statisticians. The one-parameter Lindley distribution is irrefutably one of the most eminent distributions in Statistics. The classical one-parameter Lindley distribution was proposed by Lindley,[1] Lindley[2] to encapsulate a difference between fiducial distribution and posterior distribution. The survival function (SF) of the Lindley distribution with parameter $\eta > 0$, is given by

$$\bar{F}(x) = \left[1 + \frac{\eta x}{1+\eta}\right] e^{-\eta x}; \qquad x > 0 \tag{1}$$

It has used for analysing copious lifetime data especially in applications of modelling stress-strength reliability. There is, of course, a comprehensive literature on the Lindley distribution. For example, the dominance of Lindley distribution over the exponential distribution for waiting times before service of bank customers was pointed out by Ghitany et al.[3] Shanker et al.[4] also studied a comparison study of the goodness-of-fit of exponential and Lindley distributions on modelling of lifetime data. The parameter of Lindley distribution with progressive Type-II censoring scheme was estimated by Krishna and Kumar[5] and they showed that it may fit better than exponential, lognormal and gamma distributions in some real life situations. Furthermore, the inverse and discrete versions of Lindley distribution are developed by Sharma et al.[6] and Deniz and Ojeda,[7] respectively.

But in some sense, the Lindley distribution does not provide enough tractability for analyzing different types of lifetime data. In this regard, by using various approaches, researchers have focused on discovering modified, extended and generalized Lindley distributions. We mention: generalized Lindley distribution Zakerzadeh H & Dolati A,[8] quasi Lindley distribution Shanker R & Mishra A,[9] power Lindley distribution Ghitany et al.,[10] two-parameter Lindley distribution Shanker and Mishra,[11] transmuted Lindley distribution Merovci F,[12] transmuted Lindley-geometric distribution Merovci F & Elbatal I,[13] beta-Lindley distribution Merovci F & Sharma V.K.,[14] Wrapped Lindley distribution Joshi S & Jose KK,[15] Marshall-Olkin modified Lindley distribution Gillariose J, et al.[16] Marshall-Olkin two-parameter Lindley distribution Tomy GJ,[17] etc. For more details, the reader can refer a review study by tomy[18] which highlights a survey of developments on the Lindley distribution. The review about Lindley distribution show that the literature on the Lindley distribution continues to grow. Motivated by previous review, in this paper, we give a recent expository review of the Lindley distribution, especially with a discussion of recent innovations regarding sum and difference of Lindley random variables. The rest of the paper is organized as follows. In Section 2, we discuss some recent contributions. Conclusions are presented in Section 3.

Recent developments of lindley distribution

Distribution of sum and difference

Zakerzadeh and Dolati[8] showed that the distribution of a sum of n independent random variables from Lindley distribution can be written as a mixture of gamma distribution. Hassan[19] discussed sum of n independent random variables having Lindley distribution with both same and different parameters. In addition, he showed the convolution of Lindly distribution with the same parameters is useful to obtain the uniformly minimum variance unbiased estimator (UMVUE) of the stress-strength parameter $R = P(Y < X)$ model-reliability. Recently, Chesneau et al.[20] specified the distributions of sum and differences of two independent and identically distributed random variables with the common Lindley distribution. Let X and Y be two independent random variables following the Lindley distribution with parameter $\eta > 0$. Then, the random variable $Z = X + Y$ has the SF given by (1). This result is a particular case of Hassan,[19] Theorem 2. Since X and Y are independent, the probability density function (PDF) of Z is given by the following convolution product: for $x > 0$,

$$f(x)_S = \int_{+\infty}^{+\infty} f_x(x-t) f_x(t)\,dt = \int_0^x \frac{\eta^2}{1+\eta}(1+x-t)e^{-\eta(x-t)} \frac{\eta^2}{1+\eta}(1+t)e^{-\eta t}dt$$

$$= \frac{\eta^4}{(1+\eta)^2} e^{-\eta x} \int_0^x (1+x-t)(1+t)dt = \frac{\eta^4}{(1+\eta)^2} x\left(\frac{x^2}{6} + x + 1\right) e^{-\eta x}.$$

The corresponding SF is given by

$$\bar{G}(x)_S = \frac{1}{6(1+\theta)^2}\left[\theta^3 x\left(x^2+6x+6\right) + 3\theta^2\left(x^2+4x+2\right) + 6\theta(x+2) + 6\right] e^{-\theta x}, \quad x > 0.$$

In addition, the difference of two independent random variables following the Lindley distribution with the same parameter. Then, its PDF given by

$$f(x)_D = \frac{\eta}{4(1+\eta)^2}\left[\eta(2\eta+1)|x| + 2\eta^2 + 2\eta + 1\right]e^{-\eta|x|}, \quad x \in \mathbb{R}, \eta > 0$$

The corresponding SF is given by

$$\bar{G}(x)_D = \begin{cases} 1 - \frac{1}{4(1+\eta)^2}\left[-\eta(2\eta+1)x + 2(1+\eta)^2\right]e^{\eta x} & \text{if} x < 0, \\ \frac{1}{4(1+\eta)^2}\left[\eta(2\eta+1)x + 2(1+\eta)^2\right]e^{-\eta x} & \text{if} x \geq 0. \end{cases}$$

Moreover, Chesneau et al.[20] provided several statistical and mathematical peculiarities of these models. As a continuation, Hamedani[21] showed that the assumption of independence can be replaced with a much weaker assumption of â€ sub-independenceâ€.

Modified lindley distribution

In the recent past, Chesneau et al.[22] introduced a new modified Lindley distribution, as a simple one-parameter alternative to the exponential and Lindley distributions. The SF is given as

$$\bar{G}(x) = \left[1 + \frac{\eta x}{1+\eta}e^{-\eta x}\right]e^{-\eta x}, \qquad x > 0$$

One of the eminent properties of the modified Lindley distribution is that, its PDF can be expressed as a linear combination of exponential and gamma PDFs. In addition, modified Lindley distribution is a strong one-parameter competitor to the Lindley and exponential distributions. Furthermore, Chesneau et al.,[23] Chesneau et al.[24] studied two generalizations for the modified Lindley distribution, such as the inverse modified Lindley and the wrapped modified Lindley distributions, respectively and presented their statistical properties.

Transformed lindley distributions

Maurya et al.[25] proposed exponential transformed Lindley distribution and provided an application to yarn data. Hassan et al.[26] introduced a new distribution called a new generalization of the power Lindley distribution namely the alpha power transformed power Lindley, which includes the alpha power transformed Lindley, power Lindley, Lindley, and gamma as sub-models. They proved that the model provides a better fit than the power Lindley distribution. In addition to this, Alpha-Power transformed Lindley distribution Dey et al.[28] and Alpha-Power transformed inverse Lindley distribution Dey et al.[27] are introduced in the literature.

The one-parameter unit-Lindley distribution and its associated regression model for proportion data has been proposed by Mazucheli et al.[28] Moreover, Algarni[29] suggested an extension of the generalized Lindley distribution using the Marshall-Olkin method. An extension of Lindley distribution has also been proposed by Maurya et al.[30]

Conclusions

The literature on theory and application of Lindley distribution is flourishing and rapidly growing. Several methods may be found in the literature. This paper has tried to review some recent techniques to find new Lindley distribution. These new innovations may have great promise elsewhere in Statistics.

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Conflicts of interest

None.

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