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A note on size– biased new quasi Poisson– Lindley distribution

Abstract

In this paper some important properties including coefficients of variation, skewness, kurtosis and index of dispersion of size–biased new quasi Poisson–Lindley distribution (SBNQPLD) have been discussed and their behaviors have been explained graphically for varying values of parameters. Some applications of SBNQPLD have also been discussed.

Keywords: Size–biased new Quasi Poisson–Lindley distribution, moments based measures, maximum likelihood estimation, goodness of fit

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Introduction

The size– biased version of Poisson–Lindley distribution (SBPLD) proposed by Sankaran[1] has been introduced by Ghitany and Mutairi[2] and is defined by its probability mass function (pmf)

$$P_1(x,\theta)=\frac{\theta^3}{\theta+2}\cdot\frac{x(x+\theta+2)}{(\theta+1)^{x+2}}\ ;\theta>0,x=1,2,3,...$$

Shanker et al.[3] have proposed a simple method of deriving moments of SBPLD and the applications of SBPLD to model thunderstorms events. The Poisson –Lindley distribution (PLD), a Poisson mixture of Lindley distribution of Lindley,[4] is defined by its pmf

$$P_2(x;\theta)=\frac{\theta^2(x+\theta+2)}{(\theta+1)^{x+3}}\ ;x=\ 0,\ 1,\ 2,...,>\ 0.$$

The Lindley distribution is defined by its probability density function (pdf) $f_1(x,\theta)=\frac{\theta^2}{\theta+1}(1+x)\,e^{-\theta x}\ ;x>0,\ \theta>0$

The size–biased quasi Poisson–Lindley distribution (SBQPLD), size–biased version of quasi Poisson–Lindley distribution (QPLD) of Shanker and Mishra,[5] suggested by Shanker and Mishra[6] with parameters θ and α is defined by its pmf

$$P_3(x;\theta,\alpha)=\frac{\theta^2}{\alpha+2}\frac{x(\theta x+\theta\alpha+\theta+\alpha)}{(\theta+1)^{x+2}};x=1,2,3,...,\theta>0,\alpha>-2$$

The QPLD, a Poisson mixture of quasi Lindley distribution proposed by Shanker and Mishra,[7] is defined by its pmf

$$P_4(x;\theta,\alpha)=\frac{\theta(\theta x+\theta\alpha+\theta+\alpha)}{(\alpha+1)(\theta+1)^{x+2}};x=0,1,2,...;\ \theta>0,\alpha>-1$$

The QLD is defined by its pdf

$$f_2(x;\theta,\alpha)=\frac{\theta}{\alpha+1}(\alpha+x\theta)\,e^{-\theta x};x>0,\ \theta>0,\ \alpha>-1$$

Shanker and Amanuel[8] proposed a new quasi Lindley distribution (NQLD) having pdf

$$f_3(x;\theta,\alpha)=\frac{\theta^2}{\theta^2+\alpha}(\theta+\alpha x)e^{-\theta x}$$

where $\theta+\alpha x>0$ and $\theta^2+\alpha>0$ or $\theta+\alpha x<0$ and $\theta^2+\alpha<0$ for $x>0$ $\theta>0$. Lindley distribution is a particular case of NQLD at $\alpha=\theta$. A new quasi Poisson–Lindley distribution (NQPLD), a Poisson mixture of NQLD, has been suggested by Shanker and Tekie[9] and defined by its pmf

$$P_5(x;\theta,\alpha)=\frac{\theta^2}{(\theta+1)^{x+2}}\left[1+\frac{\theta+\alpha x}{\theta^2+\alpha}\right]$$

where $\theta+\alpha x>0$ and $\theta^2+\alpha>0$ or $\theta+\alpha x<0$ and $\theta^2+\alpha<0$ for $x=0,1,2,...;\ \theta>0$.

It can be seen that the PLD is a particular case of it at $\alpha=\theta$. Shanker et al.[10] derived the pmf of size biased new quasi Poisson–Lindley distribution (SBNQPLD) having pmf

$$P_6(x;\theta,\alpha)=\frac{\theta^3}{\theta^2+2\alpha}\frac{x(\theta^2+\theta+\alpha+\alpha x)}{(\theta+1)^{x+2}};x=1,2,3,...,\theta>0,\theta^2+2\alpha>0$$

Shanker et al.[10] discussed various statistical properties, parameters estimation and applications of SBNQPLD. Shanker et al.[10] have shown that SBNQPLD can also be obtained from the size–biased Poisson distribution when its parameter λ follows a SBNQLD with pdf

$$f_4(\lambda;\theta,\alpha)=\frac{\theta^3}{\theta^2+2\alpha}\lambda(\theta+\alpha x)e^{-\theta x}$$

where $\theta+\alpha x>0$ and $\theta^2+\alpha>0$ or $\theta+\alpha x<0$ and $\theta^2+\alpha<0$ for $x>0$ $\theta>0$. That is

$$P(X=x)=\int_0^\infty\frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!}\cdot\frac{\theta^3}{\theta^2+2\alpha}\lambda(\theta+\alpha\lambda)e^{-\theta\lambda}d\lambda$$

$$= \frac{\theta^3}{\theta^2+2\alpha} \frac{x\left(\theta^2+\theta+\alpha+\alpha x\right)}{(\theta+1)^{x+2}} \quad ; x=1,2,3,...$$

The r th factorial moment about origin $\mu_{(r)}{}'$ of SBNQPLD obtained by Shanker et al.[10] as

$$\mu_{(r)}{}' = \frac{r!\left\{r\theta^3+(r+1)\theta^2+r(r+1)\alpha\theta+(r+1)(r+2)\alpha\right\}}{\theta^r\left(\theta^2+2\alpha\right)} ; r=1,2,3,...$$

Thus, the first four moments about origin obtained by Shanker et al.[10] are

$$\mu_1{}' = 1+\frac{2\left(\theta^2+3\alpha\right)}{\theta\left(\theta^2+2\alpha\right)}$$

$$\mu_2{}' = 1+\frac{6\left(\theta^2+3\alpha\right)}{\theta\left(\theta^2+2\alpha\right)}+\frac{6\left(\theta^2+4\alpha\right)}{\theta^2\left(\theta^2+2\alpha\right)}$$

$$\mu_3{}' = 1+\frac{14\left(\theta^2+3\alpha\right)}{\theta\left(\theta^2+2\alpha\right)}+\frac{36\left(\theta^2+4\alpha\right)}{\theta^2\left(\theta^2+2\alpha\right)}+\frac{24\left(\theta^2+5\alpha\right)}{\theta^3\left(\theta^2+2\alpha\right)}$$

$$\mu_4{}' = 1+\frac{30\left(\theta^2+3\alpha\right)}{\theta\left(\theta^2+2\alpha\right)}+\frac{126\left(\theta^2+4\alpha\right)}{\theta^2\left(\theta^2+2\alpha\right)}+\frac{240\left(\theta^2+5\alpha\right)}{\theta^3\left(\theta^2+2\alpha\right)}+\frac{120\left(\theta^2+6\alpha\right)}{\theta^4\left(\theta^2+2\alpha\right)}.$$

It has been observed that the central moments (moments about the mean) has not been given by et al.[10] and hence many important characteristics including coefficient of variation, skewness, kurtosis and index of dispersion of SBNQPLD has not been studied by Shanker et al.[10]

The main purpose of this paper is to derive expressions for coefficients of variation, skewness, kurtosis and index of dispersion of SBNQPLD and study their behaviour graphically. The goodness of fit of the distribution has been presented with a number of count datasets using maximum likelihood estimates from various fields of knowledge.

Moments based measures

Using the relationship between moments about the mean and the moments about the origin, the moments about mean of SBNQPLD can be obtained as

$$\mu_2 = \frac{2\left(\theta^5+\theta^4+5\alpha\theta^3+6\alpha\theta^2+6\alpha^2\theta+6\alpha^2\right)}{\theta^2\left(\theta^2+2\alpha\right)^2}$$

$$\mu_3 = \frac{2\left\{\begin{array}{l}\theta^8+3\theta^7+(7\alpha+2)\theta^6+24\alpha\theta^5+\left(16\alpha^2+18\alpha\right)\theta^4+54\alpha^2\theta^3\\+\left(12\alpha^3+36\alpha^2\right)\theta^2+36\alpha^3\theta+24\alpha^3\end{array}\right\}}{\theta^3\left(\theta^2+2\alpha\right)^3}$$

$$\mu_4 = \frac{2\left\{\begin{array}{l}\theta^{11}+13\theta^{10}+(9\alpha+24)\theta^9+(130\alpha+12)\theta^8+\left(30\alpha^2+264\alpha\right)\theta^7\\+\left(460\alpha^2+144\alpha\right)\theta^6+\left(44\alpha^3+936\alpha^2\right)\theta^5+\left(696\alpha^3+504\alpha^2\right)\theta^4\\+\left(24\alpha^4+1368\alpha^3\right)\theta^3+\left(384\alpha^4+720\alpha^3\right)\theta^2+720\alpha^4\theta+360\alpha^4\end{array}\right\}}{\theta^4\left(\theta^2+2\alpha\right)^4}.$$

The coefficient of variation (C.V), coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, coefficient of Kurtosis $\left(\beta_2\right)$ and Index of dispersion (γ) of SBNQPLD are obtained as

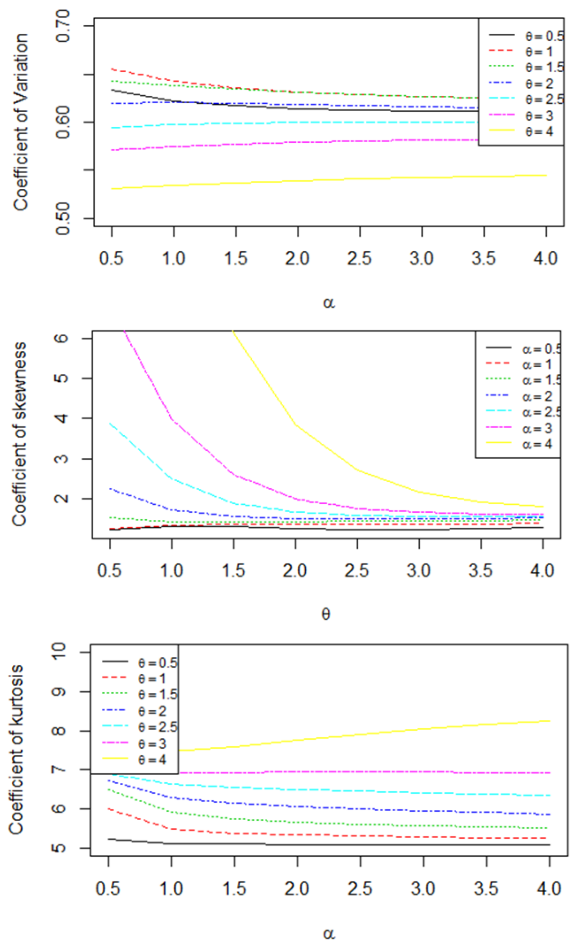
$$C.V. = \frac{\sigma}{\mu_1{}'} = \frac{\sqrt{2\left(\theta^5+\theta^4+5\alpha\theta^3+6\alpha\theta^2+6\alpha^2\theta+6\alpha^2\right)}}{\theta^3+2\theta^2+2\alpha\theta+6\alpha}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{(\mu_2)^{3/2}} = \frac{\left\{\begin{array}{l}\theta^8+3\theta^7+(7\alpha+2)\theta^6+24\alpha\theta^5+\left(16\alpha^2+18\alpha\right)\theta^4+54\alpha^2\theta^3\\+\left(12\alpha^3+36\alpha^2\right)\theta^2+36\alpha^3\theta+24\alpha^3\end{array}\right\}}{\sqrt{2}\left(\theta^5+\theta^4+5\alpha\theta^3+6\alpha\theta^2+6\alpha^2\theta+6\alpha^2\right)^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2{}^2} = \frac{\left\{\begin{array}{l}\theta^{11}+13\theta^{10}+(9\alpha+24)\theta^9+(130\alpha+12)\theta^8+\left(30\alpha^2+264\alpha\right)\theta^7\\+\left(460\alpha^2+144\alpha\right)\theta^6+\left(44\alpha^3+936\alpha^2\right)\theta^5+\left(696\alpha^3+504\alpha^2\right)\theta^4\\+\left(24\alpha^4+1368\alpha^3\right)\theta^3+\left(384\alpha^4+720\alpha^3\right)\theta^2+720\alpha^4\theta+360\alpha^4\end{array}\right\}}{2\left(\theta^5+\theta^4+5\alpha\theta^3+6\alpha\theta^2+6\alpha^2\theta+6\alpha^2\right)^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1{}'} = \frac{2\left(\theta^5+\theta^4+5\alpha\theta^3+6\alpha\theta^2+6\alpha^2\theta+6\alpha^2\right)}{\theta\left(\theta^2+2\alpha\right)\left(\theta^3+2\theta^2+2\alpha\theta+6\alpha\right)}.$$

Shapes of coefficient of variation, skewness, kurtosis and index of dispersion of SBNQPLD for varying values of parameters have been shown in figure 1.



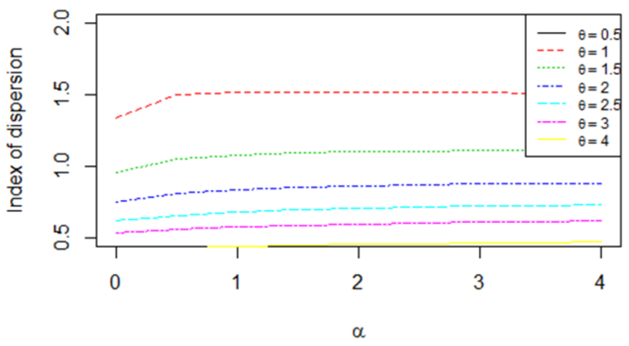


Figure 1 Behaviors of C.V, Skewness, Kurtosis and Index of dispersion of SBNQPLD for values of θ and α.

Maximum likelihood estimation of parameters

Suppose $(x_1, x_2, \ldots, x_n)$ as random samples of size n from the SBNQPLD and f_x, the observed frequency in the sample corresponding to $X = x$ $(x = 1, 2, \ldots, k)$ such that $\sum_{x=1}^{k} f_x = n$, where k being the largest observed value having non–zero frequency. The log likelihood function of SBNQPLD can be presented as

$$\log L = n\log\left(\frac{\theta^3}{\theta^2+2\alpha}\right) - \sum_{x=1}^{k} f_x (x+2)\log(\theta+1) + \sum_{x=1}^{k} f_x \log\left[\alpha x^2 + x\left(\theta^2+\theta+\alpha\right)\right]$$

The two log likelihood equations are thus obtained as

$$\frac{\partial \log L}{\partial \theta} = \frac{3n}{\theta} - \frac{2n\theta}{\theta^2+2\alpha} - \sum_{x=1}^{k}\frac{(x+2) f_x}{\theta+1} + \sum_{x=1}^{k}\frac{(2\theta+1)x f_x}{\left[\alpha x^2 + x\left(\theta^2+\theta+\alpha\right)\right]} = 0$$

$$\frac{\partial \log L}{\partial \alpha} = \frac{-2n}{\theta^2+2\alpha} + \sum_{x=1}^{k}\frac{x(x+1) f_x}{\left[\alpha x^2 + x\left(\theta^2+\theta+\alpha\right)\right]} = 0 .$$

These two log likelihood equations seems difficult to solve directly as these cannot be expressed in closed forms. The (MLE's) $(\hat{\theta}, \hat{\alpha})$ of parameters (θ, α) can be computed directly by solving the log likelihood equation using Newton–Raphson iteration available in R–software till sufficiently close values of $\hat{\theta}$ and $\hat{\alpha}$ are obtained. The initial values of parameters θ and α are the MOME $(\tilde{\theta}, \tilde{\alpha})$ of the parameters (θ, α), given in Shanker et al.[10]

Goodness of fit

To test the goodness of fit of SBNQPLD along with SBPD, SBPLD and SBQPLD, several cont datasets have been considered from various fields of knowledge. The expected frequencies of SBPD, SBPLD and SBQPLD have also been given in the tables (Table 1–10). The estimates of the parameters have been obtained by the method of maximum likelihood. It is obvious from the goodness of fit of SBNQPLD that it provides better fit over SBPD and SBPLD and competing well with SBQPLD in majority of datasets. The following datasets have been considered for testing the goodness of fit of SBNQPLD.

Table 1 Pedestrians-Eugene, Spring, Morning, available in Coleman and James[11]

Group Size	Observed frequency	Expected frequency			
		SBPD	**SBPLD**	**SBQPLD**	**SBNQPLD**
1	1486	1452.4	1532.5	1485.4	1505.5
2	694	743.3	630.6	697.2	656.8
3	195	190.2	191.9	189.7	202.5
4	37	32.4	51.3	41.1	49.2
5	10	4.1	12.8	7.8	9.0
6	1	0.6	3.9	1.8	0.0
Total	2423	2423.0	2423.0	2423	
ML Estimate		$\hat{\theta} = 0.5118$	$\hat{\theta} = 4.5082$	$\hat{\theta} = 7.14063$ $\hat{\alpha} = -0.79104$	$\hat{\theta} = 2.69606$ $\hat{\alpha} = -1.39128$
χ^2		7.370	13.760	0.776	6.1
d.f.		2	3	2	2
p-value		0.0251	0.003	0.6804	0.04735
$-2\log L$		10445.34	4622.36	4607.8	4610.0
AIC		10447.34	4624.36	4611.8	4614.0

Citation: Shanker R, Shukla KK, Shanker R. A note on size– biased new quasi Poisson– Lindley distribution. *Biom Biostat Int J.* 2020;9(3):97–104. DOI: 10.15406/bbij.2020.09.00306

Table 2 Play Groups-Eugene, Spring, Public Playground A, available in Coleman and James[11]

Group Size	Observed frequency	Expected frequency			
		SBPD	**SBPLD**	**SBQPLD**	**SBNQPLD**
1	316	306.3	322.9	315.7	313.5
2	141	156.1	132.5	142.7	141.4
3	44	39.8	40.2	40.1	44.1
4	5	6.7	10.7	9.1	10.4
5	4	1.1	3.7	2.4	0.6
Total	510	510.0	510.0	510.0	
ML Estimate		$\hat{\theta}=0.5098$	$\hat{\theta}=4.5211$	$\hat{\theta}=6.5501$ $\hat{\alpha}=-0.5069$	$\hat{\theta}=2.4693$ $\hat{\alpha}=-1.2977$
χ^2		2.39	3.07	0.94	0.38
d.f.		2	2	1	1
p-value		0.3027	0.2154	0.3322	0.5376
$-2\log L$		916.63	972.78	971.07	970.24
AIC		918.63	974.78	975.07	974.24

Table 3 Play Groups-Eugene, Spring, Public Playground A, available in Coleman and James[11]

Group Size	Observed frequency	Expected frequency			
		SBPD	**SBPLD**	**SBQPLD**	**SBNQPLD**
1	306	292.2	309.4	304.4	306.4
2	132	155.2	131.2	137.9	134.4
3	47	41.2	41.1	41.3	41.6
4	10	7.3	11.3	10.3	11.0
5	2	1.1	4.0	3.1	3.6
Total	497	497.0	497.0		
ML Estimate		$\hat{\theta}=0.5312$	$\hat{\theta}=4.3548$	$\hat{\theta}=5.71547$ $\hat{\alpha}=4.9998$	$\hat{\theta}=4.9998$ $\hat{\alpha}=25.6948$
χ^2		6.479	0.932	1.19	1.2
d.f.		2	2	1	1
p-value		0.039	0.6281	0.2753	0.2733
$-2\log L$		2142.03	971.86	970.96	971.25
AIC		2144.03	973.86	974.96	975.25

Citation: Shanker R, Shukla KK, Shanker R. A note on size– biased new quasi Poisson– Lindley distribution. *Biom Biostat Int J.* 2020;9(3):97–104. DOI: 10.15406/bbij.2020.09.00306

Table 4 Play Groups-Eugene, Spring, Public Playground D, available in Coleman and James[11]

Group Size	Observed frequency	Expected frequency			
		SBPD	SBPLD	SBQPLD	SBNQPLD
1	305	296.5	314.4	304.3	310.1
2	144	159.0	134.4	148.2	138.8
3	50	42.6	42.5	42.3	43.1
4	5	7.6	11.8	9.6	11.3
5	2	1.0	3.1	1.9	2.7
6	1	0.3	0.8	0.7	1.0
Total	507	507.0	507.0	507.0	507.0
ML Estimate		$\hat{\theta}=0.5365$	$\hat{\theta}=4.3179$	$\hat{\theta}=6.70804$ $\hat{\alpha}=-0.74907$	$\hat{\theta}=5.1516$ $\hat{\alpha}=48.6067$
χ^2		3.035	6.415	2.96	4.64
d.f.		2	2	1	1
p-value		0.219	0.040	0.0853	0.0312
$-2\log L$		2376.75	993.10	990.02	991.51
AIC		2378.75	995.1	994.02	995.51

Table 5 Play Groups-Eugene, Spring, Public Playground D, available in Coleman and James[11]

Group Size	Observed frequency	Expected frequency			
		SBPD	SBPLD	SBQPLD	SBNQPLD
1	276	292.3	319.6	276.0	313.7
2	229	200.7	166.5	228.3	173.1
3	61	68.9	63.8	61.9	65.2
4	12	15.8	21.4	12.2	20.7
5	3	3.3	9.7	2.6	8.3
Total	581	581.0	581.0	581.0	581.0
ML Estimate		$\hat{\theta}=0.6867$	$\hat{\theta}=3.4359$	$\hat{\theta}=8.6724$ $\hat{\alpha}=-1.4944$	$\hat{\theta}=4.1645$ $\hat{\alpha}=61.0287$
χ^2		6.68	37.86	0.017	29.6
d.f.		2	2	1	1
p-value		0.0354	0.00	0.8962	0.000
$-2\log L$		1146.7	1277.42	1238.11	1268.77
AIC		1148.7	1279.42	1242.11	1272.77

Table 6 Distribution of number of counts of sites with particles from Immunogold data, available in Mathews and Appleton[12]

No. of sites with particles	Observed Frequency	Expected Frequency			
		SBPD	SBPLD	SBQPLD	SBNQPLD
1	122	111.3	119.0	119.2	119.3
2	50	64.1	53.8	53.5	53.3
3	18	18.5 }	18.0	17.9	17.8
4	4	3.5 }	5.3 }	5.3	5.3
5	4	0.6 }	1.9 }	2.1	2.3
Total	198	198.0	198.0	198.0	198.0
ML estimate		$\hat{\theta}=0.575758$	$\hat{\theta}=4.050987$	$\hat{\theta}=3.7564$ $\hat{\alpha}=10.1281$	$\hat{\theta}=3.4795$ $\hat{\alpha}=0.0216$
χ^2		4.64	0.43	0.34	0.28
d.f.		1	2	1	1
p-value		0.0312	0.8065	0.5598	0.5967
$-2\log L$		393.95	409.28	409.17	409.13
AIC		395.95	411.28	413.17	413.13

Table 7 Distribution of snowshoe hares captured over 7 days, available in Keith and Meslow[13]

No. times hares caught	Observed Frequency	Expected Frequency			
		SBPD	SBPLD	SBQPLD	SBNQPLD
1	184	170.6	177.3	177.4	177.5
2	55	72.5	62.5	62.3	62.2
3	14	15.4 }	16.4 }	16.3	16.3
4	4	2.2 }	3.8 }	3.8	3.8
5	4	0.3 }	1.0 }	1.2	1.2
Total	261	261.0	261.0	261	261.0
ML estimate		$\hat{\theta}=0.425287$	$\hat{\theta}=5.351256$	$\hat{\theta}=4.9800$ $\hat{\alpha}=14.9193$	$\hat{\theta}=4.6959$ $\hat{\alpha}=-0.0302$
χ^2		6.22	1.18	3.2	3.19
d.f.		1	1	1	1
p-value		0.0126	0.2773	0.0736	0.07409
$-2\log L$		452.40	457.10	456.86	456.80
AIC		454.40	459.10	460.86	460.80

Citation: Shanker R, Shukla KK, Shanker R. A note on size– biased new quasi Poisson– Lindley distribution. *Biom Biostat Int J.* 2020;9(3):97–104. DOI: 10.15406/bbij.2020.09.00306

Table 8 Number of counts of pairs of running shoes owned by 60 members of an athletics club, reported by Simonoff[14]

Number of pairs of running shoes	Observed frequency	Expected Frequency			
		SBPD	SBPLD	SBQPLD	SBNQPLD
1	18	15.0	20.3	17.4	19.5
2	18	20.8	17.4	19.6	18.0
3	12	14.4	10.9	12.3	11.3
4	7	$\left.\begin{matrix}6.6\\3.2\end{matrix}\right\}$	5.9	6.1	6.0
5	5		5.5	4.6	5.2
Total	60	60.0	60.0	60.0	60
ML Estimate		$\hat{\theta}=1.383333$	$\hat{\theta}=1.818978$	$\hat{\theta}=2.5858$ $\hat{\alpha}=-0.7318$	$\hat{\theta}=2.08739$ $\hat{\alpha}=17.3228$
χ^2		1.87	0.64	0.31	0.33
d.f.		2	3	1	2
P-value		0.3926	0.8872	0.5777	0.8478
$-2\log L$		147.1	187.08	185.55	186.33
AIC		149.1	189.08	189.55	190.33

Table 9 The numbers of counts of flower heads as per the number of fly eggs reported by Finney and Varley[15]

Number of fly eggs	Observed Frequency	Expected Frequency			
		SBPD	SBPLD	SBQPLD	SBNQPLD
1	22	11.3	20.3	19.8	19.8
2	18	23.2	22.0	22.1	22.1
3	18	23.8	17.2	17.5	17.5
4	11	16.2	11.6	11.8	11.8
5	9	8.3	7.2	7.3	7.3
6	6	3.4	4.2	4.2	4.2
7	3	1.1	2.4	2.3	2.3
8	0	0.3	1.3	1.3	1.3
9	1	0.4	1.8	1.7	1.7
Total	88			88.0	88.0
ML estimate		$\hat{\theta}=2.0454$	$\hat{\theta}=1.2822$	$\hat{\theta}=1.3483$ $\hat{\alpha}=0.6925$	$\hat{\theta}=1.3465$ $\hat{\alpha}=2.5654$
χ^2		18.8	1.39	1.49	1.49
d.f.		4	4	3	3
p-value		0.0008	0.8459	0.6845	0.6845
$-2\log L$		206.59	329.92	329.86	329.86
AIC		208.59	331.92	333.86	333.86

Table 10 Number of households having at least one migrant according to the number of migrants, reported by Singh and Yadav[16]

x	Observed frequency	Expected Frequency			
		SBPD	SBPLD	SBQPLD	SBNQPLD
1	375	341.2	262.8	363.3	363.6
2	143	186.8	157.4	156.5	156.3
3	49	51.1	50.4	50.4	50.4
4	17	9.3	14.2	14.4	14.4
5	2	1.2	3.7	3.9	3.8
6	2	0.1	0.9	1.0	1.0
7	1	0.2	0.2	0.2	0.2
8	1	0.1	0.3	0.4	0.3
Total	590		590.0	590.0	590.0
ML Estimate		$\hat{\theta}=0.5474$	$\hat{\theta}=4.24$	$\hat{\theta}=3.8386$ $\hat{\alpha}=17.2968$	$\hat{\theta}=3.6534$ $\hat{\alpha}=0.00067$
χ^2		14.1	2.48	2.11	2.08
d.f.		2	3	2	2
P-value		0.0008	0.4789	0.3481	0.3534
$-2\log L$		1124.3	1190.4	1189.67	1189.57
AIC		1126.3	1192.4	1193.67	1193.57

Conclusion

In this paper expressions based ob central moments including coefficients of variation, skewness, kurtosis and index of dispersion of SBNQPLD have been derived and their behaviors have been explained graphically for varying values of the parameters. Some important applications of SBNQPLD have also been discussed and its goodness of fit has been compared with other discrete distributions. It has been observed that SBNQPLD provides much better fit over SBPD, SBPLD and competing well with SBQPLD in majority of datasets.

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None.

Conflicts of interest

None.

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