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A comparative study of one parameter lifetime distributions

Abstract

In this paper, a comparative study on some selected one parameter distributions has been carried out. The important properties of distributions have been compared using various datasets from engineering, biological sciences and other fields. The lifetime data have been taken from various fields of studies. Various proposed models have been applied on data to check goodness of fit and their behavior have been discussed with graphically.

Keywords: pranav distribution, akash distribution, shanker distribution, lindley distribution, exponential distribution, statistical properties, estimation of parameter, goodness of fit

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Introduction

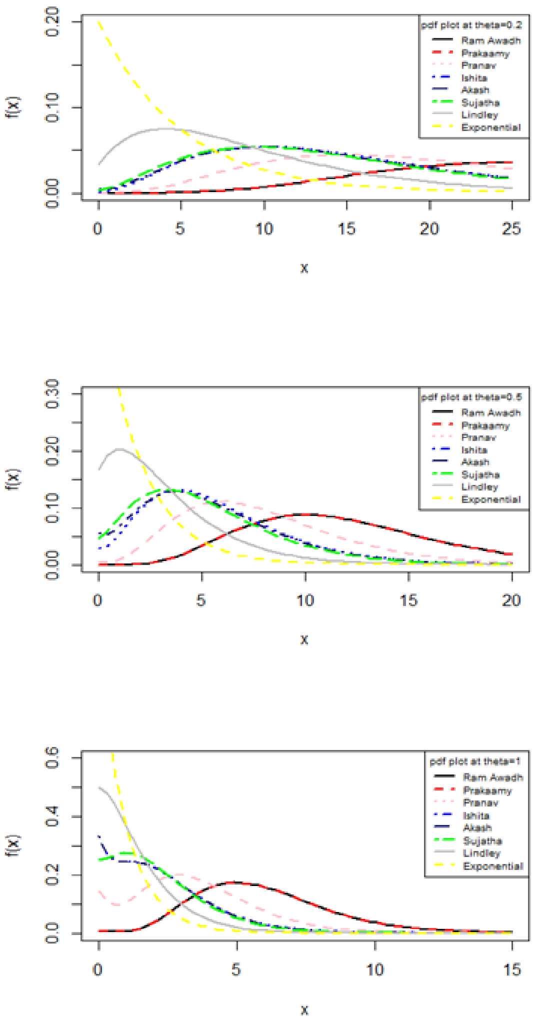
In the new era, uses of different life time distributions have been becoming more important because of increasing varieties of products and their survivors. Especially in reliability analysis, one can know failure rate as well time to survive of products, which can be calculated using different models. One parameter distribution can be applied easily way for any dataset, and its characteristics and mathematical properties can be calculated. Its applications are crucial in biostatistics as well as actuarial sciences and related field. The event may be failure of a piece of equipment, death of a person, development (or remission) of symptoms of disease, health code violation (or compliance). The modeling and statistical analysis of lifetime data are crucial for statisticians, research workers and policy makers in almost all applied sciences including engineering, medical science/biological science, insurance and finance, amongst others. Many statisticians have been proposed many distributions of one parameter and two parameters, but in this study, specially focused on some selected one parameter, most of them have been proposed recently. In this paper, author is tried to compare statistics of one parameter lifetime distributions using different lifetime data-sets from Engineering, medical sciences and social sciences. Different distributions have been proposed by different statisticians. Names of distributions of one parameter and their introducers are given in Table 1.

Table 1 Pdf of distributions and their introducers

Name of distribution	Probability distribution function (pdf)	Introducers
Exponential distribution	$f(x)=\theta e^{-\theta x}$	
Lindley distribution	$f(x)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta t}$	Lindley[1]
Akash distribution	$f(x)=\frac{\theta^3}{\theta^2+2}(1+x^2)e^{-\theta x}$	Shanker[2]
Pranav Distribution	$f(x)=\frac{\theta^4}{\theta^4+2}(\theta+x^3)e^{-\theta x}$	Shukla[3]
Ishita Distribution	$f(x)=\frac{\theta^3}{\theta^3+2}(\theta+x^2)e^{-\theta x}$	Shanker & Shukla[4]
Ram Awadh distribution	$f(x)=\frac{\theta^6}{\theta^6+120}(\theta+x^5)e^{-\theta x}$	Shukla[5]
Prakaamy distribution	$f(x)=\frac{\theta^6}{\theta^5+120}(1+x^5)e^{-\theta x}$	Shukla[6]
Sujatha distribution	$f(x)=\frac{\theta^3}{\theta^2+\theta+2}(1+x+x^2)e^{-\theta x}$	Shanker[7]

Characteristics of distributions

In this section, different distributions have been compared according to their behavior, moments and dispersion numerically as well as graphically. This section covers behavior of distributions (pdf), coefficient of variation, and coefficient of skewness, kurtosis and index of distribution respectively. Basically these distributions are continuous and known as lifetime distributions can be applied for biological, engineering and agricultural studies. Detailed studies including behavior, moments, stress & strength reliability, parameter estimation and etc. about above mentioned distributions can be shown in their paper. In statistical literature, exponential distribution was first studied by Epstein (1940) and widely used as lifetime model in different fields. The main reason for its wide use and applicability as lifetime model because it is simple to apply on any datasets, and another important use of this distribution is in the reliability field. Lindley distribution is introduced by Lindley[1-7] and further it's studied by Ghitani et al.[8] where nature and behavior of Lindley distribution including mathematical properties can be shown in their paper. They applied Lindley distribution in waiting time of customer in Bank and showed that its suitability over other distributions. Ram Awadh distribution has been introduced and studied by Shukla (2018b),[5] and he showed its superiority over other one parameter life time distribution in his paper. Similarly other one parameter distributions as above mentioned are also studied in detailed by different researchers and shows their superiority over other one parameter distributions. Let X be a continuous random variable representing the lifetimes of individuals in some population The expressions for probability density function, $f(x)$, cumulative distribution function, $F(x)$, have been presented in Figure 1 and 2. From the Figure 1, it is clear that pdf of almost all distributions are increasing as increased value of parameter. Especially pdf of exponential distribution is increasing more in comparison to other distributions as increased value of parameter. Pattern of almost all distributions are same except exponential distribution.



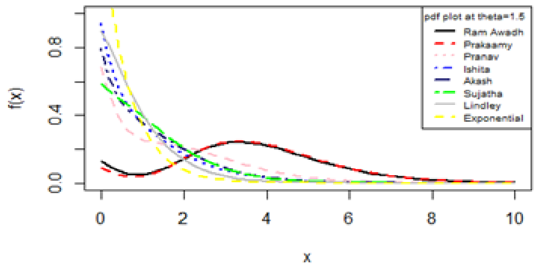
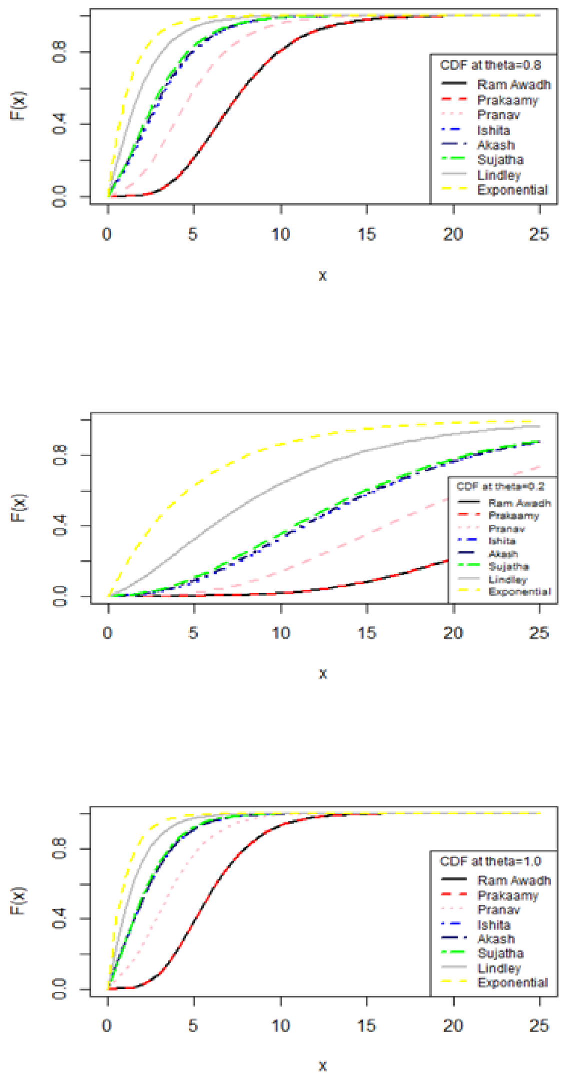


Figure 1 Pdf plots of distributions varying values of parameter.



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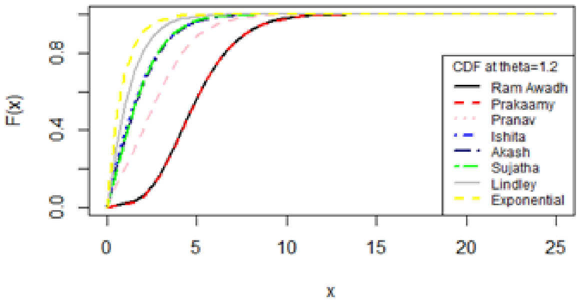


Figure 2 Cdf plots of different distributions for varying value of parameter.

Mathematical constants

Coefficient of variation (C.V.), coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, coefficient of Kurtosis $\left(\beta_2\right)$, and index of dispersion (γ) of above mentioned distributions in the Table 1 have been compared. The graphs of C.V, $\sqrt{\beta_1}$, β_2 and γ of distributions for varying values of the parameter θ are shown in Figure 2.

From the Figure 3, it is observed that coefficient of variation, coefficient of Skewness and coefficient of kurtosis of Ram Awadh, Prakaamy and Pranav distributions are increasing vastly up to certain points then decreasing as increased value of parameter. As we know that coefficient of variation, coefficient of skewness and coefficient of kurtosis of exponential distribution are independent from theta whereas value of index of dispersion is decreasing for all distributions as increased value of theta. The conditions under which Akash, Shanker and Lindley distributions are over-dispersed $\left(\mu < \sigma^2\right)$, equi-dispersed $\left(\mu = \sigma^2\right)$, and under-dispersed $\left(\mu = \sigma^2\right)$ are summarized in Table 2.

Parameter estimation

In this section, estimation of parameter using maximum likelihood method for Prakaamy, Sujatha, Ram Awadh, Pranav, Akash, Ishita, Lindley and Exponential distributions have been given respectively.

Table 2 Over-dispersion, equi-dispersion and under-dispersion of distributions for varying values of their parameter θ

Distribution	Over-dispersion $\left(\mu < \sigma^2\right)$	Equi-dispersion $\left(\mu = \sigma^2\right)$	Under-dispersion $\left(\mu > \sigma^2\right)$
Ram Awadh	$\theta < 1.044533$	$\theta = 1.044533$	$\theta > 1.044533$
Prakaamy	$\theta = 1.0421856$	$\theta = 1.0421856$	$\theta > 1.0421856$
Sujatha	$\theta < 1.364271$	$\theta = 1.364271$	$\theta > 1.364271$
Pranav	$\theta < 1.9853197$	$\theta = 1.9853197$	$\theta > 1.9853197$
Ishita	$\theta < 1.53565315$	$\theta = 1.53565315$	$\theta > 1.53565315$
Akash	$\theta < 1.515400063$	$\theta = 1.515400063$	$\theta > 1.515400063$
Lindley	$\theta < 1.170086487$	$\theta = 1.170086487$	$\theta > 1.170086487$
Exponential	$\theta < 1$	$\theta = 1$	$\theta > 1$

Prakaamy distribution

Let $(t_1, t_2, t_3, \ldots, t_n)$ be a random sample of size n from Prakaamy distribution. The maximum likelihood function, L of Prakaamy is given by

$$L = \left(\frac{\theta^6}{\left(\theta^5 + 120\right)}\right)^n \prod_{i=1}^{n} (1 + t^5) e^{-n\theta\bar{x}}$$

$$\ln L = n\left(\ln\theta^6 - \ln(\theta^5 + 120)\right) + \sum_{i=1}^{n} (1 + t^5) - n\theta\bar{x}$$

Sujatha distribution

Let $(t_1, t_2, t_3, \ldots, t_n)$ be a random sample of size n from Sujatha distribution. The maximum likelihood function, L of Sujatha is given by

$$L = \left(\frac{\theta^3}{\left(\theta^2 + \theta + 2\right)}\right)^n \prod_{i=1}^{n} (1 + t + t^2) e^{-n\theta\bar{x}}$$

The natural log likelihood function can be obtained as

$$\ln L = n\left(\ln\theta^3 - \ln(\theta^2 + \theta + 2)\right) + \sum_{i=1}^{n} (1 + t + t^2) - n\theta\bar{x}$$

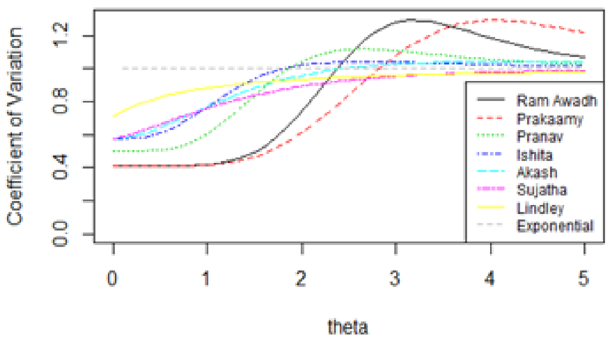
Ram awadh distribution

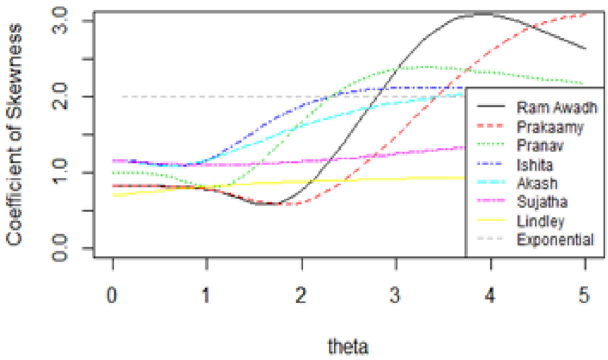
Let $(t_1, t_2, t_3, \ldots, t_n)$ be a random sample of size n from Ram Awadh distribution. The maximum likelihood function, L of Ram Awadh is given by

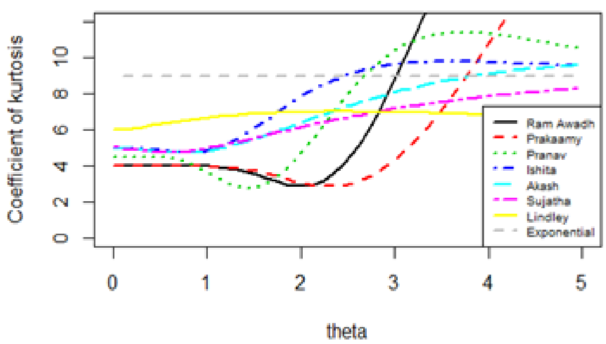
$$L = \left(\frac{\theta^6}{\left(\theta^6 + 120\right)}\right)^n \prod_{i=1}^{n} (\theta + t^5) e^{-n\theta\bar{x}}$$

The natural log likelihood function can be obtained as

$$\ln L = n\left(\ln\theta^6 - \ln(\theta^6 + 120)\right) + \sum_{i=1}^{n}(\theta + t^5) - n\theta\overline{x}$$







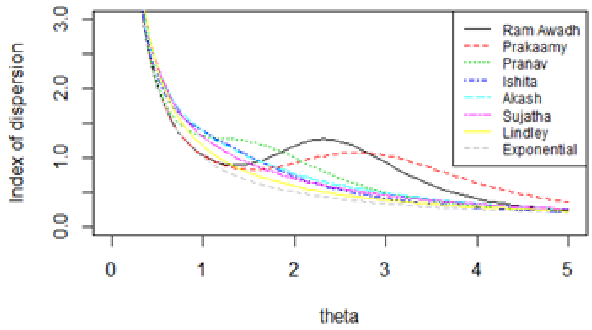


Figure 3 Graphs of C.V, and distributions for varying values of the parameter.

Pranav distribution

Let $(t_1, t_2, t_3, \ldots, t_n)$ be a random sample of size n from Pranav distribution. The maximum likelihood function, L of Pranav is given by

$$L = \left(\frac{\theta^4}{(\theta^4 + 6)}\right)^n \prod_{i=1}^{n}(\theta + t^3)e^{-n\theta\overline{x}}$$

The natural log likelihood function can be obtained as

$$\ln L = n\left(\ln\theta^4 - \ln(\theta^4 + 6)\right) + \sum_{i=1}^{n}(\theta + t^3) - n\theta\overline{x}$$

Akash distribution

Let $(t_1, t_2, t_3, \ldots, t_n)$ be a random sample of size n from Akash distribution. The maximum likelihood function, L of Akash is given by

$$L = \left(\frac{\theta^2}{(\theta^2 + 1)}\right)^n \prod_{i=1}^{n}(1 + t^2)e^{-n\theta\overline{x}}$$

The natural log likelihood function can be obtained as

$$\ln L = n\left(\ln\theta^2 - \ln(\theta^2 + 1)\right) + \sum_{i=1}^{n}(1 + t^2) - n\theta\overline{x}$$

Ishita distribution

Let $(t_1, t_2, t_3, \ldots, t_n)$ be a random sample of size n from Ishita distribution. The maximum likelihood function, L of Ishita is given by

$$L = \left(\frac{\theta^3}{(\theta^3 + 2)}\right)^n \prod_{i=1}^{n}(\theta + t^2)e^{-n\theta\overline{x}}$$

$$\ln L = n\left(\ln\theta^3 - \ln(\theta^3 + 2)\right) + \sum_{i=1}^{n}(\theta + t^2) - n\theta\overline{x}$$

Lindley distribution

Let $(t_1, t_2, t_3, \ldots, t_n)$ be a random sample of size n from Lindley distribution. The maximum likelihood function, L of Lindley is given by

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$$L = \left(\frac{\theta^2}{(\theta+1)}\right)^n \prod_{i=1}^{n}(1+t)e^{-n\theta\bar{x}}$$

The natural log likelihood function can be obtained as

$$\ln L = n\left(\ln\theta^2 - \ln(\theta+1)\right) + \sum_{i=1}^{n}(1+t) - n\theta\overline{x}$$

Exponential distribution

Let $(t_1, t_2, t_3, \ldots, t_n)$ be a random sample of size n from Exponential distribution. The maximum likelihood function, L of Exponential is given by

$$L = \theta^n e^{-n\theta\bar{x}}$$

The natural log likelihood function can be obtained as

$$\ln L = n\ln(\theta) - n\theta\overline{x}$$

Applications and goodness of Fit

In this section the goodness of fit test of Prakaamy, Sujatha, Ram Awadh, Pranav, Akash, Ishita, Lindley and exponential distributions for following eighteen real lifetime data- sets using maximum likelihood estimate have been discussed.

Data set 1: The data set represents the strength of 1.5cm glass fibers measured at the National Physical Laboratory, England. Unfortunately, the units of measurements are not given in the paper, and they are taken from Smith and Naylor[9]

0.55	0.93	1.25	1.36	1.49	1.52	1.58	1.61	1.64	1.68	1.73	1.81	2.00	0.74
1.04	1.27	1.39	1.49	1.53	1.59	1.61	1.66	1.68	1.76	1.82	2.01	0.77	1.11
	1.28	1.42	1.50	1.54	1.60	1.62	1.66	1.69	1.76	1.84	2.24	0.81	1.13
	1.29	1.48	1.50	1.55	1.61	1.62	1.66	1.70	1.77	1.84	0.84	1.24	1.30
	1.48	1.51	1.55	1.61	1.63	1.67	1.70	1.78	1.89				

Data set 2: The data is given by Birnbaum and Saunders[10] on the fatigue life of 6061 – T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second. The data set consists of 101 observations with maximum stress per cycle 31,000 psi. The data ($\times\ 10^{-3}$) are presented below (after subtracting 65).

5	25	31	32	34	35	38	39	39	40	42	43	43
43	44	44	47	47	48	49	49	49	51	54	55	55
55	56	56	56	58	59	59	59	59	59	63	63	64
64	65	65	65	66	66	66	66	66	67	67	67	68
69	69	69	69	71	71	72	73	73	73	74	74	76
76	77	77	77	77	77	77	79	79	80	81	83	83
84	86	86	87	90	91	92	92	92	92	93	94	97
98	98	99	101	103	105	109	136	147				

Data Set 3: The data set is from Lawless.[11] The data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life tests and they are:

17.88	28.92	33.00	41.52	42.12	45.60	48.80	51.84	51.96	54.12	55.56	67.80
68.44	68.64	68.88	84.12	93.12	98.64	105.12	105.84	127.92	128.04	173.40	

Data set 4: The data is from Picciotto[12] and arose in test on the cycle at which the Yarn failed. The data are the number of cycles until failure of the yarn and they are:

86	146	251	653	98	249	400	292	131	169	175	176	76
264	15	364	195	262	88	264	157	220	42	321	180	198
38	20	61	121	282	224	149	180	325	250	196	90	229
166	38	337	65	151	341	40	40	135	597	246	211	180
93	315	353	571	124	279	81	186	497	182	423	185	229
400	338	290	398	71	246	185	188	568	55	55	61	244
20	284	393	396	203	829	239	236	286	194	277	143	198
264	105	203	124	137	135	350	193	188				

Data set 5: This data represents the survival times (in days) of 72 guinna pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal.[13]

12	15	22	24	24	32	32	33	34	38	38	43	44
48	52	53	54	54	55	56	57	58	58	59	60	60
60	60	61	62	63	65	65	67	68	70	70	72	73
75	76	76	81	83	84	85	87	91	95	96	98	99
109	110	121	127	129	131	143	146	146	175	175	211	233
258	258	263	297	341	341	376						

Data set 6: This data is related with behavioral sciences, collected by N. Balakrishnan, Victor Leiva and Antonio Sanhueza:[14] The scale "General Rating of Affective Symptoms for Preschoolers (GRASP)" measures behavioral and emotional problems of children, which can be classified with depressive condition or not according to this scale. A study conducted by the authors in a city located at the south part of Chile has allowed collecting real data corresponding to the scores of the GRASP scale of children with frequency in parenthesis, which are:

19(16)	20(15)	21(14)	22(9)	23(12)	24(10)	25(6)	26(9)	27(8)	28(5)	29(6)	30(4)
31(3)	32(4)	33 34	35(4)	36(2)	37(2)	39	42 44				

Data set 7: The data set reported by Efron[15] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using radiotherapy (RT)

6.53	7	10.42	14.48	16.10	22.70	34	41.55	42	45.28	49.40	53.62	63
64	83	84	91	108	112	129	133	133	139	140	140	146
149	154	157	160	160	165	146	149	154	157	160	160	165
173	176	218	225	241	248	273	277	297	405	417	420	440
523	583	594	1101	1146	1417							

Data set 8: The data set reported by Efron[15] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using a combination of radiotherapy and chemotherapy (RT+CT).

12.20	23.56	23.74	25.87	31.98	37	41.35	47.38	55.46	58.36	63.47	68.46	78.26
74.47	81.43	84	92	94	110	112	119	127	130	133	140	146
155	159	173	179	194	195	209	249	281	319	339	432	469
519	633	725	817	1776								

Data set 9: This data set represents remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee and Wang.[16]

0.08	2.09	3.48	4.87	6.94	8.66	13.11	23.63	0.20	2.23	3.52	4.98	6.97
9.02	13.29	0.40	2.26	3.57	5.06	7.09	9.22	13.80	25.74	0.50	2.46	3.64
5.09	7.26	9.47	14.24	25.82	0.51	2.54	3.70	5.17	7.28	9.74	14.76	6.31
0.81	2.62	3.82	5.32	7.32	10.06	14.77	32.15	2.64	3.88	5.32	7.39	10.34
14.83	34.26	0.90	2.69	4.18	5.34	7.59	10.66	15.96	36.66	1.05	2.69	4.23
5.41	7.62	10.75	16.62	43.01	1.19	2.75	4.26	5.41	7.63	17.12	46.12	1.26
2.83	4.33	5.49	7.66	11.25	17.14	79.05	1.35	2.87	5.62	7.87	11.64	17.36
1.40	3.02	4.34	5.71	7.93	11.79	18.10	1.46	4.40	5.85	8.26	11.98	19.13
1.76	3.25	4.50	6.25	8.37	12.02	2.02	3.31	4.51	6.54	8.53	12.03	20.28
2.02	3.36	6.76	12.07	21.73	2.07	3.36	6.93	8.65	12.63	22.69		

Data Set 10: This data set is given by Linhart and Zucchini,[17] which represents the failure times of the air conditioning system of an airplane:

23	261	87	7	120	14	62	47	225	71	246	21	42
20	5	12	120	11	3	14	71	11	14	11	16	90 1
	16	52	95									

Data set 11: This data set used by Bhaumik et al.,[18] is vinyl chloride data obtained from clean upgradient monitoring wells in mg/l:

5.1	1.2	1.3	0.6	0.5	2.4	0.5	1.1	8	0.8	0.4	0.6	0.9
0.4	2	0.5	5.3	3.2	2.7	2.9	2.5	2.3	1	0.2	0.1	0.1
1.8	0.9	2	4	6.8	1.2	0.4	0.2					

Data set 12: This data set represents the waiting times (in minutes) before service of 100 Bank customers and examined and analyzed by Ghitany et al.,[8] for fitting the Lindley (1958)[1] distribution.

0.8,	0.8,	1.3,	1.5,	1.8,	1.9,	1.9,	2.1,	2.6,	2.7,	2.9,	3.1,	3.2,
3.3,	3.5,	3.6,	4.0,	4.1,	4.2,	4.2,	4.3,	4.3,	4.4,	4.4,	4.6,	4.7,
4.7,	4.8,	4.9,	4.9,	5.0,	5.3,	5.5,	5.7,	5.7,	6.1,	6.2,	6.2,	6.2,
6.3,	6.7,	6.9,	7.1,	7.1,	7.1,	7.1,	7.4,	7.6,	7.7,	8.0,	8.2,	8.6,
8.6,	8.6,	8.8,	8.8,	8.9,	8.9,	9.5,	9.6,	9.7,	9.8,	10.7,	10.9,	11.0,
11.0,	11.1,	11.2,	11.2,	11.5,	11.9,	12.4,	12.5,	12.9,	13.0,	13.1,	13.3,	13.6,
13.7,	13.9,	14.1,	15.4,	15.4,	17.3,	17.3,	18.1,	18.2,	18.4,	18.9,	19.0,	19.9,
20.6,	21.3,	21.4,	21.9,	23.0,	27.0,	31.6,	33.1,	38.5				

Data set 13: This data is for the times between successive failures of air conditioning equipment in a Boeing 720 airplane, Proschan:[19]

74	57	48	29	502	12	70	21	29	386	59	27	153
26	326											

Data set 14: This data set represents the lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by Gross and Clark.[20]

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2	1.7	2.7	4.1	1.8	1.5
1.2	1.4	3	1.7	2.3	1.6	2						

Data Set 15: This data set is the strength data of glass of the aircraft window reported by Fuller et al:[21]

18.83	20.8	21.657	23.03	23.23	24.05	24.321	25.5	25.52	25.8	26.69	26.77	26.78
27.05	27.67	29.9	31.11	33.2	33.73	33.76	33.89	34.76	35.75	35.91	36.98	37.08
37.09	39.58	44.045	45.29	45.381								

Data set 16: The following data represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20mm (Bader and Priest:[22]

1.312	1.314	1.479	1.552	1.700	1.803	1.861	1.865	1.944	1.958	1.966
1.997	2.006	2.021	2.027	2.055	2.063	2.098	2.140	2.179	2.224	2.240
2.253	2.270	2.272	2.274	2.301	2.301	2.359	2.382	2.382	2.426	2.434
2.435	2.478	2.490	2.511	2.514	2.535	2.554	2.566	2.570	2.586	2.629
2.633	2.642	2.648	2.684	2.697	2.726	2.770	2.773	2.800	2.809	2.818
2.821	2.848	2.880	2.954	3.012	3.067	3.084	3.090	3.096	3.128	3.233
3.433	3.585	3.858								

Data set 17: The first set of data represents the failure times (in minutes) for a sample of 15 electronic components in an accelerated life test Lawless[11] and the data are

1.4, 5.1, 6.3, 10.8, 12.1, 18.5, 19.7, 22.2, 23.0, 30.6, 37.3, 46.3, 53.9, 59.8, and 66.2

Data set 18: The following data set represents the number of cycles to failure for 25 100-cm specimens of yarn, tested at a particular strain level.[11]

15	20	38	42	61	76	86	98	121	146	149	157
	175	176	180	180	198	220	224	251	264	282	321
325	653										

Goodness of Fit

In order to compare the goodness of fit of all distributions, $-2\ln L$, AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion), K-S Statistics (Kolmogorov-Smirnov Statistics) for all eighteen real lifetime data- sets have been computed and presented in Table 3. The formulae for computing AIC, BIC, and K-S Statistics are as follows:

$AIC = -2\ln L + 2k$, $BIC = -2\ln L + k\ln n$ and

$$D = \sup_x \left| F_n(x) - F_0(x) \right|$$

Where k = the number of parameters, n = the sample size and $F_n(x)$ is the empirical distribution function. The best distribution is the distribution which corresponds to lower values of $-2\ln L$, AIC, BIC, and K-S statistics.

Table 3 MLE's, -2ln L, AIC, BIC, K-S Statistics of the fitted distributions of datasets 1-18

	Model	Parameter estimate	-2ln L	AIC	BIC	K-S statistic
	Prakaamy	2.49738	186.05	188.05	190.29	0.467
	Sujatha	**1.3500**	**154.80**	**156.80**	**158.72**	**0.430**
	Ram Awadh	2.0794	212.42	214.42	214.62	0.468
Data 1	Pranav	1.56071	180.96	182.96	184.87	0.488
	Akash	1.35544	163.73	165.73	169.93	0.355
	Ishita	1.25202	168.28	170.28	172.19	0.499
	Lindley	0.99611	162.56	164.56	166.70	0.371
	Exponential	0.66364	177.66	179.66	181.80	0.402
	Prakaamy	0.087813	918.72	920.72	922.95	0.116
	Sujatha	0.04357	951.78	953.78	955.69	0.195
	Ram Awadh	**0.08781**	**918.70**	**920.70**	**920.91**	**0.116**
Data 2	Pranav	0.05854	934.06	936.06	937.97	0.161
	Akash	0.043876	950.97	952.97	955.58	0.184
	Ishita	0.043906	950.90	952.90	954.83	0.194
	Lindley	0.028859	983.11	985.11	987.71	0.242
	Exponential	0.014635	1044.87	1046.87	1049.48	0.357
	Prakaamy	0.083070	228.29	230.29	232.53	0.161
	Sujatha	0.041229	227.17	229.17	231.08	0.121
	Ram Awadh	0.083070	228.29	230.29	230.49	0.161
Data 3	**Pranav**	**0.055384**	**226.05**	**228.05**	**229.96**	**0.122**
	Akash	0.041510	227.06	229.06	230.20	0.107
	Ishita	0.041533	227.03	229.03	230.95	0.118
	Lindley	0.027321	231.47	233.47	234.61	0.149
	Exponential	0.013845	242.87	244.87	246.01	0.263
	Prakaamy	0.027162	1329.86	1331.86	1334.10	0.969
	Sujatha	0.013491	1255.53	1257.53	1259.44	0.962
	Ram Awadh	0.027041	1327.50	1329.50	1329.76	0.969
Data 4	Pranav	0.01803	1273.63	1275.63	1277.54	0.967
	Akash	0.013514	1255.83	1257.83	1260.43	0.110
	Ishita	0.013514	1255.84	1257.84	1259.75	0.962
	Lindley	**0.008970**	**1251.34**	**1253.34**	**1255.95**	**0.098**

Table Continued

	Exponential	0.004505	1280.52	1282.52	1285.12	0.190
	Prakaamy	0.033935	873.53	875.53	877.77	0.168
	Sujatha	**0.016919**	**851.58**	**853.58**	**855.49**	**0.096**
	Ram Awadh	0.033935	873.54	875.54	875.73	0.168
Data 5	Pranav	0.022622	854.52	856.54	858.44	0.107
	Akash	0.016966	851.62	853.62	855.53	0.095
	Ishita	0.016985	851.63	853.63	855.54	0.095
	Lindley	0.01127	858.55	860.55	862.46	0.162
	Exponential	0.005684	889.22	891.22	893.13	0.296
	Prakaamy	0.24035	899.93	901.93	904.53	0.308
	Sujatha	0.11745	985.69	987.69	989.60	0.403
	Ram Awadh	**0.240359**	**899.92**	**901.92**	**902.12**	**0.308**
Data 6	Pranav	0.16022	945.03	947.03	948.94	0.362
	Akash	0.119610	981.28	983.28	986.18	0.393
	Ishita	0.120089	980.02	982.02	983.93	0.399
	Lindley	0.077247	1041.64	1043.64	1046.54	0.448
	Exponential	0.040060	1130.26	1132.26	1135.16	0.525
	Prakaamy	0.026533	955.97	957.97	960.20	0.400
	Sujatha	0.013257	802.84	804.84	806.75	0.297
	Ram Awadh	0.026534	955.97	957.97	958.16	0.400
Data 7	Pranav	0.017704	851.016	853.16	855.07	0.339
	Akash	0.013263	803.96	805.96	810.01	0.298
	Ishita	0.013269	804.08	806.08	807.99	0.298
	Lindley	0.008804	763.75	765.75	767.81	0.245
	Exponential	**0.004421**	**744.87**	**746.87**	**748.93**	**0.166**
	Prakaamy	0.026860	726.69	728.69	730.93	0.393
	Sujatha	0.013415	609.38	611.38	613.29	0.278
	Ram Awadh	0.026860	726.69	728.69	728.89	0.393
Data 8	Pranav	0.01791	646.17	648.17	650.08	0.327
	Akash	0.013423	609.93	611.93	613.71	0.280
	Ishita	0.013448	609.95	611.95	613.86	0.279
	Lindley	0.008910	579.16	581.16	582.95	0.219

Table Continued

	Exponential	**0.004475**	**564.02**	**566.02**	**567.80**		**0.145**
	Prakaamy	0.65098	1116.77	1118.77	1121.00		0.957
	Sujatha	0.303635	873.22	875.22	877.13		0.922
	Ram Awadh	0.65728	1123.19	1125.19	1125.39		0.957
Data 9	Pranav	0.43771	962.42	964.42	966.33		0.943
	Akash	0.310500	887.89	889.89	892.74		0.198
	Ishita	0.326152	894.12	896.12	898.03		0.928
	Lindley	0.196045	839.06	841.06	843.91		0.116
	Exponential	**0.106773**	**828.68**	**830.68**	**833.54**		**0.077**
	Prakaamy	0.10067	464.94	466.94	469.18		0.966
	Sujatha	0.04989	352.46	354.46	356.37		0.966
	Ram Awadh	0.10072	466.15	468.15	468.34		0.966
Data 10	Pranav	0.067146	391.23	393.23	395.15		0.966
	Ishita	0.050362	356.52	358.52	360.43		0.966
	Shanker	0.033569	325.74	327.74	329.14		0.351
	Lindley	0.033021	323.27	325.27	326.67		0.345
	Exponential	**0.016779**	**305.26**	**307.26**	**308.66**		**0.213**
	Prakaamy	2.28430	129.14	131.14	133.37		0.980
	Sujatha	1.14606	115.54	117.54		119.45	0.963
	Ram Awadh	2.10944	123.88	125.88	126.08		0.975
Data 11	Pranav	1.46645	116.67	118.67	120.58		0.965
	Akash	1.165719	115.15	117.15	118.68		0.156
	Ishita	1.157035	114.60	116.60	118.51		0.961
	Lindley	0.823821	112.61	114.61	116.13		0.133
	Exponential	**0.532081**	**110.91**	**112.91**	**114.43**		**0.089**
	Prakaamy	0.60712	727.94	729.94	732.17		0.221
	Sujatha	0.28461	639.63	641.63	643.55		0.088
	Ram Awadh	0.60887	729.70	731.70	731.90		0.221
Data 12	Pranav	0.46478	665.91	667.91	669.82		0.129
	Akash	0.295277	641.93	643.93	646.51		0.100
	Ishita	0.30157	643.69	645.69	647.61		0.108
	Lindley	**0.186571**	**638.07**	**640.07**	**642.68**		**0.058**

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Table Continued

	Exponential	0.101245	658.04	660.04	662.65	0.163
Data 13	Prakaamy	0.049484	241.20	243.20	245.43	0.931
	Sujatha	0.024637	193.93	195.93	197.85	0.904
	Ram Awadh	0.04948	241.20	243.20	243.40	0.931
	Pranav	0.03298	209.03	211.03	212.94	0.921
	Akash	0.024734	194.30	196.30	197.01	0.456
	Ishita	0.024745	194.32	196.32	198.23	0.905
	Lindley	0.016360	181.34	183.34	184.05	0.386
	Exponential	**0.008246**	**173.94**	**175.94**	**176.65**	**0.277**
Data 14	Prakaamy	2.27350	61.43	63.43	65.67	0.515
	Sujatha	**1.13674**	**57.49**	**59.49**	**61.40**	**0.442**
	Ram Awadh	2.04587	68.52	70.52	70.72	0.514
	Pranav	1.401401	62.38	64.38	66.29	0.485
	Akash	1.156923	59.52	61.52	62.51	0.320
	Ishita	1.094847	60.16	62.16	64.07	0.325
	Lindley	0.816118	60.50	62.50	63.49	0.341
	Exponential	0.526316	65.67	67.67	68.67	0.389
Data 15	Prakaamy	0.194733	223.07	225.07	227.31	0.197
	Sujatha	0.095613	241.50	243.50	245.41	0.302
	Ram Awadh	**0.19473**	**223.07**	**225.07**	**225.27**	**0.197**
	Pranav	0.12981	232.77	234.77	236.68	0.253
	Akash	0.097062	240.68	242.68	244.11	0.266
	Ishita	0.097328	240.48	242.48	244.39	0.297
	Lindley	0.062988	253.99	255.99	257.42	0.333
	Exponential	0.032455	274.53	276.53	277.96	0.426
Data 16	**Prakaamy**	**2.00984**	**188.77**	**190.77**	**193.00**	**0.261**
	Sujatha	0.936119	221.60	223.60	225.52	0.364
	Ram Awadh	1.84921	207.41	209.41	209.60	0.303
	Pranav	1.225139	217.12	219.12	221.03	0.303
	Akash	0.964726	224.28	226.28	228.51	0.348
	Ishita	0.931565	223.14	225.14	227.05	0.330
	Lindley	0.659000	238.38	240.38	242.61	0.390
	Exponential	0.407941	261.74	263.74	265.97	0.434

Data17	Prakaamy	0.21781	158.03	160.03	162.26	0.281
	Sujatha	0.10668	132.86	134.86	136.78	0.177
	Ram Awadh	0.21790	158.29	160.29	160.49	0.281
	Pranav	0.145325	141.44	143.44	145.35	0.231
	Akash	0.108478	133.68	135.68	137.59	0.184
	Ishita	0.10898	134.40	136.40	138.31	0.185
	Lindley	**0.070223**	**128.81**	**130.81**	**132.72**	**0.110**
	Exponential	0.036300	129.47	131.47	133.38	0.155
Data18	Prakaamy	0.033657	336.97	338.97	341.21	0.206
	Sujatha	0.01677	309.23	311.23	313.14	0.124
	Ram Awadh	0.03365	336.97	338.97	339.17	0.206
	Pranav	**0.023255**	**249.54**	**251.54**	**253.45**	**0.144**
	Akash	0.016822	309.41	311.41	313.32	0.125
	Ishita	0.016839	309.42	311.42	313.33	0.124
	Lindley	0.011183	305.01	307.01	308.92	0.129
	Exponential	0.005622	309.17	311.17	313.09	0.199

The best fitting has been shown by making -2ln L, AIC, BIC, and K-S Statistics in bold.

Conclusion

In this paper an attempt has been made to find the suitability of Prakaamy, Sujatha, Ram Awadh, Pranav, Akash, Ishita, Lindley and exponential distributions for modeling real lifetime data from engineering, medical science and other fields of knowledge. Nature and behavior of distributions have been presented graphically. Coefficient of Variation, Coefficient of Skewness, coefficient of kurtosis and Index of dispersion of distributions have also been presented graphically. The conditions under which different distributions are over-dispersed, equi-dispersed, and under-dispersed have also been given. The goodness of fit has been tested of above mentioned distributions on eighteen real lifetime datasets for their suitability for modeling lifetime data. It is observed that Exponential distribution gives good fits over other distributions for six datasets, whereas three datasets are related to biological fields, and three datasets are related chemical and engineering fields. Lindley distribution give better fit than other considered distributions for three datasets, whereas one of them is related to biological field and two of them are related to engineering fields. Ram Awadh distribution gives closer fits over other considered distributions for three datasets, whereas two of them are related to engineering and one dataset is related to biological field. Pranav distribution gives good fit over other considered distributions for two datasets, which are related to engineering fields. Sujatha distribution gives closure fit over other considered distributions for three datasets, which are related to medical science and engineering fields. Prakaamy distribution gives good fit over other considered distributions for one dataset which is related to engineering field. From the goodness of fit test, it can be observed that exponential and Lindley distribution can be considered as good model for biological as well as engineering studies. Ram Awadh and Sujatha distribution can also be considered good model for biological and engineering fields whereas Pranav and Prakaamy distribution can been considered good model for engineering filed.

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Conflicts of interest

Author declares that there are no conflicts of interest.

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