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Ram Awadh distribution with properties and applications

Abstract

In this paper, a new one parameter life lime distribution has been proposed and named Ram Awadh distribution. Its moments and moments based measures have been derived. Statistical properties including stochastic ordering, mean deviations, Bonferroni and Lorenz curves, order statistics, Renyi entropy and stress–strength measure have been discussed. Simulation study of proposed distribution has also been discussed. For estimating its parameter method of moments and method of maximum likelihood have been discussed. Goodness of fit of Ram Awadh distribution has been presented and compared with other lifetime distributions of one parameter. It was found superior than other one parameter life time distributions.

Keywords: moments, reliability measures, stochastic ordering, mean deviation, bonferroni and lorenz curves, order statistics, renyi entropy measure, estimation of parameters, goodness of fit

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Introduction

One parameter new life time distribution having parameters λ is defined by its pdf

$$f(x;\lambda)=\frac{\lambda^6}{\lambda^6+120}\left(\lambda+x^5\right)e^{-\lambda x}\quad ;x>0,\,\lambda>0\,. \tag{1.1}$$

We would name pdf (1.1) Ram Awadh distribution' which is a mixture of two–component, exponential distribution having scale parameter λ and gamma distribution having shape parameter 6 and scale parameter λ, and their mixing proportions of $\frac{\lambda^6}{\left(\lambda^6+120\right)}$ and $\frac{120}{\left(\lambda^6+120\right)}$ respectively.

$$f_2(x;\lambda)=p\,g_1(x;\lambda)+(1-p)g_2(x;\lambda,6)$$

Where $p=\frac{\lambda^6}{\left(\lambda^6+120\right)},g_1(x)=\lambda e^{-\lambda x}$ and $g_2(x)=\frac{\lambda^6x^5e^{-\lambda x}}{120}$

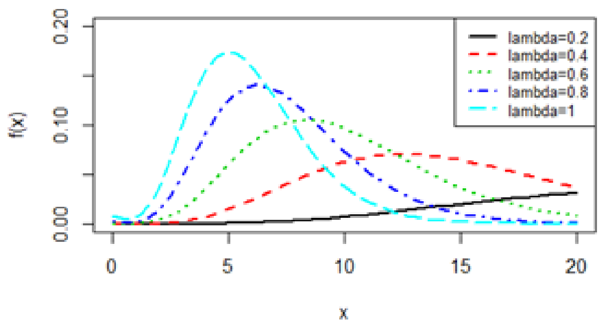
The corresponding cumulative distribution function (cdf) of (1.1) is given by

$$F(x;\lambda)=1-\left[1+\frac{\lambda x\left(\lambda^4x^4+5\lambda^3x^3+20\lambda^2x^2+60\lambda x+120\right)}{\lambda^6+120}\right]e^{-\lambda x}\;;x>0,\lambda>0 \tag{1.2}$$

The main objective of this paper is to propose a new life time distribution, which may be flexible than other distributions of one parameter proposed by different researchers. Ghitany et al.,[1] reported in their paper that Lindley is superior to exponential distribution with reference to data relating to waiting time before service of the bank customers. One parameter lifetime distributions namely Pranav, Ishita, Akash, Shanker, Sujatha and Lindley distributions are proposed by Shukla,[2] Shanker & Shukla,[3] Shanker,[4] Shanker,[5] Shanker[6] and Lindley[7] respectively and applied on biological and engineering data. Statistical properties, estimation of parameter and application of these lifetime distributions have been discussed in the respective papers. It is observed the superiority of proposed distribution over above mentioned distributions, which can be seen in section–10.

In this paper, new one parameter life time distribution has been proposed and named Ram Awadh distribution. Its raw moments and central moments have been obtained and coefficients of variation, skewness, kurtosis and index of dispersion have been discussed. Its hazard rate function, mean residual life function, stochastic ordering, mean deviations, Bonferroni and Lorenz curves, order statistics , Renyi entropy measure and stress – strength have been discussed. Both the method of moments and the method of maximum likelihood have been discussed for estimating the parameter of Ram Awadh distribution. A simulation study of distribution has also been carried out. The goodness of fit of the proposed distribution has been presented and compared with other lifetime distributions of one parameter.

Graphs of the pdf and the cdf of Ram Awadh distributionn for varying values of parameter are presented in Figure 1&2.



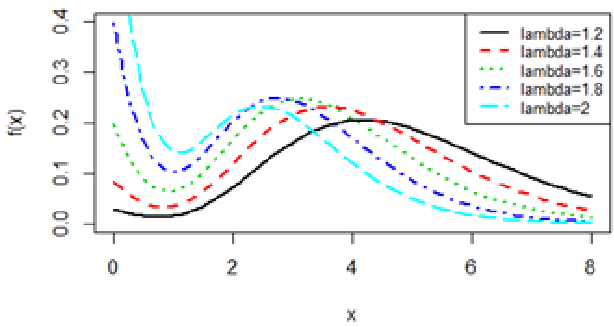


Figure 1 Pdf plots of Ram Awadh distribution for varying values of parameter λ.

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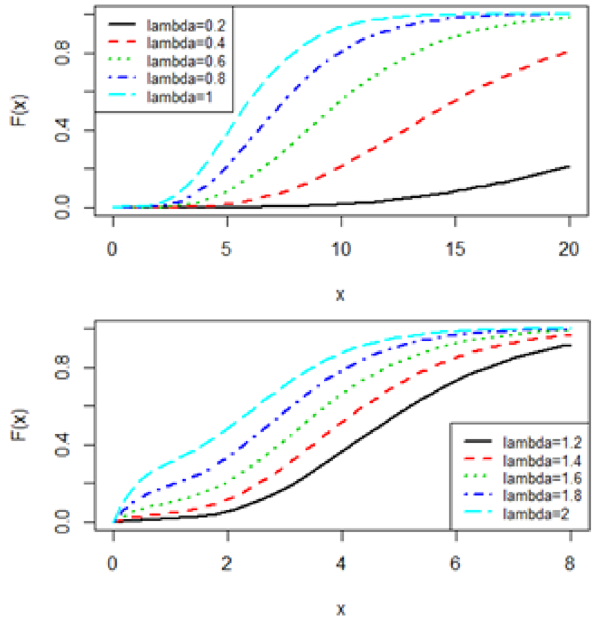

Figure 2 Cdf plots of Ram Awadh distribution for varying values of parameter λ.

Statistical constants

The r th moment about origin of Ram Awadh distribution can be obtained as

$$\mu_r' = \frac{r!\left[\lambda^6 + (r+1)(r+2)(r+3)(r+4)(r+5)\right]}{\lambda^r\left(\lambda^6+120\right)};r=1,2,3,... \quad (2.1)$$

Thus the first four moments about origin of Ram Awadh distribution are given by

$$\mu_1' = \frac{\lambda^6+720}{\lambda\left(\lambda^6+120\right)},\ \mu_2' = \frac{2(\lambda^6+2520)}{\lambda^2\left(\lambda^6+120\right)},$$

$$\mu_3' = \frac{6(\lambda^6+6720)}{\lambda^3\left(\lambda^6+120\right)},\ \mu_4' = \frac{24(\lambda^6+15120)}{\lambda^4\left(\lambda^6+120\right)}$$

And central moments of Ram Awadh distribution are obtained as follows:

$$\mu_2 = \frac{(\lambda^{12}+3840\lambda^6+86400)}{\lambda^2\left(\lambda^6+120\right)^2}$$

$$\mu_3 = \frac{2(\lambda^{18}+12960\lambda^{12}-172800\lambda^6+1036800)}{\lambda^3\left(\lambda^6+120\right)^3}$$

$$\mu_4 = \frac{9(\lambda^{24}+25280\lambda^{18}+2054400\lambda^{12}+271872000\lambda^6+3317760000)}{\lambda^4\left(\lambda^6+120\right)^4}$$

The coefficient of variation $(C.V)$, coefficient of skewness $(\sqrt{\beta_1})$, coefficient of kurtosis (β_2) and index of dispersion (γ) of Ram Awadh distribution are calculated as

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{(\lambda^{12}+3840\lambda^6+86400)}}{\left(\lambda^6+720\right)}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{2(\lambda^{18}+12960\lambda^{12}-172800\lambda^5+1036800)}{\left(\lambda^{12}+3840\lambda^6+86400\right)^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{9(\lambda^{24}+25280\lambda^{18}+2054400\lambda^{12}+271872000\lambda^6+3317760000)}{\left(\lambda^{12}+3840\lambda^6+86400\right)^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\left(\lambda^{12}+3840\lambda^6+86400\right)}{\lambda(\lambda^6+120)(\lambda^6+720)}$$

The value of index of dispersion will be one at $\lambda = 1.044533$. To study the nature of C.V, $\sqrt{\beta_1}$, β_2, and γ of Ram Awadh distribution, graphs of C.V, $\sqrt{\beta_1}$, β_2, and γ of Ram Awadh distribution have been drawn for varying values of the parameter and presented in Figure 3.

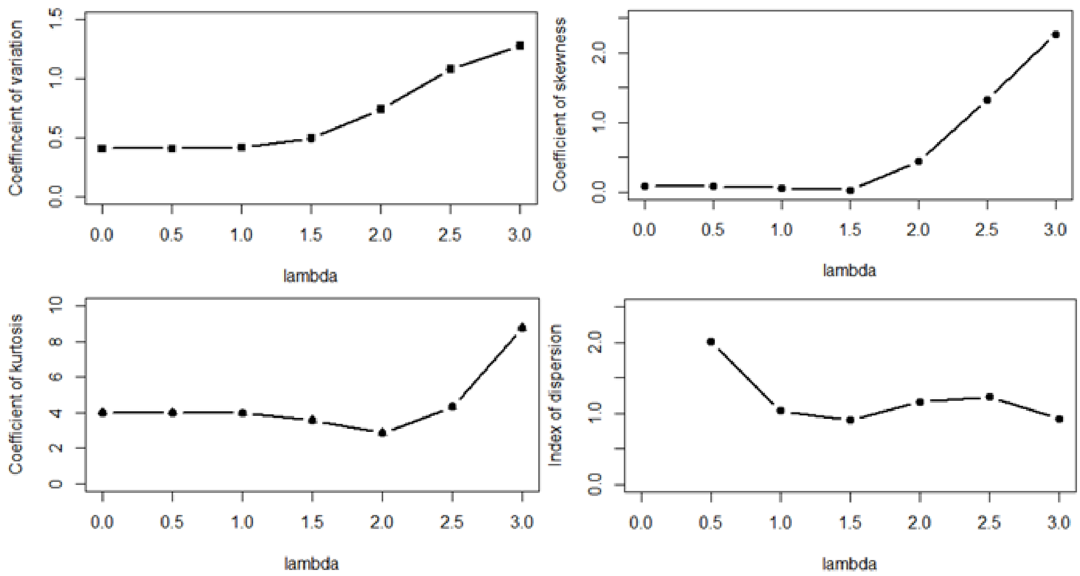

Figure 3 CV, CS, CK and Index of dispersion of Ram Awadh distribution.

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Reliability measures

There are two important reliability measures namely hazard rate function and mean residual life function. Let X be a continuous random variable with pdf $f(x)$ and cdf $F(x)$. The hazard rate function and the mean residual life function of X are respectively defined as

$$h(x) = \lim_{\Delta x \to 0} \frac{P(X < x + \Delta x | X > x)}{\Delta x} = \frac{f(x)}{1 - F(x)} \quad (3.1)$$

and $m(x) = E\left[X - x | X > x\right] = \frac{1}{1 - F(x)} \int_x^{\infty} \left[1 - F(t)\right] dt \quad (3.2)$

The corresponding $h(x)$ and $m(x)$ of Ram Awadh distribution (1.1) are as follows:

$$h(x) = \frac{\lambda^6 \left(\lambda + x^5\right)}{\left(\lambda^5 x^5 + 5\lambda^4 x^4 + 20\lambda^3 x^3 + 60\lambda^2 x^2 + 120\lambda x + \lambda^6 + 120\right)} \quad (3.3)$$

and

$$m(x) = \frac{1}{\left(\begin{array}{l}\lambda^5 x^5 + 5\lambda^4 x^4 + 20\lambda^3 x^3 \\ +60\lambda^2 x^2 + 120\lambda x + \lambda^6 + 120\end{array}\right)} \int_x^{\infty} \left(\begin{array}{l}\lambda^5 t^5 + 5\lambda^4 t^4 + 20\lambda^3 t^3 + 60\lambda^2 t^2 \\ +120\lambda t + \lambda^6 + 120\end{array}\right) e^{-\lambda t} dt$$

$$= \frac{\left(\lambda^5 x^5 + 10\lambda^4 x^4 + 60\lambda^3 x^3 + 240\lambda^2 x^2 + 600\lambda x + \lambda^6 + 720\right)}{\lambda\left(\lambda^5 x^5 + 5\lambda^4 x^4 + 20\lambda^3 x^3 + 60\lambda^2 x^2 + 120\lambda x + \lambda^6 + 120\right)} \quad (3.4)$$

It can be verified that $h(0) = \frac{\lambda^7}{\lambda^6 + 120} = f(0)$ and $m(0) = \frac{\lambda^6 + 720}{\lambda\left(\lambda^6 + 120\right)} = \mu_1'$.

The graphs of $h(x)$ and $m(x)$ of Ram Awadh distribution for varying values of parameter are presented in Figure 4 & 5.

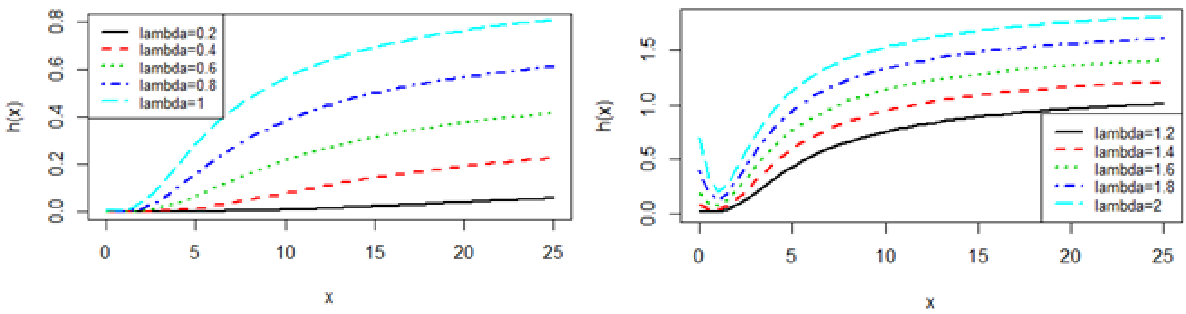

Figure 4 $h(x)$ Plots of Ram Awadh distribution for varying values of λ.

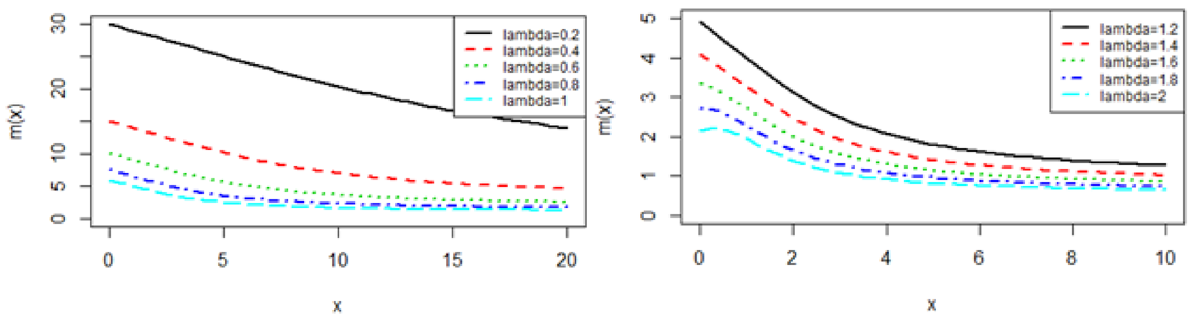

Figure 5 $m(x)$ Plots of Ram Awadh distribution for varying values of λ.

Stochastic orderings

For judging the comparative behavior of continuous distribution, it is important tool.

A random variable X is said to be smaller than a random variable Y in the

Stochastic order $(X \le_{st} Y)$ if $F_X(x) \ge F_Y(x)$ for all x

Hazard rate order $(X \le_{hr} Y)$ if $h_X(x) \ge h_Y(x)$ for all x

Mean residual life order $(X \le_{mrl} Y)$ if $m_X(x) \le m_Y(x)$ for all x

Likelihood ratio order $(X \le_{lr} Y)$ if $\frac{f_X(x)}{f_Y(x)}$ decreases in x.

The following results due to Shaked & Shanthikumar[8] are well known for establishing stochastic ordering of distributions.

$$X \le_{lr} Y \Rightarrow X \le_{hr} Y \Rightarrow X \le_{mrl} Y$$
$$\Downarrow$$
$$X \le_{st} Y$$

The Ram Awadh distribution is ordered with respect to the strongest 'likelihood ratio ordering' as established in the following theorem:

Theorem: Let X and $Y \sim$ Ram Awadh distribution (λ_1) and (λ_2) respectively. If $\lambda_1 \ge \lambda_2$ then $X \le_{lr} Y$ and hence $X \le_{hr} Y$, $X \le_{mrl} Y$ and $X \le_{st} Y$.

Proof: We have

$$\frac{f_X(x;\lambda_1)}{f_Y(x;\lambda_2)} = \frac{\lambda_1^6\left(\lambda_2^6 + 120\right)}{\lambda_2^6\left(\lambda_1^6 + 120\right)} e^{-(\lambda_1 - \lambda_2)x} \quad ; x > 0$$

Now

$$\ln\frac{f_X(x;\lambda_1)}{f_Y(x;\lambda_2)}=\ln\left[\frac{\lambda_1^6\left(\lambda_2^6+120\right)}{\lambda_2^6\left(\lambda_1^6+120\right)}\right]+\ln\left(\frac{\lambda_1+x^5}{\lambda_2+x^5}\right)-\left(\lambda_1-\lambda_2\right)x\,.$$

This gives $\frac{d}{dx}\ln\frac{f_X(x;\lambda_1)}{f_Y(x;\lambda_2)}=\frac{-2(\lambda_1+\lambda_2)}{(\lambda_1+x^5)(\lambda_2+x^5)}-\left(\lambda_1-\lambda_2\right)$.

Thus if $\theta_1>\theta_2$, $\frac{d}{dx}\ln\frac{f_X(x;\theta_1)}{f_Y(x;\theta_2)}<0$. This means that $X\leq_{lr}Y$ and hence $X\leq_{hr}Y$, $X\leq_{mrl}Y$ and $X\leq_{st}Y$.

Mean deviations

The mean deviation about mean and median defined by

$$\delta_1(X)=\int_0^\infty\left|x-\mu\right|f(x)dx \quad\text{and}\quad \delta_2(X)=\int_0^\infty\left|x-M\right|f(x)dx,$$

respectively, where $\mu=E(X)$ and $M=$ Median (X). The measures $\delta_1(X)$ and $\delta_2(X)$ can be calculated using the following simplified relationships

$$\delta_1(X)=\int_0^\mu(\mu-x)f(x)dx+\int_\mu^\infty(x-\mu)f(x)dx$$

$$=\mu F(\mu)-\int_0^\mu x\,f(x)dx-\mu\left[1-F(\mu)\right]+\int_\mu^\infty x\,f(x)dx$$

$$=2\mu F(\mu)-2\mu+2\int_\mu^\infty x\,f(x)dx$$

$$=2\mu F(\mu)-2\int_0^\mu x\,f(x)dx \quad (5.1)$$

and

$$\delta_2(X)=\int_0^M(M-x)f(x)dx+\int_M^\infty(x-M)f(x)dx$$

$$=M\,F(M)-\int_0^M x\,f(x)dx-M\left[1-F(M)\right]+\int_M^\infty x\,f(x)dx$$

$$=-\mu+2\int_M^\infty x\,f(x)dx$$

$$=\mu-2\int_0^M x\,f(x)dx \quad (5.2)$$

Using pdf (1.1) and the mean of Ram Awadh distribution, it can be written as:

$$\int_0^\mu x\,f(x;\lambda)dx=\mu-\frac{\left\{\begin{array}{l}\lambda^7\mu+\lambda^6(\mu^6+1)+6\lambda^4\mu^4(\lambda\mu+5)+120\mu^2\lambda^2(\lambda\mu+5)+\\120\lambda^2\mu^2(\lambda\mu+3)+720(\lambda\mu+1)\end{array}\right\}e^{-\lambda\mu}}{\lambda\left(\lambda^6+120\right)} \quad (5.3)$$

$$\int_0^M x\,f(x;\theta)dx=\mu-\frac{\left\{\begin{array}{l}\lambda^7M+\lambda^6(M^6+1)+6\lambda^4M^4(\lambda M+5)+120M^2\lambda^2(\lambda M+5)+\\120\lambda^2M^2(\lambda M+3)+720(\lambda M+1)\end{array}\right\}e^{-\lambda M}}{\lambda\left(\lambda^6+120\right)} \quad (5.4)$$

Using expressions from (5.1), (5.2), (5.3), and (5.4), the mean deviation about mean, $\delta_1(X)$ and the mean deviation about median, $\delta_2(X)$ of Ram Awadh distribution are obtained as

$$\delta_1(X)=\frac{2\left\{\lambda^5\mu^5+10\lambda^3\mu^3(\lambda\mu+6)+120\lambda\mu(2\lambda\mu+5)+(\lambda^6+720)\right\}e^{-\lambda\mu}}{\lambda\left(\lambda^6+120\right)} \quad (5.5)$$

$$\delta_2(X)=\frac{2\left\{\begin{array}{l}\lambda^7M+\lambda^6(M^6+1)+6\lambda^4M^4(\lambda M+5)+120M^2\lambda^2(\lambda M+5)+\\120\lambda^2M^2(\lambda M+3)+720(\lambda M+1)\end{array}\right\}e^{-\lambda M}}{\lambda\left(\lambda^6+120\right)}-\mu \quad (5.6)$$

Bonferroni and lorenz curves

It was given by Bonferroni,[9] and Bonferroni and Gini indices have applications not only in economics to study income and poverty, but it has also in many applications in different fields, such as demography, insurance and medicine. It can be defined as

$$B(p)=\frac{1}{p\mu}\int_0^q x\,f(x)dx=\frac{1}{p\mu}\left[\int_0^\infty x\,f(x)dx-\int_q^\infty x\,f(x)dx\right]=\frac{1}{p\mu}\left[\mu-\int_q^\infty x\,f(x)dx\right] \quad (6.1)$$

a n d

$$L(p)=\frac{1}{\mu}\int_0^q x\,f(x)dx=\frac{1}{\mu}\left[\int_0^\infty x\,f(x)dx-\int_q^\infty x\,f(x)dx\right]=\frac{1}{\mu}\left[\mu-\int_q^\infty x\,f(x)dx\right] \quad (6.2)$$

respectively or equivalently

$$B(p)=\frac{1}{p\mu}\int_0^p F^{-1}(x)dx \quad (6.3)$$

$$\text{and}\quad L(p)=\frac{1}{\mu}\int_0^p F^{-1}(x)dx \quad (6.4)$$

respectively, where $\mu=E(X)$ and $q=F^{-1}(p)$.

The Bonferroni and Gini indices are thus defined as

$$B=1-\int_0^1 B(p)dp \quad (6.5)$$

$$\text{and}\quad G=1-2\int_0^1 L(p)dp \quad (6.6)$$

respectively.

Using pdf of Ram Awadh distribution (1.1), it can be written

$$\int_q^\infty x\,f(x;\lambda)dx=\frac{\left\{\begin{array}{l}\lambda^7q+\lambda^6(q^6+1)+6\lambda^4q^4(\lambda q+5)+120q^2\lambda^2(\lambda q+5)\\+120\lambda^2q^2(\lambda q+3)+720(\lambda q+1)\end{array}\right\}e^{-\lambda q}}{\lambda\left(\lambda^6+120\right)} \quad (6.7)$$

Now using equation (6.7) in (6.1) and (6.2),

$$B(p)=\frac{1}{p}\left[1-\frac{\left\{\begin{array}{l}\lambda^7q+\lambda^6(q^6+1)+6\lambda^4q^4(\lambda q+5)+120q^2\lambda^2(\lambda q+5)\\+120\lambda^2q^2(\lambda q+3)+720(\lambda q+1)\end{array}\right\}e^{-\lambda q}}{\left(\lambda^6+720\right)}\right] \quad (6.8)$$

and

$$L(p)=1-\frac{\left\{\begin{array}{l}\lambda^7 q+\lambda^6(q^6+1)+6\lambda^4 q^4(\lambda q+5)+120q^2\lambda^2(\lambda q+5)\\+120\lambda^2 q^2(\lambda q+3)+720(\lambda q+1)\end{array}\right\}e^{-\lambda q}}{\left(\lambda^6+720\right)} \quad (6.9)$$

Now using equations (6.8) and (6.9) in (6.5) and (6.6), the Bonferroni and Gini indices of Ram Awadh distribution are thus given as

$$B=1-\frac{\left\{\begin{array}{l}\lambda^7 q+\lambda^6(q^6+1)+6\lambda^4 q^4(\lambda q+5)+120q^2\lambda^2(\lambda q+5)\\+120\lambda^2 q^2(\lambda q+3)+720(\lambda q+1)\end{array}\right\}e^{-\lambda q}}{\left(\lambda^6+720\right)} \quad (6.10)$$

$$G=\frac{2\left\{\begin{array}{l}\lambda^7 q+\lambda^6(q^6+1)+6\lambda^4 q^4(\lambda q+5)+120q^2\lambda^2(\lambda q+5)\\+120\lambda^2 q^2(\lambda q+3)+720(\lambda q+1)\end{array}\right\}e^{-\lambda q}}{\left(\lambda^6+720\right)}-1 \quad (6.11)$$

Order statistics and renyi entropy measure

Order statistics

Let $X_1, X_2, ..., X_n$ be a random sample of size n from Ram Awadh distribution (1.1). Let $X_{(1)} < X_{(2)} < ... < X_{(n)}$ denote the corresponding order statistics. The pdf and the cdf of the k th order statistic, say $Y = X_{(k)}$ are given by

$$f_Y(y)=\frac{n!}{(k-1)!(n-k)!}F^{k-1}(y)\{1-F(y)\}^{n-k}f(y)$$

$$=\frac{n!}{(k-1)!(n-k)!}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l F^{k+l-1}(y)f(y)$$

and

$$F_Y(y)=\sum_{j=k}^{n}\binom{n}{j}F^j(y)\{1-F(y)\}^{n-j}$$

$$=\sum_{j=k}^{n}\sum_{l=0}^{n-j}\binom{n}{j}\binom{n-j}{l}(-1)^l F^{j+l}(y),$$

respectively, for $k=1,2,3,...,n$.

Thus, the pdf and the cdf of k th order statistic of Ram Awadh distribution (1.1) are obtained as

$$f_Y(y)=\frac{n!\lambda^6\left(1+x^5\right)e^{-\lambda x}}{\left(\lambda^6+120\right)(k-1)!(n-k)!}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l$$

$$\times\left[1-\frac{\left\{\lambda x\left(\lambda^4x^4+5\lambda^3x^3+20\lambda^2x^2+60\lambda x+120\right)\right\}e^{-\lambda x}}{\lambda^6+120}\right]^{k+l-1}$$

and

$$F_Y(y)=\sum_{j=k}^{n}\sum_{l=0}^{n-j}\binom{n}{j}\binom{n-j}{l}(-1)^l\left[1-\frac{\left\{\lambda x\left(\lambda^4x^4+5\lambda^3x^3+20\lambda^2x^2+60\lambda x+120\right)\right\}e^{-\lambda x}}{\lambda^6+120}\right]^{j+l}$$

Entropy measure

Entropy of a random variable X is a measure of variation of uncertainty. A popular entropy measure is Renyi entropy.[10] If X is a continuous random variable having probability density function $f(.)$, then Renyi entropy is defined as

$$T_R(\gamma)=\frac{1}{1-\gamma}\log\left\{\int f^\gamma(x)dx\right\}$$

where $\gamma>0$ and $\gamma\neq 1$.

Thus, the Renyi entropy for Ram Awadh (1.1) can be obtained as

$$=\frac{1}{1-\gamma}\log\left[\int_0^\infty\frac{\lambda^{6\gamma}}{\left(\lambda^6+120\right)^\gamma}\left(\lambda+x^5\right)^\gamma e^{-\lambda\gamma x}dx\right]$$

$$=\frac{1}{1-\gamma}\log\left[\int_0^\infty\frac{\lambda^{7\gamma}}{\left(\lambda^6+120\right)^\gamma}\left(x^5\right)^\gamma\sum_{j=0}^{\infty}\binom{\gamma}{j}\left(\frac{x^5}{\lambda}\right)^j e^{-\lambda\gamma x}dx\right]$$

$$=\frac{1}{1-\gamma}\log\left[\sum_{j=0}^{\infty}\binom{\gamma}{j}\frac{\lambda^{7\gamma}}{\left(\lambda^6+120\right)^\gamma}\int_0^\infty e^{-\lambda\gamma x}x^{5j+1-1}dx\right]$$

$$=\frac{1}{1-\gamma}\log\left[\sum_{j=0}^{\infty}\binom{\gamma}{j}\frac{\lambda^{7\gamma}}{\left(\lambda^6+120\right)^\gamma}\frac{\Gamma(5j+1)}{(\lambda\gamma)^{5j+1}}\right]$$

$$=\frac{1}{1-\gamma}\log\left[\frac{\lambda^{7\gamma-5j-1}}{\left(\lambda^6+120\right)^\gamma}\sum_{j=0}^{\infty}\binom{\gamma}{j}\frac{\Gamma(5j+1)}{(\gamma)^{5j+1}}\right]$$

A simulation study

This process consists in generating N=10,000 pseudo–random samples of sizes 20, 40, 60, 80 and 100 from Ram Awadh distribution. Acceptance and rejection method has been used for this study. Average bias and mean square error of the MLEs of the parameter λ are estimated using the following formulae

Average Bias $=\frac{1}{N}\sum_{j=1}^{n}(\hat{\lambda}_j-\lambda)$, MSE= $\frac{1}{N}\sum_{j=1}^{n}(\hat{\lambda}_j-\lambda)^2$

The following algorithm can be used to generate random sample from Ram Awadh distribution.

Algorithm

Rejection method: To simulate from the density f_X, it is assumed that envelope density h from which it can simulate, and that have some $k<\infty$ such that $\sup_x\frac{f_X(x)}{h(x)}\leq k$ Simulate X from h.

Generate *Y~U(0,kh(X)*, where $k=\frac{\lambda^6}{(\lambda^6+120)}$

If $Y<f_X(x)$ then return X, otherwise go back to step 1.

The average bias (mean square error) of simulated estimate of parameter λ for different values of n and λ are presented in Table 1.

The graphs of estimated mean square error of the maximum likelihood estimate (MLE) for different values of parameter λ and n have been shown in Figure 6.

Stress–strength reliability

It explains the life of a component which has random strength X

that is subjected to a random stress Y. When the stress applied to it exceeds the strength, the component fails instantly and the component will function adequately till $X > Y$. Therefore, $R = P(Y < X)$ is a measure of component reliability.

Let X and Y be independent strength and stress random variables having Ram Awadh (1.1) with parameter λ_1 and λ_2 respectively. Then the stress–strength reliability R can be obtained as

$$R = P(Y < X) = \int_0^\infty P(Y < X \mid X = x) f_X(x)\,dx$$

$$= \int_0^\infty f(x;\lambda_1)\, F(x;\lambda_2)\,dx$$

$$= 1 - \frac{\lambda_1^6 \left[\begin{array}{l} \lambda_2^{16} + 10\lambda_2^{15}\lambda_1 + 45\lambda_2^{14}\lambda_1^2 + 120\lambda_2^{13}\lambda_1^3 + 210\lambda_2^{12}\lambda_1^4 \\ +(252\lambda_1^5 + 120)\lambda_2^{11} + (210\lambda_1^6 + 600\lambda_1 + 720)\lambda_2^{10} \\ +(120\lambda_1^7 + 1200\lambda_1^2 + 5400\lambda_1)\lambda_2^9 + (1200\lambda_1^3 + 18600\lambda_1^2 + 45\lambda_1^8)\lambda_2^8 \\ +10\lambda_1^3(\lambda_1^6 + 60\lambda_1 + 3900)\lambda_2^7 + \lambda_1^4(\lambda_1^6 + 120\lambda_1 + 55320)\lambda_2^6 \\ +(55440\lambda_1^5 + 6652800)\lambda_2^5 + (39600\lambda_1^6 + 4752000\lambda_1)\lambda_2^4 \\ +(19800\lambda_1^7 + 2376000\lambda_1^2)\lambda_2^3 + (66600\lambda_1^8 + 792000\lambda_1^3)\lambda_2^2 \\ +(1320\lambda_1^9 + 158400\lambda_1^4)\lambda_2 + 120\lambda_1^{10} + 14400\lambda_1^5 \end{array}\right]}{\left(\lambda_1^6 + 120\right)\left(\lambda_2^6 + 120\right)\left(\lambda_1 + \lambda_2\right)^{11}}$$

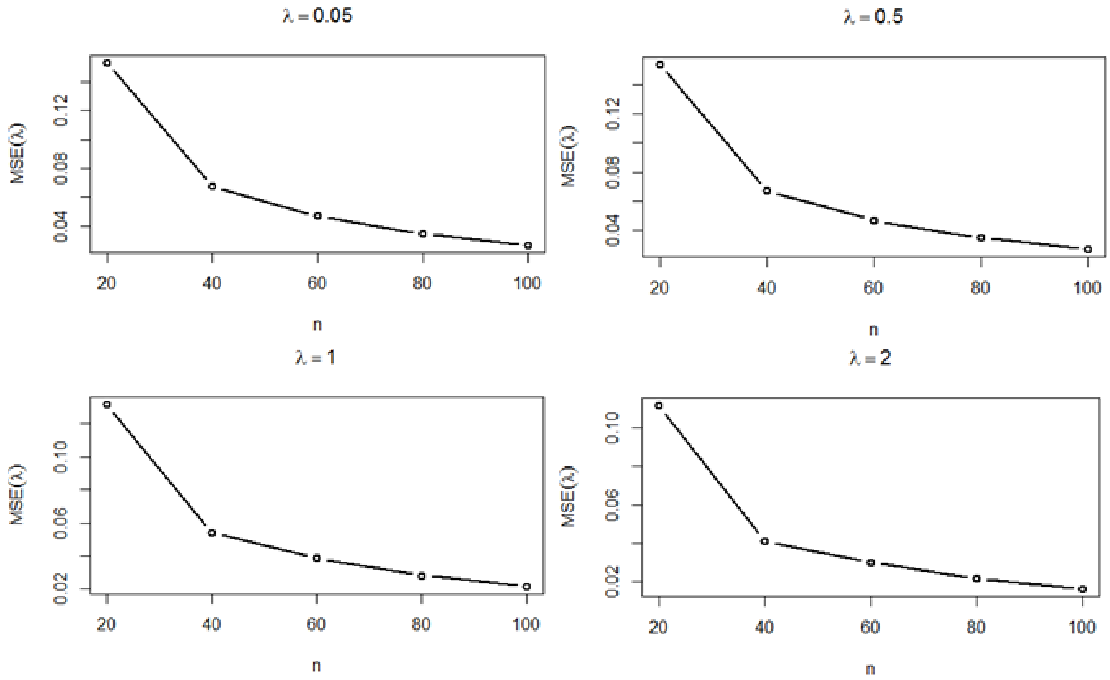

Figure 6 Estimated mean squared error of the MLEs for different values of λ and n

Table 1 Average bias (mean square error) of the simulated estimates of parameter λ

n	**Parameter** λ			
	0.05	**0.5**	**1**	**2**
20	0.08744(0.152915)	0.08775(0.154001)	0.081133(0.13165)	0.07470(0.11161)
40	0.041025(0.067322)	0.040931 (0.06701)	0.036754(0.05403)	0.032039(0.04106)
60	0.027958(0.04690)	0.027833(0.046482)	0.025377 (0.03864)	0.022434(0.03019)
80	0.02082(0.034680)	0.020767(0.034504)	0.018765(0.028171)	0.016455(0.02166)
100	0.016428(0.026989)	0.016368(0.026792	0.014731(0.021702)	0.012791(0.01636)

Parameters estimation

Method of moments estimates (MOME) of parameters

Equating population mean of Ram Awadh distribution to the corresponding sample mean,

MOME $\hat{\lambda}$ of λ is the solution of following non–linear equation

$$\lambda^7\bar{x} + 120\lambda\bar{x} - (\lambda^6 + 720) = 0 \quad (10.1)$$

Maximum likelihood estimates (MLE) of parameters

Let $(x_1, x_2, x_3, \ldots, x_n)$ be a random sample of size n from Ram Awadh (1.1)). The likelihood function, L of Ram Awadh distribution is given by

$$L = \left(\frac{\lambda^6}{\lambda^6 + 120}\right)^n \prod_{i=1}^n \left(\lambda + x_i^5\right) e^{-n\lambda\bar{x}}$$

and so its natural log likelihood function is thus obtained as

$$\ln L = n\ln\left(\frac{\lambda^6}{\lambda^6+120}\right)+\sum_{i=1}^{n}\ln\left(\lambda+x_i^5\right)-n\lambda\bar{x}$$

The maximum likelihood estimates (MLEs) $\hat{\lambda}$ of λ to the solution of the following non–linear equation

$$\frac{\partial \ln L}{\partial\lambda}=\frac{6n}{\lambda}-\frac{6n\lambda^5}{(\lambda^6+120)}+\sum\frac{1}{(\lambda+x^5)}-n\bar{x}=0 \quad (10.2)$$

where $\bar{x}$ is the sample mean. Equation (10.2) can solve directly for parameter λ using Newton–Raphson method. Its parameter is estimated using R–software.

Illustrative example

Data set 1: This data is related with behavioral sciences, collected by N. Balakrishnan, Victor Leiva & Antonio Sanhueza,[11] the detailed about the data are given in Balkrishnan et al.,[12] The scale "General Rating of Affective Symptoms for Preschoolers (GRASP)" measures, which are

19(16)	20(15)	21(14)	22(9)	23(12)	24(10)	25(6)	26(9)
27(8)	28(5)	29(6)	30(4)	31(3)	32(4)	33 34	35(4)
36(2)	37(2)	39 42 44					

Data set 2: This data set is the strength data of glass of the aircraft window reported by Fuller et al.,[12]

18.83	20.8	21.657	23.03	23.23	24.05	24.321
25.5	25.52	25.8	26.69	26.77	26.78	27.05
27.67	29.9	31.11	33.2	33.73	33.76	33.89
34.76	35.75	35.91	36.98	37.08	37.09	39.58
44.045	45.29	45.381				

Data Set 3: The following data represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20mm (Bader and Priest)[13]:

1.312	1.314	1.479	1.552	1.700	1.803	1.861
1.865	1.944	1.958	1.966	1.997	2.006	2.021
2.027	2.055	2.063	2.098	2.140	2.179	2.224
2.240	2.253	2.270	2.272	2.274	2.301	2.301
2.359	2.382	2.382	2.426	2.434	2.435	2.478
2.490	2.511	2.514	2.535	2.554	2.566	2.570
2.586	2.629	2.633	2.642	2.648	2.684	2.697
2.726	2.770	2.773	2.800	2.809	2.818	2.821
2.848	2.880	2.954	3.012	3.067	3.084	3.090
3.096	3.128	3.233	3.433	3.585	3.858	

For the above three data sets, Ram Awadh distribution has been fitted along with one parameter exponential, Lindley and Akash, Shanker, Sujatha, Ishita and Pranav distribution. The pdf and cdf of one parameter fitted distributions are presented in Table 2. The ML estimates, values of $-2\ln L$ and K–S statistics of the fitted distributions are presented in Table 3. As we know that the best distribution corresponds to the lower values of $-2\ln L$ and K–S.

Table 2 The p.d.f. and the c.d.f. of fitted distributions

Distribution	pdf	Cdf
Pranav	$f(x;\lambda)=\frac{\lambda^4}{\lambda^4+6}\left(\lambda+x^3\right)e^{-\lambda x}$	$F(x;\lambda)=1-\left[1+\frac{\lambda x\left(\lambda^2x^2+3\lambda x+6\right)}{\lambda^4+6}\right]e^{-\lambda x}$
Akash	$f(x,\lambda)=\frac{\lambda^3}{\lambda^2+2}\left(1+x^2\right)e^{-\lambda x}$	$F(x,\lambda)=1-\left[1+\frac{\lambda x(\lambda x+2)}{\lambda^2+2}\right]e^{-\lambda x}$
Shanker	$f(x,\lambda)=\frac{\lambda^2}{\lambda^2+1}(\lambda+x)e^{-\lambda x}$	$F(x,\lambda)=1-\left[1+\frac{\lambda x}{\lambda^2+1}\right]e^{-\lambda x}$
Sujatha	$f(x;\lambda)=\frac{\lambda^3}{\lambda^2+\lambda+2}\left(1+x+x^2\right)e^{-\lambda x}$	$F(x,\lambda)=1-\left[1+\frac{\lambda x(\lambda x+\lambda+2)}{\lambda^2+\lambda+2}\right]e^{-\lambda x}$
Ishita	$f(x;\lambda)=\frac{\lambda^3}{\lambda^3+2}\left(\lambda+x^2\right)e^{-\lambda x}$	$F(x,\lambda)=1-\left[1+\frac{\lambda x(\lambda x+2)}{\lambda^3+2}\right]e^{-\lambda x}$
Lindley	$f(x;\lambda)=\frac{\lambda^2}{\lambda+1}(1+x)e^{-\lambda x}$	$F(x;\lambda)=1-\left[\frac{\lambda+1+\lambda x}{\lambda+1}\right]e^{-\lambda x}$
Exponential	$f(x;\lambda)=\lambda e^{-\lambda x}$	$F(x;\lambda)=1-e^{-\lambda x}$

Profile plot of parameter and fitted plot for dataset–1, 2 and 3 are presented in Figures 7–9 respectively. From the graph, it is observed that Ram Awadh distribution is closer to observed dataset in comparison to other distributions of one parameter.

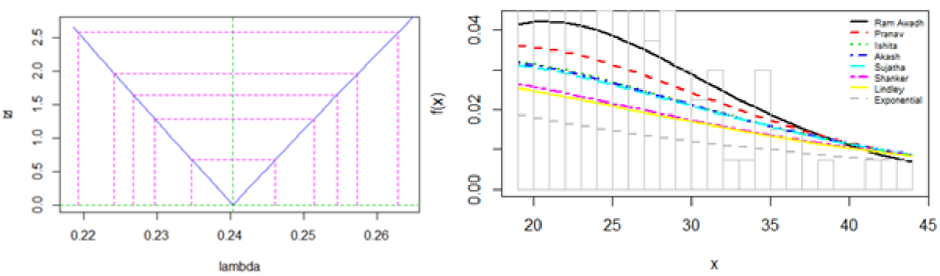

Figure 7 Profile of parameter and fitted probability plots for data set-1.

Table 3 MLE's, -2ln L, AIC, BIC, K-S Statistics of the fitted distributions of data-sets 1-3

Data set	Model	Parameter Estimate	-2ln L	AIC	BIC	K-S Statistic
Data 1	RamAwadh	0.240358	899.93	901.93	904.53	0.308
	Pranav	0.160222	945.03	947.03	948.94	0.362
	Ishita	0.120083	980.02	982.02	984.62	0.399
	Sujatha	0.117456	985.69	987.69	990.29	0.403
	Akash	0.11961	981.28	983.28	986.18	0.4
	Shanker	0.079746	1033.1	1035.1	1037.99	0.442
	Lindley	0.077247	1041.64	1043.64	1046.54	0.448
	Exponential	0.04006	1130.26	1132.26	1135.16	0.525
Data2	RamAwadh	0.194733	223.07	225.07	227.31	0.197
	Pranav	0.129818	232.77	234.77	236.68	0.253
	Ishita	0.097325	240.48	242.48	244.39	0.298
	Sujatha	0.09561	241.5	243.5	245.41	0.302
	Akash	0.097062	240.68	242.68	244.11	0.266
	Shanker	0.064712	252.35	254.35	255.78	0.326
	Lindley	0.062988	253.99	255.99	257.42	0.333
	Exponential	0.032455	274.53	276.53	277.96	0.426
Data 3	RamAwadh	2.009849	188.77	190.77	193	0.261
	Pranav	1.225138	217.12	219.12	221.03	0.303
	Ishita	0.931571	223.14	225.14	227.05	0.33
	Sujatha	0.936119	221.6	223.6	225.52	0.364
	Akash	0.964726	224.28	226.28	228.51	0.348
	Shanker	0.658029	233.01	235.01	237.24	0.355
	Lindley	0.659	238.38	240.38	242.61	0.39
	Exponential	0.407941	261.74	263.74	265.97	0.434

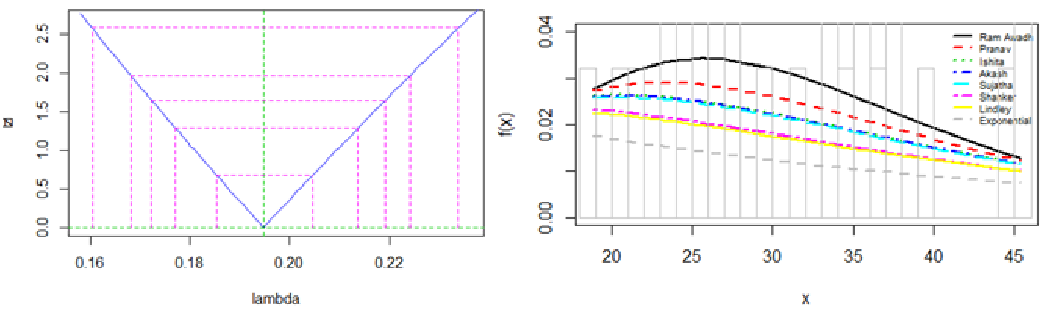

Figure 8 Profile of parameter and fitted probability plots for data set-2.

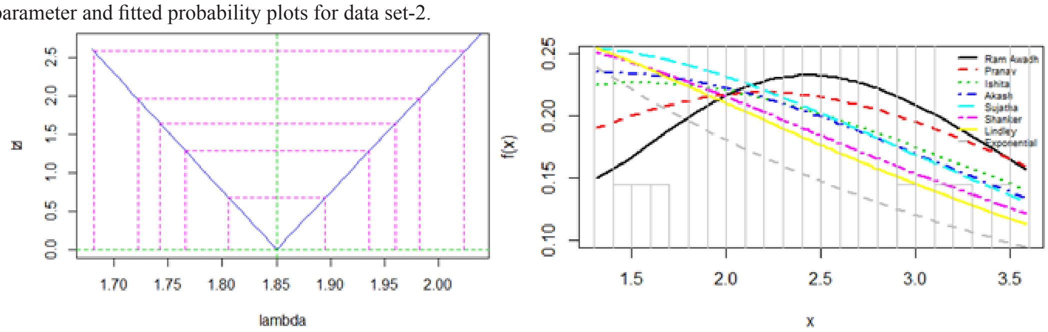

Figure 9 Profile of parameter and fitted probability plots for data set-3.

Conclusion

In this paper, a new one parameter lifetime distribution named Ram Awadh distribution has been proposed. Its mathematical properties including moments, measure of dispersion, hazard rate function, mean residual life function, stochastic ordering, mean deviations, order statistics, Bonferroni and Lorenz curves, and stress–strength reliability have been discussed. Simulation study of Ram Awadh distribution has also been discussed. The method of moments and the method of maximum likelihood estimation have been derived for estimating the parameter. In the last, three numerical examples of real lifetime data sets have been illustrated to test the goodness of fit of the Ram Awadh distribution. Its fit was found satisfactory over exponential, Lindley, Sujatha, Ishita, Akash , Shanker and Pranav distribution.

Note: The paper is named Ram Awadh distribution in the name of my Father Shri Ram Awadh Shukla.

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Conflict of interest

Author declares that there is no conflict of interest.

References

1. Ghitany ME, Atieh B, Nadarajah S. Lindley distribution and its Applications. *Mathematics Computing and Simulation*. 2008;78:493–506.
2. Shukla KK. Pranav distribution with Properties and Applications. *Biom Biostat Int J*. 2018;7(3):244–254.
3. Shanker R, Shukla KK. Ishita distribution and Its Applications. *Biom Biostat Int J*. 2017;5(2):1–9.
4. Shanker R. Akash distribution and Its Applications. *International Journal of Probability and Statistics*. 2015a;4(3):65–75.
5. Shanker R. Shanker distribution and Its Applications. *International journal of Statistics and Applications*. 2015b;5(6):338–348.
6. Shanker R. Sujatha Distribution and Its Applications. *Statistics in Transition– New series*. 2016;17(3):1–20.
7. Lindley DV. Fiducial distributions and Bayes' theorem. *Journal of the Royal Statistical Society*. 1958;20:102–107.
8. Shaked M, Shanthikumar JG. *Stochastic Orders and Their Applications*. New York: Academic Press; 1994.
9. Bonferroni CE. *Elementi di Statistca generale*. Firenze: Seeber; 1930.
10. Renyi entropy. On measures of entropy and information, in proceedings of the 4th Berkeley symposium on Mathematical statistics and Probability. 1961;1:547–561.
11. Balakrishnan N, Victor L, Antonio S. A mixture model based on Saunders Distributions, A study conducted by Authors regarding the Scores of the GRASP (General Rating of Affective Symptoms for Preschoolers), in a city located at South Part of the Chile. 2010.
12. Fuller EJ, Frieman S, Quinn J, et al. Fracture mechanics pproach to the design of glass aircraft windows: A case study. *SPIE Proc*. 1994;2286:419–430.
13. Bader MG, Priest AM. Statistical aspects of fiber and bundle strength in hybrid composites. In: Hayashi T and Kawata K, Umekawa, editors. Tokyo: Progress in Science in Engineering Composites. 1982. p. 1129–1136.