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Modeling and forecasting norway mortality rates using the lee carter model

Abstract

Mortality data is an important element in the fields of actuarial science, health, epidemiology and national planning. Mortality levels are generally regarded as indicators of a general welfare of a national population and its subgroups. It reflects the quality of life within quantity. Therefore, developing a model for forecasting mortality rate will help a nation to develop its quality of life. The Lee and Carter stochastic mortality model has been used for fitting and forecasting the mortality rate of Norway which is considered as the country with the highest living standards based on the human development index. The data set contains Norway mortality data from 1946-2014. The Singular Value Decomposition (SVD) approach is used for estimating the parameters of Lee Carter model. Auto Regressive Integrated Moving Average (ARIMA) time series model is used for forecasting the mortality values. In this study 97.5 % variance of Norway mortality data could be explained by the proposed Lee Carter model. The best fitting ARIMA model for Norway data is identified as ARIMA (3,2,1) with drift. The predicted Lee Carter model gives a good fit to Norway data over a wide range of ages but shows poor performance below age 4 years and after age 55 years. Therefore, improvements in the Lee Carter model is needed to obtain better predictions for the ages below 4 years and after 55 years. This proposed model can be used to construct the life tables, pension scheme planning and actuarial science applications.

Keywords: auto regressive integrated moving average, forecasting, lee carter model, mortality, single value decomposition

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Abbreviations: SVD, singular value decomposition; ARIMA, auto regressive integrated moving average; SSA, singular spectrum analysis; LC, lee and carter; AIC, akaiko information criterion; BIC, bayesian information criterion; SSE, sum of squares of errors

Introduction

In the fields of actuarial science, health, epidemiology and national planning, mortality data is an important element. Mortality levels reflect the quality of life within quantity. Population forecasting is essential for all long term planning for the provision of services of a nation. Therefore, developing a model for forecasting mortality rate can facilitate a nation to develop their quality of life.[1]

The fundamental aspect of the mankind is to live healthy long life. Recent enormous advancements in technology have provided tremendous support to fulfill this aspect. However, wide disparities are visible in levels of mortality across countries and regions. The reduction of mortality, particularly infant and maternal mortality, is part of the internationally agreed development goals in the 21st century.[2]

With the human lifespan increasing, several researchers in numerous fields have recently become inquisitive about finding out quantitative models of mortality rates.[3] Once we are able to model human aging, we are able to hunt for ways that to increase our life and counteract the negative aspects of aging.[4]

Singular spectrum analysis (SSA) and Hyndman-Ullah models were used in the literature to obtain 10 forecasts for the period 2000-2009 in nine European countries including Belgium, Denmark, Finland, France, Italy, the Netherlands, Norway, Sweden and Switzerland.[5] Computational results show a superior accuracy of the SSA forecasting algorithms, when compared with the Hyndman-Ullah approach. In most previous studies Lee Carter model has been used for modeling mortality rate in other countries and has used Bayesian approach to forecast mortality rate. Lee carter method is used to model the variability.[6] With this approach, missing data is handled and the sampling error is automatically incorporated within the model and its mortality forecasts. Here the author has described the 20th century trends of mortality for developed countries with the US as an example. A Bayesian approach was used to model mortality data for males and females from England and Whales.[7] The author has developed Lee Carter mortality including age-period and age-cohort interactions and random effects on mortality. Moreover, Poisson distribution in advanced statistical analysis of mortality is used in.[8] This technique helps to compare low numbers of deaths in a stratum, thereby deriving more meaningful conclusions from the information.

This study was mainly focused on modeling and forecasting mortality rates using Lee Carter Model for Norway which is considered as the country with the highest living standards based on the human development index. The predicted model can be used to construct life tables for Norway and also can be used in actuarial science applications and pension scheme planning.

This article is organized as follows. In Materials and methods section, the methodology and the statistical framework behind the analysis is discussed. We describe the Lee Carter model for modeling and forecasting mortality rates. Theories used are also discussed in this section. In the results and discussion section the methodology is illustrated by analyzing the Norway mortality data. Finally, the article is concluded with a discussion. The statistical software R[9] has been used for all the computations in this article.

Materials and methods

Line graphs are used to visualize the mortality pattern of each age group and birth pattern of Norway. Through the graphs the outliers can be observed clearly. The Lee and Carter (LC) stochastic mortality model[1] has been used in this study for fitting and forecasting the mortality rate of Norway. The Singular Value Decomposition approach is used for estimating the a_x and b_x parameters of Lee Carter model.[2]

The singular value decomposition is a factorization of a real or complex matrix. Generally, SVD of a m * n real or complex matrix A is a factorization of the form UDV^T where U is a m*m matrix, V is a n*n matrix and D is a m*n rectangular diagonal matrix with non-negative real numbers on the diagonal. The diagonal entries of D, are called the singular values of A. The columns of U and V are the left and right singular vectors of A.[10]

Lee carter model for mortality data

The Lee-Carter model is a numerical formula utilized in mortality prediction and life expectancy forecasting. The input to the model is a matrix of age specific mortality rates ordered monotonically by time, typically with ages in columns and years in rows. The output is another forecasted matrix of mortality rates.[11,12] The Lee Carter model for mortality data is as follows[11]

$$ln\left(m_{x,t}\right) = a_x + b_x k_t + \varepsilon_{x,t} \quad (1)$$

where, $m_{x,t}$ – central rate of mortality for age group x at time t, a_x – coefficient which describes average age specific pattern of mortality, k_t – time trend for the general mortality, b_x – coefficient which measures sensitivity of $ln\left(m_{x,t}\right)$ at age- grouping x as the k_t varies, $\varepsilon_{x,t}$ – error associated with age grouping x and time t. Error terms assumed to follow a normal distribution with mean zero and to be independent of age x and time t.[13]

The model uses the singular value decomposition (SVD) approach to find a univariate time series vector k_t that describes mortality trend in a given time (t), captures 80-90% of the mortality trend, a vector b_x that describes the amount of mortality change at a given age(x) for a unit of yearly total mortality change. Life expectancy being fairly constant yearly is implied with the linearity of k_t. First age specific mortality rates are transformed into $a_{x,t}$ which spans both age(x) and time(t) by taking their logarithms, and then centering them by subtracting their age-specific means which is calculated over time before being input to the SVD .

To forecast mortality, the above k_t which may be adjusted into the future using ARIMA time series methods[11] the corresponding future $a_{x,t+n}$ is recovered by multiplying k_{t+n} by b_x and the appropriate diagonal element of S (when [U S V] = svd(mort)), and the actual mortality rates are recovered by taking exponentials of this vector. Because of the linearity of k_t, it is generally modeled as a random walk with trend. Life expectancy and other life table measures can be calculated from this forecasted matrix after adding back the means and taking exponentials to yield regular mortality rates.

This Lee Carter model is considered as the golden model for modeling mortality due to the simplicity in parameter estimation and it gives a good fit over a wide range of ages. Lee and Carter used U.S. mortality rates for conventional 5-year age groups.[1] The same procedure is used in this study. As the initial step mortality data of 158 years is plotted for 19 age groups.

The estimated parameter vector $\hat{a}_x$ is determined as the average over time of the logarithm of the central death rates as

$$\hat{a}_x = 1/\left(158\sum_t ln\left(m_{x,t}\right)\right) \quad (2)$$

The results were stored in a matrix of 158 years by 19 age groups. Then subtract the average age pattern $\hat{a}_x$ from all years. To obtain estimated parameters b_x and k_t, singular value decomposition is applied on matrix Z, where $Z = ln\left(m_{x,t}\right) - \hat{a}_x$. By applying SVD to matrix Z, $SVD\left(Z\right) = \lambda_1 P_{x,1} Q_{t,1} + \lambda_2 P_{x,2} Q_{t,2} + \ldots = \lambda_1 P_{x,k} Q_{t,k}$ where $k = rank\left(z\right)$, $\lambda_i\left(i = 1,2,\ldots,k\right)$ is the singular values with increasing order $P_{x,i}$ and $Q_{t,i}$ left and right singular vectors respectively. Also,

$$\hat{b}_x = P_{x,1} \quad (3)$$

and

$$\hat{k}_t = \lambda_1 Q_{t,1} \quad (4)$$

This $\hat{b}_x$ vector models how the different age groups react to mortality change. Moreover, this $\hat{k}_t$ vector captures overall mortality change over time. The proportion of variance described by the 1st component of SVD is calculated as

$$\lambda_1^2 / \Sigma\lambda_i^2 \quad (5)$$

This is used as a diagnostic test for Lee Carter model.[14] Model validation is presumably the foremost vital step within the model building sequence. Information criteria are model selection tools which are used to compare any models fit to the same data. Basically, information criteria are likelihood-based measures of model fit that embody a penalty for complexity, specifically, the number of parameters. Different information criteria are distinguished by the form of the penalty, and can prefer different models. Akaiko Information Criterion (AIC) and Bayesian Information Criterion (BIC) were used to identify the best fitting parameters. For accurate prediction, predicted values are compared with the actual values. The Human development index is a composite statistic developed by the United Nations to measure and rank the level of social and economic development based on life expectancy, education and income per capita in countries.

The mortality data was extracted from the life tables. Life tables are tables of statistics relating to life expectancy and mortality for a given category of people.[15] First the data set was divided in to training and testing set. The training data set contained the data from 1846 to 2004. Then the data from 2005 - 2014 were used as test set. The Singular Value Decomposition (SVD) approach is used for estimating the parameters of LC model. Auto Regressive Integrated Moving Average (ARIMA) time series model is used for forecasting the mortality values.

To produce mortality forecasts, Lee and Carter assume that b_x remains constant over time and use forecasts of from a univariate time series model.[16] After testing several ARIMA specifications most appropriate model was identified using AIC and BIC values. Then the Actual and Predicted Value plots were used to identify how well the model behaved.[17]

Results and discussion

In this section birth and death pattern of Norway was plotted to identify how the death rate varied in each age group for the past 168 years. To identify the trend and outliers of mortality rate the preliminary analysis was carries out.

First log (number of births) were plotted against Year. The resulting plot is shown in Figure 1. Dramatic decline around 1940s and 1980s was visible.

The plots of mortality rate versus age indicated a decreasing trend of mortality in each age group. (Plots for each age group are included in the Figure 7-14).

And it was clearly visible that male mortality rate is always higher than the female rate. After the 1950s there was a significant increase in male mortality. This was directly linked with the diseases due to tobacco use like cancers and chronic lower respiratory diseases. Child mortality dropped rapidly after 1950s. The dominant factor was that people were aware of the personal hygiene. Specially the infant mortality was directly connected with the mothers' education status. With the growth of mothers' education status infant and maternal mortality declined rapidly.[14]

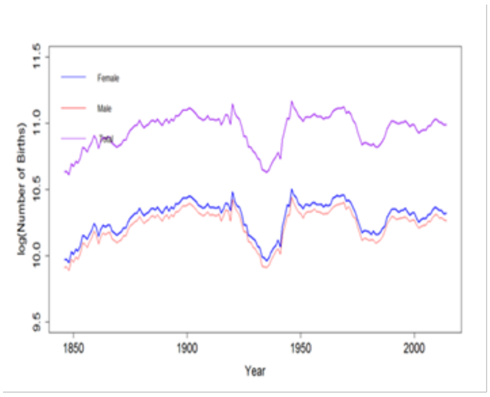

Figure 1 Birth pattern of norway.

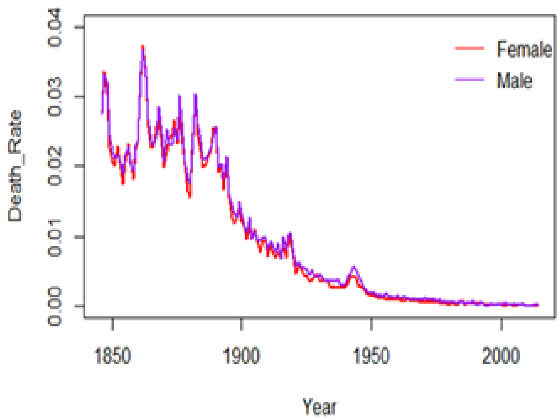

Figure 2 Death rate of age 2.

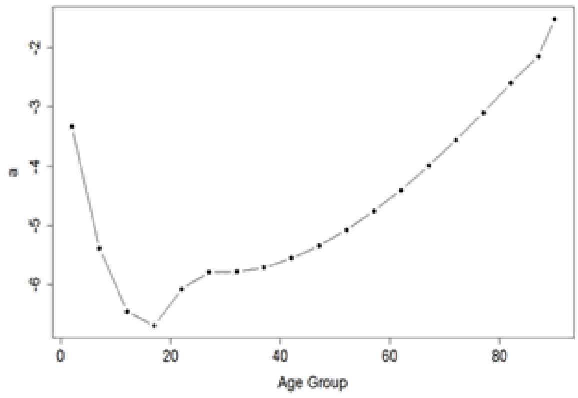

Figure 3 General mortality pattern of norway.

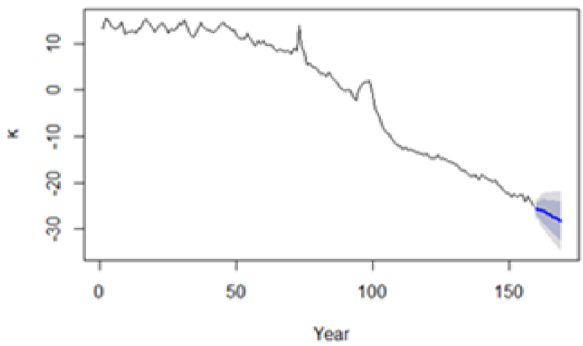

Figure 4 Forecasted values from ARIMA (3,2,1) with drift.

In this research Lee and Carter model stated in the equation1 was used to model the Norway data. The parameters were estimated according to the equation 2, 3 and 4. The values obtained for the parameters a_x and b_x of Lee Carter model for Norway data are summarized in the Table 1.

Table 1 Values of the a_x and b_x parameters for 19 age groups

Age group	a_x	b_x
00-04	-3.3329	0.07864
05-09	-5.3973	0.1156
10-14	-6.4578	0.10207
15-19	-6.693	0.0935
20-24	-6.0772	0.07436
25-29	-5.7985	0.07753
30-34	-5.7849	0.07694
35-39	-5.7175	0.0716
40-44	-5.5552	0.06395
44-49	-5.3472	0.053
50-54	-5.079	0.04177
55-59	-4.7593	0.03331
60-64	-4.407	0.02671
65-69	-3.9991	0.02254
70-74	-3.5596	0.01992
75-79	-3.0991	0.01668
80-84	-2.6013	0.0154
85-89	-2.1478	0.01087
90+	-1.5219	0.00564

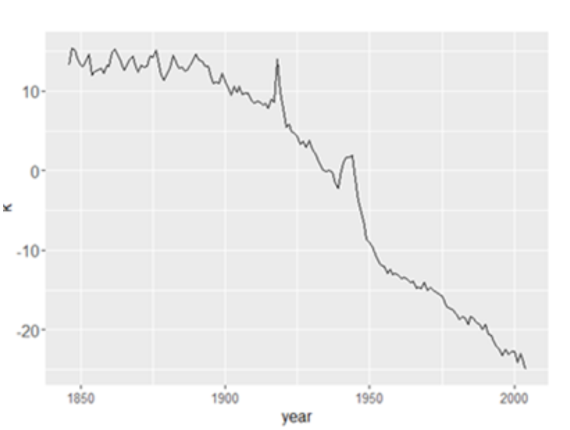

Figure 5 Trend of mortality index k_t over the period 1946-2004.

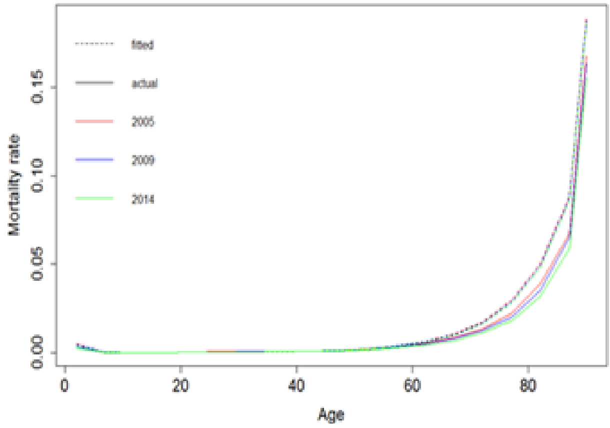

Figure 6 Fitted and actual value plot.

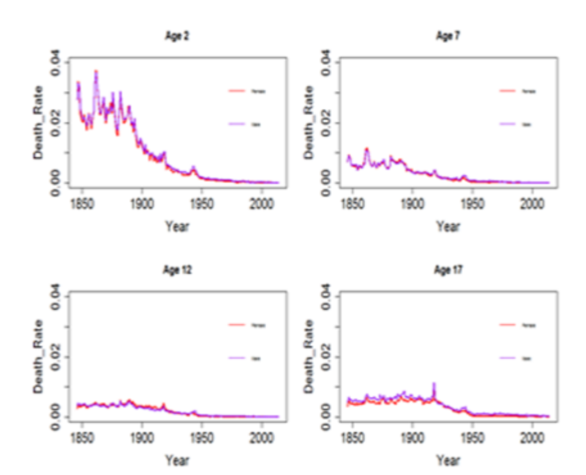

Figure 7 Morality pattern of norway for age 2, 7, 12, 17.

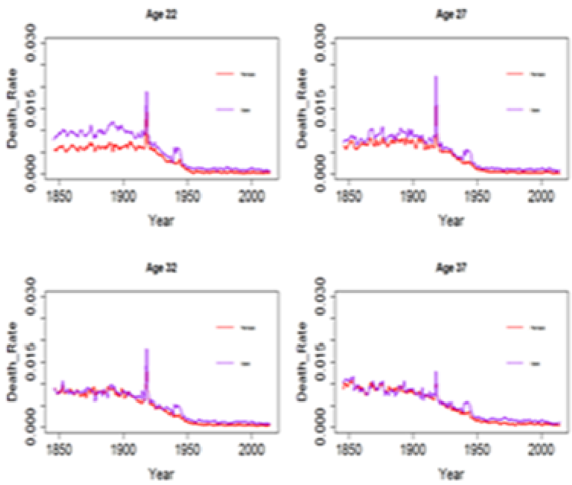

Figure 8 Morality pattern of norway for ages 22, 27, 32, 37.

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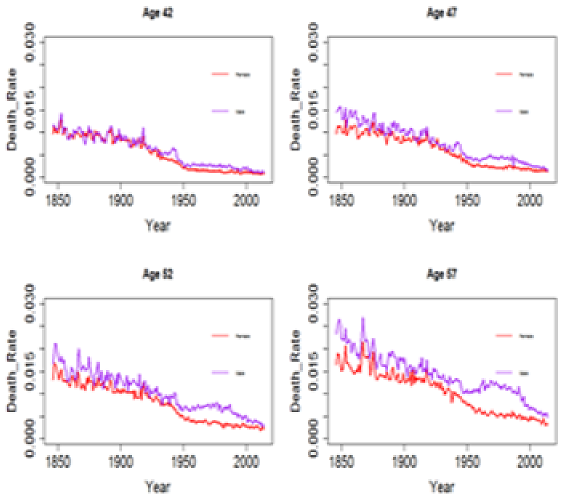

Figure 9 Morality pattern of norway for ages 42, 47, 52, 57.

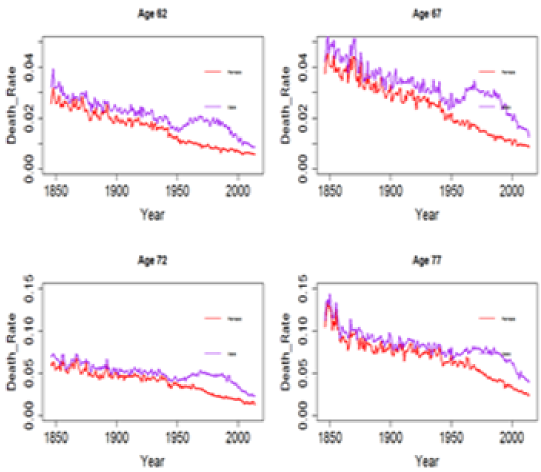

Figure 10 Morality pattern of norway for ages 62, 67, 72, 77.

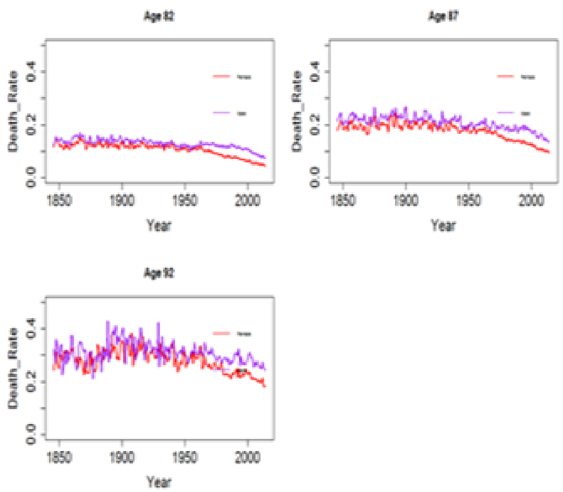

Figure 11 Morality pattern of norway for ages 82, 87, 92.

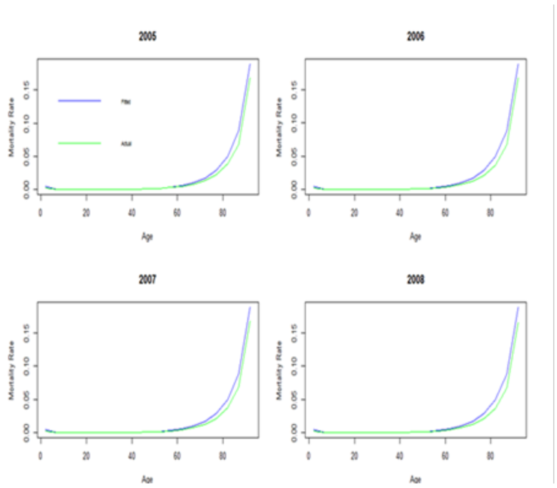

Figure 12 Actual vs fitted value plots for 2005 to 2008.

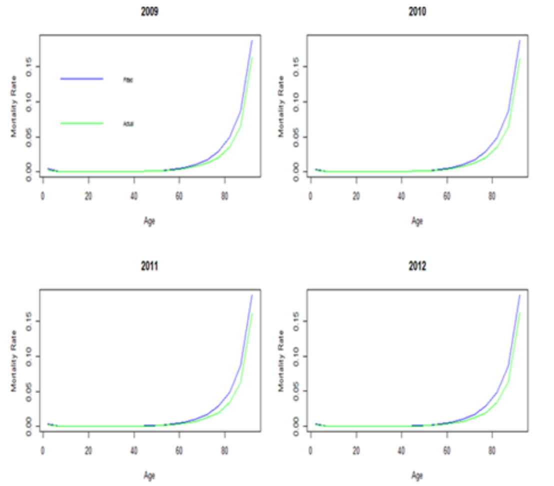

Figure 13 Actual and fitted value plots for 2009 to 2012.

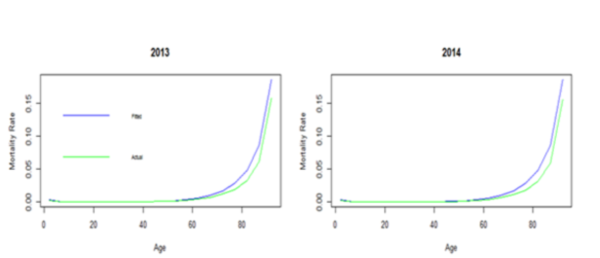

Figure 14 Actual and fitted value plots for 2013 to 2014.

Then the estimated parameters were plotted against the age group. Differences in relative rates of change by age are captured by b_x. Differences in mortality by age are captured by a_x.

From the Figure 3, higher child mortality below age 5 years is observed and after the age of 25 mortality increases nearly exponentially. Lowest mortality rate is observed at the age group of 15-19.

The proportion of variance described by the 1st component of SVD is calculated using the equation 5. By applying SVD 97.5462 %

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temporal variance of Norway mortality data could be explained by the 1st component of SVD. After testing AIC and BIC values for several ARIMA specifications the best fitting parameters of ARIMA model for Norway data was identified. The AIC and BIC values for fitted ARIMA models are shown in Table 2.

The best fitting parameters of ARIMA model were identified as ARIMA (3,2,1) with drift which gave the lowest AIC and BIC values 457.47, 472.75 respectively.

Values of k for next 10 years are forecasted from ARIMA (3,2,1) with drift with the 80% and 95% confidence interval. This was done using the "FORECAST" package of R. The forecasted values are shown in the Table 3.

Table 2 AIC and BIC values for fitted ARIMA models

ARIMA	**(0,0,0)**	**(0,0,1)**	**(0,1,0)**	**(0,1,1)**	**(1,0,0)**	**(1,1,0)**	**(1,1,1)**	
AIC	1282.56	1084	463.2	465.2	475.8	465.2	461	
BIC	1288.7	463.2	466.3	471.3	485	471.3	470.2	
ARIMA	**(0,2,0)**	**(0,2,2)**	**(0,0,2)**	**(2,2,0)**	**(2,0,0)**	**(2,0,2)**	**(1,2,1)**	
AIC	565.25	460.8	928.5	505.3	477.8	475.2	460.7	
BIC	568.3	470	940.8	514.4	490.1	493.6	469.9	
ARIMA	**(1,2,2)**	**(1,1,2)**	**(2,2,1)**	**(2,2,2)**	**(2,1,1)**	**(2,1,2)**	**(3,2,1)**	**(1,2,3)**
AIC	460.1	462.7	461.4	460.7	463.4	464.4	457.5	460.1
BIC	472.32	474.9	473.6	476	475.6	479.7	472.8	475.3

Table 3 Forecasted values for k_t and confidence interval

		Confidence interval			
		80%		**95%**	
Year	**Forecast**	**Lower bound**	**Upper bound**	**Lower bound**	**Upper bound**
2005	-25.595	-26.8876	-24.3025	-27.5718	-23.6183
2006	-25.869	-27.6966	-24.0422	-28.6638	-23.0749
2007	-26.047	-28.3651	-23.728	-29.5925	-22.5007
2008	-26.305	-28.9161	-23.6947	-30.2981	-22.3126
2009	-26.629	-29.5247	-23.7327	-31.0577	-22.1996
2010	-26.976	-30.1275	-23.8243	-31.7958	-22.1559
2011	-27.311	-30.7315	-23.8905	-32.5422	-22.0798
2012	-27.635	-31.3132	-23.9575	-33.2602	-22.0105
2013	-27.954	-31.8881	-24.0205	-33.9706	-21.938
2014	-28.275	-32.4556	-24.0944	-34.6687	-21.8813

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Table 4 Age group wise SSE (sum of squared error)

Age group	SSE
00-04	2.47E-05
05-09	3.00E-08
10-14	3.74E-09
15-19	5.72E-09
20-24	4.69E-08
25-29	2.78E-07
30-34	3.54E-07
35-39	1.95E-07
40-44	1.08E-07
44-49	5.93E-08
50-54	5.82E-07
55-59	2.69E-06
60-64	1.52E-05
65-69	5.23E-05
70-74	0.000182498
75-79	0.000776518
80-84	0.001869918
85-89	0.005167924
90+	0.006279726

The trajectory of k is shown in the Figure 5. k_t reflects year-to-year changes in the general level of mortality.

From the Figure 5 decreasing trend is observed for mortality index (k_t). Two distinguishable peaks are observed during 1914-1918 and 1939- 1945 due to the world war I and World War II respectively. The forecasted k values are used together with calculated a and b values to predict the mortality rate for next 10 years. The fitted and actual value plots are shown in the Figure 6.

(Actual and fitted value plots and values for all predicted 10 years are shown in the Figure 5-7. Then for each age group Sum of Squared error values were calculated as below.

From the fitted and actual value plots shown in Figure 6 and from the age group wise Sum of Squares of Errors (SSE) values in Table 4 it was clear that higher deviation is observed for 00-04 years and after 50 years (Tables 5-7).

Conclusion

The Lee and Carter (LC) stochastic mortality model has been used in this study for fitting and forecasting the mortality rate of Norway which is considered as the country with the highest living standards based on the human development index. LC model is used since it is regarded as the golden model for mortality data due to the simplicity in parameter estimation and it gives a good fit over a wide range of ages. The data set contains Norway mortality data from 1846-2014. The Singular Value Decomposition (SVD) approach is used for estimating the parameters of LC model. Auto Regressive Integrated Moving Average (ARIMA) time series model is used for forecasting the mortality values.

Table 5 Actual and fitted values for 2005 to 2007

Year	**2005**		**2006**		**2007**	
k	**-25.59506215**		**-25.86935037**		**-26.04657306**	
Age group	**Fitted**	**Actual**	**Fitted**	**Actual**	**Fitted**	**Actual**
0-4	0.004768	0.00307	0.004667	0.0032	0.004602	0.00307
5-9	0.000235	0.00021	0.000228	0.00018	0.000223	0.00016
10-14	0.000115	0.00011	0.000112	0.00012	0.00011	0.00007
15-19	0.000113	0.00012	0.00011	0.00006	0.000109	0.00012
20-24	0.000342	0.00036	0.000335	0.00037	0.000331	0.0003
25-29	0.000417	0.0007	0.000408	0.00058	0.000403	0.00056
30-34	0.000429	0.00061	0.00042	0.00062	0.000414	0.00065
35-39	0.000526	0.00066	0.000516	0.00069	0.000509	0.00057
40-44	0.000753	0.00096	0.00074	0.00084	0.000731	0.00077
44-49	0.001226	0.00128	0.001209	0.00111	0.001197	0.00112
50-54	0.002137	0.00205	0.002113	0.00206	0.002097	0.00199
55-59	0.003654	0.00327	0.003621	0.00319	0.0036	0.00311

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Table continued...

Year	2005		2006		2007	
k	-25.59506215		-25.86935037		-26.04657306	
Age group	Fitted	Actual	Fitted	Actual	Fitted	Actual
60-64	0.006154	0.00518	0.006109	0.00502	0.00608	0.00487
65-69	0.010296	0.0083	0.010233	0.0085	0.010192	0.00818
70-74	0.017088	0.01318	0.016995	0.01259	0.016935	0.01311
75-79	0.029421	0.02214	0.029287	0.02106	0.0292	0.0211
80-84	0.050015	0.03884	0.049805	0.03711	0.049668	0.03776
85-89	0.088397	0.06745	0.088134	0.06781	0.087964	0.06839
90+	0.188957	0.167835	0.188665	0.167123	0.188476	0.167564

Table 6 Actual and fitted values for 2008- 2010

Year	2008		2009		2010	
k	-26.30536036		-26.62866133		-26.97586597	
Age group	Fitted	Actual	Fitted	Actual	Fitted	Actual
0-4	0.004509	0.00273	0.004396	0.00313	0.004278	0.00276
5-9	0.000216	0.00017	0.000209	0.00015	0.0002	0.00012
10-14	0.000107	0.0001	0.000104	0.00011	9.99E-05	0.00009
15-19	0.000106	0.0001	0.000103	0.0001	9.95E-05	0.00009
20-24	0.000325	0.00036	0.000317	0.00039	0.000309	0.00031
25-29	0.000395	0.00061	0.000385	0.00051	0.000375	0.00058
30-34	0.000406	0.00062	0.000396	0.00063	0.000386	0.00064
35-39	0.0005	0.0006	0.000489	0.00064	0.000477	0.00059
40-44	0.000719	0.00081	0.000704	0.00082	0.000689	0.00068
44-49	0.001181	0.00118	0.001161	0.00115	0.00114	0.0011
50-54	0.002075	0.00181	0.002047	0.00178	0.002017	0.00177
55-59	0.003569	0.00311	0.003531	0.00316	0.00349	0.00301
60-64	0.006038	0.0047	0.005986	0.00477	0.005931	0.0048
65-69	0.010133	0.00812	0.010059	0.00779	0.009981	0.00776
70-74	0.016848	0.01284	0.01674	0.01256	0.016624	0.01262
75-79	0.029074	0.02081	0.028918	0.02	0.028751	0.01994
80-84	0.049471	0.03713	0.049225	0.03519	0.048963	0.0351
85-89	0.087718	0.06697	0.08741	0.06535	0.08708	0.06402
90+	0.188202	0.164542	0.187859	0.163347	0.187491	0.161117

Table 7 Actual and fitted values for 2011 to 2014

Year	2011		2012		2013		2014	
k	-27.31098443		-27.63534578		-27.95428909		-28.27498506	
Age group	Fitted	Actual	Fitted	Actual	Fitted	Actual	Fitted	Actual
0-4	0.004166	0.00232	0.004061	0.00248	0.003961	0.00245	0.003862	0.00243
5_9	0.000193	0.00014	0.000186	0.00014	0.000179	0.00012	0.000172	0.00012
10_14	9.66E-05	0.00007	9.34E-05	0.00008	9.04E-05	0.00006	8.75E-05	0.00008
15_19	9.64E-05	0.00011	9.36E-05	0.00012	9.08E-05	0.00005	8.81E-05	0.00007
20_24	0.000301	0.00047	0.000294	0.00023	0.000287	0.00027	0.00028	0.00021
25_29	0.000365	0.00053	0.000356	0.00042	0.000347	0.00042	0.000339	0.00039
30_34	0.000376	0.00054	0.000367	0.00046	0.000358	0.00047	0.000349	0.00046
35_39	0.000465	0.0007	0.000455	0.00057	0.000444	0.00056	0.000434	0.00056
40_44	0.000674	0.00075	0.000661	0.00074	0.000647	0.00075	0.000634	0.00073
44_49	0.00112	0.00103	0.001101	0.001	0.001082	0.00108	0.001064	0.00092
50_54	0.001989	0.00186	0.001963	0.00164	0.001937	0.0016	0.001911	0.00156
55_59	0.003451	0.0028	0.003414	0.00282	0.003378	0.00275	0.003342	0.00274
60-64	0.005878	0.00479	0.005827	0.0044	0.005778	0.00464	0.005729	0.00414
65_69	0.009906	0.00737	0.009834	0.00749	0.009763	0.00719	0.009693	0.00677
70_74	0.016514	0.01202	0.016407	0.01199	0.016303	0.01193	0.0162	0.01122
75_79	0.028591	0.01917	0.028436	0.01936	0.028285	0.01875	0.028135	0.018
80_84	0.048711	0.03452	0.048468	0.03378	0.048231	0.03329	0.047993	0.03186
85_89	0.086764	0.06267	0.086459	0.06349	0.086159	0.06083	0.08586	0.05876
90+	0.187137	0.160931	0.186795	0.161875	0.18646	0.15791	0.186123	0.15525

In this study 97.5 % temporal variance of Norway mortality data could be explained by the 1st SVD component. The best fitting ARIMA model for Norway data is identified as ARIMA (3,2,1) with drift which gave the lowest Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values. The general pattern of mortality showed higher child mortality for ages below 4 years and an accidental hump around ages 20 and nearly exponential increase after the age of 25. The sensitivity of mortality has shown mortality decline at high rate for ages 20-25 years. Mortality index has shown decreasing trend and two spikes due to World War I and World War II. The predicted Lee Carter model gives a good fit to Norway data over a wide range of ages but shows poor performance below age of 4 years and after age of 55 years. Therefore, an improvement in the LC model is needed to obtain better predictions for these two age categories.

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Conflicts of interest

The authors declare that they have no conflict interests.

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