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Discrete shanker distribution and its derived distributions

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Introduction

One parameter continuous Shanker distribution introduced by Shanker (2015 b) with parameter θ is defined by its probability density function (pdf).

$$f(x;\theta)=\frac{\theta^2}{\theta^2+1}(\theta+x)e^{-\theta x}.\ x>0.\theta>0. \quad (1.1)$$

Discretization of continuous distribution

Discretization of continuous distribution can be done using different methodologies. In this paper we deal with the derivation of a new discrete distribution which takes values in $\{0,1,...\}$. This new distribution is generated by discretizing the continuous survival function of the Shanker distribution, which is may be obtained as

$$S(x)=\int_x^\infty f(x;\theta)\,dx$$

$$=\frac{\theta^2+1+\theta x}{\theta^2+1}e^{-\theta x},\quad x>0.\ \theta>0. \quad (2.1)$$

$$S(x+1)=\frac{\theta^2+1+\theta(x+1)}{\theta^2+1}e^{-\theta(x+1)},\quad x>0.\ \theta>0. \quad (2.2)$$

The probability mass function (pmf) of discrete Shanker distribution may be obtained as

$$P(X=x)=S(x)-S(x+1)$$

$$=\frac{(\theta^2+1+\theta x)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}{\theta^2+1}e^{(-\theta x)},\quad x=0,1,2,3 \quad (2.3)$$

Probability recurrence relation

Probability recurrence relation of discrete Shanker distribution may be obtained as

$$P_{(r+2)}=e^{-\theta}(2P_{r+1}-e^{-\theta}P_r),\ r\ \geq 1 \quad (2.5)$$

Where $P_0=\frac{(\theta^2+1)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}{\theta^2+1}$, and

$$P_1=\frac{(\theta^2+1+\theta)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}{\theta^2+1}e^{-\theta} \quad (2.6)$$

Factorial moment recurrence relation

Factorial moment generating function (fmgf) may be obtained as

$$M(t)=G(1+t)$$

$$=\frac{\left(\theta^2+1\right)\left(1-e^{-\theta}\right)-\theta}{\left(\theta^2+1\right)\left(1-e^{-\theta}-e^{-\theta}t\right)}+\frac{\theta\left(1-e^{-\theta}\right)}{\left(\theta^2+1\right)\left(1-e^{-\theta}-e^{-\theta}t\right)^2} \quad (2.7)$$

The more general form of factorial moment may also be written as

$$\mu'_{[r]}=\frac{r!e^{-\theta r}\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta r\right]}{\left(\theta^2+1\right)\left(1-e^{-\theta}\right)^{r+1}} \quad (2.8)$$

Size-biased discrete shanker (SBDJ) distribution

If a random variable X have discrete Shanker distribution with parameter θ then the pmf of the size-biased distribution may be derived as

$$f_s(x;\theta)=\frac{xP_x}{\mu},\ x=1,\ 2,\ 3,\ \ldots \quad (3.1)$$

Where P_x and μ denote respectively pmf and the mean of discrete Shanker distribution.

The pmf $f_s(x;\theta)$ of size- biased discrete Shanker distribution with parameters θ may be derived from (3.1) as

$$f_s(x,\alpha)=\frac{xp_x}{\mu}$$

$$= xe^{-\theta(x-1)} \frac{\left[(\theta^2+1+\theta x)\left(1-e^{-\theta}\right)-\theta e^{-\theta}\right]\left(1-e^{-\theta}\right)^2}{\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta\right]} \quad x = 1, 2, 3, \ldots \tag{3.2}$$

Recurrence relation of size- biased discrete shanker distribution

Probability generating function $G_s(t)$ for Size- biased Discrete Shanker Distribution may be obtained as

$$G_s(t) = \sum_{x=0}^{\infty} t^x P_x$$

$$= t\frac{\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)-\theta e^{-\theta}\right]\left(1-e^{-\theta}\right)^2\left(1-e^{-\theta}t\right)+\theta\left(1-e^{-\theta}\right)^3\left(1+e^{-\theta}t\right)}{\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta\right]\left(1-e^{-\theta}t\right)^3} \tag{3.3}$$

Probability recurrence relation for size- biased discrete shanker distribution

Probability recurrence relation of Size- biased Discrete Shanker Distribution distribution may be obtained as

$$P_r = e^{-\theta}\left[3P_{r-1}-3e^{-\theta}P_{r-2}+e^{-\theta}P_{r-3}\right] \text{for } r > 2 \tag{3.4}$$

where

$$P_1 = \frac{\left[(\theta^2+1+\theta)\left(1-e^{-\theta}\right)-\theta e^{-\theta}\right]\left(1-e^{-\theta}\right)^2}{\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta\right]} \text{ and} \tag{3.5}$$

$$P_2 = 2e^{-}\frac{\left[(\theta^2+1+2\theta)\left(1-e^{-\theta}\right)-\theta e^{-\theta}\right]\left(1-e^{-\theta}\right)^2}{\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta\right]}$$

Factorial moment recurrence relation for size- biased discrete shanker distribution

Factorial moment generating function $M_s(t)$ of Size- biased discrete Shanker distribution may be obtained as

$$M_s(t) = G_s(1+t)$$

$$\mathrm{M}(\mathrm{t}) = (1+t)\frac{\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)-\theta e^{-\theta}\right]\left(1-e^{-\theta}\right)^2\left(1-e^{-\theta}-e^{-\theta}t\right)+\theta\left(1-e^{-\theta}\right)^3\left(1+e^{-\theta}+e^{-\theta}t\right)}{\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta\right]\left(1-e^{-\theta}-e^{-\theta}t\right)^3} \tag{3.6}$$

More general form $$\mu_{[r]} = \frac{r!e^{-\theta(r-1)}\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)\left(r+e^{-\theta}\right)+\theta\left(r^2-e^{-\theta}\right)\right]}{\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta\right]\left(1-e^{-\theta}\right)^r} \tag{3.7}$$

Factorial moment recurrence relation of Size- biased discrete Shanker distribution may be obtained as

$$\mu'_{[r]} = \frac{e^{-\theta}}{A^3}\left[3A^2r\mu'_{[r-1]}-3Ar(r-1)e^{-\theta}\mu'_{[r-2]}+Ar(r-1)(r-2)e^{-2\theta}\mu'_{[r-3]}\right], \tag{3.7}$$

Where $A = 1-e^{-\theta}$.

$$\mu_{[1]} = \frac{\left[\left(\theta^2+1\right)\left(1+e^{-\theta}\right)+\theta\right]}{\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta\right]}$$

$$\mu_{[2]} = \frac{2e^{-\theta}\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)\left(2+e^{-\theta}\right)+\theta\left(4-e^{-\theta}\right)\right]}{\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta\right]\left(1-e^{-\theta}\right)^2} \tag{3.8}$$

$$\mu_{[3]} = \frac{6e^{-2\theta}\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)\left(3+e^{-\theta}\right)+\theta\left(9-e^{-\theta}\right)\right]}{\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta\right]\left(1-e^{-\theta}\right)^3}$$

Method of estimation of shanker distribution

The parameter θ of Shanker distribution has been estimated using Newton's –Raphson method by considering appropriate initial guest value for θ. The function of θ can be expressed as

Fitting of discrete shanker distribution

Shanker et al.,[1] fitted Poisson distribution (PD), Poisson- Lindley distribution (PLD) and Poisson-Akash distribution (PAD) to eleven numbers of data sets covering ecology, genetics and thunderstorms. In this investigation discrete Shanker (DS) distribution has been fitted to all 11 data sets have been considered for a comparison (Tables 1-11).[2-36]

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Table 1 Observed and expected number of homocytometer yeast cell counts per square observed by gosset

No. of yeast cell per square	Observed	Expected frequency			
	Frequency	DS	PD	PLD	PAD
0	213	213	202.1	234	236.8
1	128	109.15	138	99.4	95.6
2	37	47.44	47.1	40.5	39.9
3	18	19	10.7	16	16.6
4	3	7.25	1.8	6.2	6.7
5	1	2.67	0.2	2.4	2.7
6	0	1.48	0.1	1.5	1.7
Total	400	400	400	400	400
	Estimated θ	1.1621	0.6825	1.950236	2.260342
	χ^2	7.89	10.08	11.04	14.68
	d.f.	3	2	2	2
	p- vale	0.0468	0.0065	0.004	0.0006

Table 2 Observed and expected number of red mites on apple leaves

No. of yeast cell per square	Observed	Expected frequency			
	Frequency	DS	PD	PLD	PAD
0	38	38	25.3	35.8	36.3
1	17	22.4	29.1	20.7	20.1
2	10	11.02	16.7	11.4	11.2
3	9	4.97	6.4	6	6.1
4	3	2.13	1.8	3.1	3.2
5	2	0.88	0.4	1.6	1.6
6	1	0.36	0.2	0.8	0.8
7	0	0.14	0.1	0.6	0.7
Total	80	80	80	80	80
	Estimated θ	1.0494	1.15	1.255891	1.620588
	χ^2	6.246	18.27	2.47	2.07
	d.f.	4	2	3	3
	p- vale	0.1815	0.0001	0.4807	0.558

Table 3 Observed and expected number of european corn-border of McGuire et al

No. of yeast cell per square	Observed	Expected frequency			
	Frequency	DS	PD	PLD	PAD
0	188	187.98	169.4	194	196.3
1	83	85.03	109.8	79.5	76.5
2	36	33.03	35.6	31.3	30.8
3	14	11.88	7.8	12	12.4
4	2	4.07	1.2	4.5	4.9
5	1	1.99	0.2	2.7	3.1
Total	324	324	324	324	324
	Estimated θ	1.2644	0.648148	2.043252	2.345109
	χ^2	0.367	15.19	1.29	2.33
	d.f.	3	2	2	2
	p- vale	0.9470	0.0005	0.5247	0.3119

Table 4 Distribution of number of chromatid aberrations

No. of yeast cell per square	Observed	Expected frequency			
	Frequency	DS	PD	PLD	PAD
0	268	268	231.3	257	260.4
1	87	87	92.8	126.7	93.4
2	26	28.23	34.7	32.8	32.1
3	9	8.01	6.3	11.2	11.5
4	4	2.18	0.8	3.8	4.1
5	2	0.58	0.1	1.2	1.4
6	1	0.15	0.1	0.4	0.5
7	3	0.05	0.1	0.2	0.3
Total	400	400	400	400	400
	Estimated θ	1.4870	0.5475	2.380442	2.659408
	χ^2	6.417	38.21	6.21	4.17
	d.f.	3	2	3	3
	p- vale	0.0930	0	0.1018	0.2437

Table 5 Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), exposure-60 $\mu g / kg$

No. of yeast cell per square	Observed	Expected frequency			
	Frequency	DS	PD	PLD	PAD
0	413	413.01	374	405.7	409.5
1	124	134.97	177.4	133.6	128.7
2	42	38.92	42.1	42.6	42.1
3	15	10.49	6.6	13.3	13.9
4	5	2.71	0.8	4.1	4.6

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Table continued...

No. of yeast cell per square	Observed	Expected frequency			
	Frequency	DS	PD	PLD	PAD
5	0	0.68	0.1	1.2	1.5
6	2	0.22	0	0.5	0.7
Total	601	601	601	601	601
	Estimated θ	1.5385	0.47421	2.685373	2.915059
	χ^2	5.562	48.17	1.34	0.29
	d.f.	3	2	3	3
	p- vale	0.1350	0	0.7196	0.9619

Table 6 Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), exposure-75 $\mu g\ /\ kg$

No. of yeast cell per square	Observed	Expected frequency			
	Frequency	DS	PD	PLD	PAD
0	200	200.01	172.5	191.8	194.1
1	57	70.01	95.4	70.3	67.6
2	30	21.51	26.4	24.9	24.5
3	7	6.17	4.9	8.6	8.9
4	4	1.69	0.7	2.9	3.2
5	0	0.45	0.1	1	1.1
6	2	0.16	0	0.5	0.6
Total	300	300	300	300	300
	Estimated θ	1.4798	0.55333	2.35334	2.62674
	χ^2	8.191	29.68	3.91	3.12
	d.f.	3	2	2	2
	p- vale	0.0422	0	0.1415	0.2101

Table 7 Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), exposure-90 $\mu g\ /\ kg$

No. of yeast cell per square	Observed	Expected frequency			
	Frequency	DS	PD	PLD	PAD
0	155	155.01	127.8	158.3	160.7
1	83	82.63	109	77.2	74.3
2	33	37.2	46.5	35.9	35.3
3	14	15.41	13.2	16.1	16.5
4	11	6.08	2.8	7.1	7.5
5	3	2.32	0.5	3.1	3.3
6	1	1.35	0.2	2.3	2.4
Total	300	300	300	300	300
	Estimated θ	1.1301	0.853333	1.617611	1.963313
	χ^2	3.432	24.97	1.51	1.98
	d.f.	4	2	3	3
	p- vale	0.4883	0	0.6799	0.5766

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Table 8 Observed abd expected number of days that experienced X thunderstroms event at cape kennedy, florida for 11-year period of record for the month of june, january 1957 to december 1967

No. of yeast cell per square	Observed	Expected frequency			
	Frequency	DS	PD	PLD	PAD
0	187	187.01	155.6	185.3	187.9
1	77	87.72	117	83.5	80.2
2	40	35.21	43.9	35.9	35.3
3	17	13.07	11	15	15.4
4	6	4.62	2.1	6.1	6.6
5	2	1.58	0.3	2.5	2.7
6	1	0.79	0.1	1.7	1.9
Total	330	330	330	330	330
	Estimated χ^2	1.2345	0.751515	1.804268	2.139736
	χ^2	3.721	31.93	1.43	1.35
	d.f.	4	2	3	3

Table 9 Observed abd expected number of days that experienced X thunderstroms event at cape kennedy, florida for 11-year period of record for the month of july, january 1957 to december 1967

No. of yeast cell per square	Observed	Expected frequency			
	Frequency	DS	PD	PLD	PAD
0	177	177.01	142.3	177.7	180
1	80	93.79	124.4	88	84.7
2	47	42.01	54.3	41.5	40.9
3	26	17.32	15.8	18.9	19.4
4	9	7.79	3.5	8.4	8.9
5	2	3.08	0.7	6.5	7.1
Total	341	341	341	341	341
	Estimated θ	1.1348	0.8739	1.583536	1.938989
	χ^2	6.972	39.74	5.15	5.02
	d.f.	4	2	3	3
	p- vale	0.1374	0	0.1611	0.1703

Table 10 Observed abd expected number of days that experienced X thunderstroms event at cape kennedy, florida for 11-year period of record for the month of august, january 1957 to december 1967

No. of yeast cell per square	Observed	Expected frequency			
	Frequency	DS	PD	PLD	PAD
0	185	184.99	151.8	184.8	187.5
1	89	92.42	122.9	87.2	83.9
2	30	39.27	49.7	39.3	38.6
3	24	15.39	13.4	17.1	17.5
4	10	6.74	2.7	7.3	7.6
5	3	2.19	0.5	5.3	5.9
Total	341	341	341	341	341
	Estimated θ	1.1828	0.809384	1.693425	2.038417
	χ^2	8.987	49.49	5.03	4.69
	d.f.	4	2	3	3
	p- vale	0.0414	0	0.1696	0.196

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Table 11 Observed abd expected number of days that experienced X thunderstroms event at cape kennedy, florida for 11-year period of record for summesr, january 1957 to december 1967

No. of yeast cell per square	Observed	Expected frequency			
	Frequency	DS	PD	PLD	PAD
0	549	549.01	449	547.5	555.1
1	246	274.27	364.8	259	249.2
2	117	117	116.54	148.2	116.9
3	67	45.67	40.1	51.2	52.3
4	25	17.04	8.1	21.9	23.2
5	7	7.16	1.3	9.2	10
6	1	2.31	0.5	6.3	7.3
Total	1012	1012	1012	1012	1012
	Estimated θ	1.1828	0.812253	1.68899	2.033715
	χ^2	16.824	119.45	9.6	9.4
	d.f.	5	3	4	4
	p- vale	0.0048	0	0.0477	0.0518

Conclusions

In this article, the discrete Shanker distribution has been introduced by discretizing the continuous Shanker distribution. We have studied some properties of the distributions. Further the applications of the distribution and goodness of fit of the distribution.

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Conflict of interest

None.

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