On Modeling of Lifetime Data Using Aradhana, Sujatha, Lindley and Exponential Distributions

Abstract

The modeling and statistical analysis of lifetime data are crucial for statisticians and research workers in almost all applied sciences including engineering, medical sciences/biological sciences, insurance, finance, amongst others. One parameter lifetime distributions that are popular in Statistics literature for modeling lifetime data are exponential and Lindley distributions. An extensive study has been carried out by Shanker et al. [1] for modeling lifetime data using Lindley and exponential distributions and observed that there are many lifetime data where these distributions are not suitable from theoretical and applied point of view. Recently Shanker [2,3] has introduced one parameter Lifetime distributions namely "Aradhana distribution" and "Sujatha distribution" for modeling lifetime data.

In the present paper the interrelationships and comparative studies of Aradhana, Sujatha, Lindley and exponential distributions have been made to model lifetime data. The relationships, their distributional properties and estimation of parameter have been discussed. The applications and goodness of fit of these distributions for modeling lifetime data through various examples from engineering, medical science and other fields have also been discussed and explained.

Keywords: Aradhana distribution; Sujatha distribution; Lindley distribution; Exponential distribution; Statistical properties; Estimation of parameter; Goodness of fit

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Introduction

The time to the occurrence of event of interest is known as lifetime or survival time or failure time in reliability analysis. The event may be failure of a piece of equipment, death of a person, development (or remission) of symptoms of disease, health code violation (or compliance). The modeling and statistical analysis of lifetime data are crucial for statisticians and research workers in almost all applied sciences including engineering, medical science/biological science, insurance and finance, amongst others.

Shanker [2,3] has introduced one parameter continuous distributions named, "Aradhana distribution" and "Sujatha distribution"for modeling lifetime data from engineering and medical science and studied its various mathematical properties, estimation of its parameter, and its applications. A number of continuous distributions for modeling lifetime data have been introduced in statistical literature including exponential, Lindley, gamma, lognormal and Weibull, amongst others. The exponential, Lindley and the Weibull distributions are more popular in practice than the gamma and the lognormal distributions because the survival functions of the gamma and the lognormal distributions cannot be expressed in closed forms and both require numerical integration. Though Aradhana, Sujatha, Lindley and exponential distributions are of one parameter, Aradhana, Sujatha and Lindley distributions have advantage over the exponential distribution that the exponential distribution has constant hazard rate and mean residual life function whereas the Aradhana, Sujatha, and Lindley distributions have increasing hazard rate and decreasing mean residual life function. Further, Aradhana and Sujatha distributions of Shanker [2,3] have flexibility over both Lindley and exponential distributions.

Aradhana, Sujatha, Lindley and Exponential Distributions

Shanker [2] introduced a new one parameter continuous distribution named, 'Aradhana distribution' for modeling lifetime data from engineering and medical science. This distribution is a three- component mixture of an exponential (θ) distribution, a gamma $(2,\theta)$ distribution and a gamma $(3,\theta)$ distribution with their mixing proportions $\frac{\theta^2}{\theta^2+2\theta+2}$, $\frac{2\theta}{\theta^2+2\theta+2}$ and $\frac{2}{\theta^2+2\theta+2}$ respectively. It has been shown by Shanker [2] that Aradhana distribution is flexible than the Lindley distribution for modeling lifetime data in reliability and in terms of its hazard rate shapes and it gives better fit than Akash, Shanker, Lindley and exponential distributions in modeling lifetime data. Shanker [2] has discussed its various mathematical and statistical properties including its shape, moment generating function, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic orderings, mean deviations, distribution of order statistics, Bonferroni and Lorenz curves, Renyi entropy measure, stress-strength reliability, amongst others. Shanker [4] has also obtained a Poisson mixture of Aradhana distribution named, “Poisson-Aradhana distribution (PAD)”for modeling count data.

Shanker [3] introduced another one parameter continuous distribution named, ‘Sujatha distribution’ for modeling lifetime data from engineering and medical science. This distribution is also a three-component mixture of an exponential (θ) distribution, a gamma $(2,\theta)$ distribution and a gamma $(3,\theta)$ distribution with their mixing proportions $\frac{\theta^2}{\theta^2+\theta+2}$, $\frac{\theta}{\theta^2+\theta+2}$ and $\frac{2}{\theta^2+\theta+2}$ respectively. It has been shown by Shanker [3] that Sujatha distribution is flexible than the Lindley distribution for modeling lifetime data in reliability and in terms of its hazard rate shapes and it gives better fit than Lindley and exponential distributions in modeling lifetime data. Shanker [3] has discussed its various mathematical and statistical properties including its shape, moment generating function, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic orderings, mean deviations, distribution of order statistics, Bonferroni and Lorenz curves, Renyi entropy measure, stress-strength reliability, amongst others. Shanker [5] has also obtained a Poisson mixture of Sujatha distribution named, “Poisson-Sujatha distribution (PSD)”for modeling count data.

The Lindley distribution is a two-component mixture of an exponential (θ) distribution and a gamma $(2,\theta)$ distribution with their mixing proportions $\frac{\theta}{\theta+1}$ and $\frac{1}{\theta+1}$ respectively and is given by Lindley [6] in the context of Bayesian Statistics as a counter example of fiducial Statistics. A detailed study about its various mathematical properties, estimation of parameter and application showing the superiority of Lindley distribution over exponential distribution for the waiting times before service of the bank customers has been done by Ghitany et al. [7]. The Lindley distribution has been generalized, extended, mixed, modified and its detailed applications in reliability and other fields of knowledge by different researchers including Sankaran [8], Zakerzadeh & Dolati [9], Nadarajah et al. [10], Deniz & Ojeda [11], Bakouch et al. [12], Shanker & Mishra [13,14,15], Shanker & Amanuel [16], Ghitany et al. [17], Shanker et al. [1,18-21], are some among others.

In statistical literature, exponential distribution was the first widely used lifetime distribution model in areas ranging from studies on the lifetimes of manufactured items to research involving survival or remission times in chronic diseases. The main reason for its wide usefulness and applicability as lifetime model is partly because of the availability of simple statistical methods for it and partly because it appeared suitable for representing the lifetimes of many phenomenons such as various types of manufactured items.

Let T be a continuous random variable representing the lifetimes of individuals in some population. The expressions for probability density function, $f(t)$, cumulative distribution function, $F(t)$, hazard rate function, $h(t)$, mean residual life function, $m(t)$, mean μ_1', variance μ_2, coefficient of variation (C.V.), coefficient of Skewness $(\sqrt{\beta_1})$, coefficient of Kurtosis (β_2), and index of dispersion (γ) of Aradhana and Sujatha distributions are summarized in Table 1 and of Lindley and exponential distributions in Table 2.

A table of values for coefficient of variation (C.V.), coefficient of Skewness $(\sqrt{\beta_1})$, coefficient of Kurtosis (β_2), and index of dispersion (γ) for Aradhana, Sujatha and Lindley distributions for various values of their parameter for comparative study are summarized in the Table 3.

The condition under which Aradhana, Sujatha, Lindley and exponential distributions are Over-dispersion $(\mu<\sigma^2)$, equi-dispersion $(\mu=\sigma^2)$ and under-dispersion $(\mu>\sigma^2)$ of Aradhana, Sujatha, Lindley and exponential distributions for varying values of their parameter are presented in Table 4.

Graphs of coefficient of variation (C.V), coefficient of skewness $(\sqrt{\beta_1})$ coefficient of kurtosis (β_2) and index of dispersion (γ) for Aradhana, Sujatha, and Lindley distributions are presented for varying values of their parameter θ in Figure 1.

Estimation of Parameter

Estimate of the parameter of Aradhana distribution

Let $(t_1,t_2,t_3,\ldots,t_n)$ be a random sample from Aradhana distribution. The maximum likelihood estimate (MLE) $\hat{\theta}$ of θ and the method of moment estimate (MOME) of is the solution of the following cubic equation

$$\bar{t}\,\theta^3+\left(2\bar{t}-1\right)\theta^2+2\left(\bar{t}-2\right)\theta-6=0\,.$$

Estimate of the parameter of Sujatha distribution

Let $(t_1,t_2,t_3,\ldots,t_n)$ be random sample from Sujatha distribution. The maximum likelihood estimate (MLE) $\hat{\theta}$ of θ and the method of moment estimate (MOME) of is the solution of the following cubic equation

$$\bar{t}\,\theta^3+\left(\bar{t}-1\right)\theta^2+2\left(\bar{t}-1\right)\theta-6=0$$

Estimate of the parameter of Lindley distribution

Let $(t_1,t_2,t_3,\ldots,t_n)$ be a random sample of size n from Lindley distribution. The MLE $\hat{\theta}$ of θ and MOME $\hat{\theta}$ of θ is given by $\hat{\theta}=\frac{-(\bar{t}-1)+\sqrt{(\bar{t}-1)^2+8\bar{t}}}{2\bar{t}};\quad \bar{t}>0$, where $\bar{t}$ is the sample mean.

Estimate of the parameter of Exponential distribution

Let $(t_1,t_2,t_3,\ldots,t_n)$ be a random sample of size n from exponential distribution. The MLE $\hat{\theta}$ of θ and MOME $\hat{\theta}$ of θ is given by $\hat{\theta}=\frac{1}{\bar{t}}$, where $\bar{t}$ is the sample mean.

Applications and Goodness of fit

In this section the goodness of fit test of Aradhana, Sujatha, Lindley and exponential distributions for following sixteen real lifetime data- sets using maximum likelihood estimate have been discussed.

In order to compare Aradhana, Sujatha, Lindley and exponential distributions, $-2\ln L$, AIC (Akaike Information Criterion), AICC (Akaike Information Criterion Corrected), BIC (Bayesian Information Criterion), K-S Statistics (Kolmogorov-Smirnov Statistics) for all sixteen real lifetime data- sets have been computed and presented in Table 5. The formulae for computing AIC, AICC, BIC, and K-S Statistics are as follows:

$$AIC = -2\ln L + 2k,\quad AICC = AIC + \frac{2k(k+1)}{(n-k-1)},\quad BIC = -2\ln L + k\ln n$$

and $D = \underset{x}{\text{Sup}}\left|F_n(x) - F_0(x)\right|$, where k = the number of parameters, n = the sample size and $F_n(x)$ is the empirical distribution function.

The best distribution is the distribution which corresponds to lower values of $-2\ln L$, AIC, AICC, BIC, and K-S statistics.

The best fitting has been shown by making -2ln L, AIC, AICC, BIC, and K-S Statistics in bold

Concluding Remarks

In this paper an attempt has been made to find the suitability of Aradhana, Sujatha, Lindley and exponential distributions for modeling real lifetime data from engineering, medical science and other fields. Firstly a table for values of the various characteristics of Aradhana, Sujatha, Lindley and exponential distributions has been presented for different values of their parameter which reflects their nature and behavior. The condition under which Aradhana, Sujatha, Lindley and exponential distributions are over-dispersed, equi-dispersed, and under-dispersed has been given. Several lifetime data from medical science, engineering and other fields of knowledge have been fitted using Aradhana, Sujatha, Lindley and exponential distributions to study the advantages and disadvantages of these distributions. The goodness of fit test of these distributions using Kolmogorov-Smirnov tests indicate that each has advantages and disadvantages for modeling lifetime data.

Table 1: Characteristics of Aradhana and Sujatha Distributions.

Aradhana Distribution	Sujatha Distribution
$f(t) = \frac{\theta^3}{\theta^2+2\theta+2}(1+t)^2 e^{-\theta t}$	$f(t) = \frac{\theta^3}{\theta^2+\theta+2}\left(1+t+t^2\right)e^{-\theta t}$
$F(t) = 1-\left[1+\frac{\theta t(\theta t+2\theta+2)}{\theta^2+2\theta+2}\right]e^{-\theta t}$	$F(t) = 1-\left[1+\frac{\theta t(\theta t+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta t}$
$h(t) = \frac{\theta^3(1+t)^2}{\theta t(\theta t+2\theta+2)+\left(\theta^2+2\theta+2\right)}$	$h(t) = \frac{\theta^3\left(1+t+t^2\right)}{\theta^2\left(1+t+t^2\right)+2\theta t+\theta+2}$
$m(t) = \frac{\theta^2 t^2+2\theta t(\theta+2)+\left(\theta^2+4\theta+6\right)}{\theta\left[\theta t(\theta t+2\theta+2)+\left(\theta^2+2\theta+2\right)\right]}$	$m(t) = \frac{\theta^2\left(t^2+t+1\right)+2\theta(t+1)+6}{\theta\left[\left(\theta^2+\theta+2\right)+\theta t(\theta t+\theta+2)\right]}$
$\mu_1' = \frac{\theta^2+4\theta+6}{\theta\left(\theta^2+2\theta+2\right)}$	$\mu_1' = \frac{\theta^2+2\theta+6}{\theta\left(\theta^2+\theta+2\right)}$
$\mu_2 = \frac{\theta^4+4\theta^3+18\theta^2+12\theta+12}{\theta^2\left(\theta^2+\theta+2\right)^2}$	$\mu_2 = \frac{\theta^4+4\theta^3+18\theta^2+12\theta+12}{\theta^2\left(\theta^2+\theta+2\right)^2}$
$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\theta^4+8\theta^3+24\theta^2+24\theta+12}}{\theta^2+4\theta+6}$	$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\theta^4+4\theta^3+18\theta^2+12\theta+12}}{\theta^2+2\theta+6}$

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$\sqrt{\beta_1}=\dfrac{2\left(\begin{array}{l}\theta^6+12\theta^5+54\theta^4+100\theta^3\\+108\theta^2+72\theta+24\end{array}\right)}{\left(\theta^4+8\theta^3+24\theta^2+24\theta+12\right)^{3/2}}$	$\sqrt{\beta_1}=\dfrac{2\left(\begin{array}{l}\theta^6+6\theta^5+36\theta^4+44\theta^3\\+54\theta^2+36\theta+24\end{array}\right)}{\left(\theta^4+4\theta^3+18\theta^2+12\theta+12\right)^{3/2}}$
$\beta_2=\dfrac{3\left(\begin{array}{l}3\theta^8+48\theta^7+304\theta^6+944\theta^5+1816\theta^4\\+2304\theta^3+1920\theta^2+960\theta+240\end{array}\right)}{\left(\theta^4+8\theta^3+24\theta^2+24\theta+12\right)^2}$	$\beta_2=\dfrac{3\left(\begin{array}{l}3\theta^8+24\theta^7+172\theta^6+376\theta^5+736\theta^4\\+864\theta^3+912\theta^2+480\theta+240\end{array}\right)}{\left(\theta^4+4\theta^3+18\theta^2+12\theta+12\right)^2}$
$\gamma=\dfrac{\sigma^2}{\mu_1'}=\dfrac{\theta^4+8\theta^3+24\theta^2+24\theta+12}{\theta\left(\theta^2+2\theta+2\right)\left(\theta^2+4\theta+6\right)}$	$\gamma=\dfrac{\sigma^2}{\mu_1'}=\dfrac{\theta^4+4\theta^3+18\theta^2+12\theta+12}{\theta\left(\theta^2+\theta+2\right)\left(\theta^2+2\theta+6\right)}$

Table 2: Characteristics of Lindley and Exponential Distributions.

Lindley Distribution	Exponential Distribution
$f(t)=\dfrac{\theta^2}{\theta+1}(1+t)e^{-\theta t}$	$f(t)=\theta e^{-\theta t}$
$F(t)=1-\dfrac{\theta+1+\theta t}{\theta+1}e^{-\theta t}$	$F(t)=1-e^{-\theta t}$
$h(t)=\dfrac{\theta^2(1+t)}{\theta+1+\theta t}$	$h(t)=\theta$
$m(t)=\dfrac{\theta+2+\theta t}{\theta(\theta+1+\theta t)}$	$m(t)=\dfrac{1}{\theta}$
$\mu_1'=\dfrac{\theta+2}{\theta(\theta+1)}$	$\mu_1'=\dfrac{1}{\theta}$
$\mu_2=\dfrac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}$	$\mu_2=\dfrac{1}{\theta^2}$
$C.V=\dfrac{\sigma}{\mu_1'}=\dfrac{\sqrt{\theta^2+4\theta+2}}{\theta+2}$	$C.V=\dfrac{\sigma}{\mu_1'}=1$
$\sqrt{\beta_1}=\dfrac{2\left(\theta^3+6\theta^2+6\theta+2\right)}{\left(\theta^2+4\theta+2\right)^{3/2}}$	$\sqrt{\beta_1}=2$

$\beta_2 = \dfrac{3\begin{pmatrix}3\theta^4+24\theta^3+44\theta^2\\+32\theta+8\end{pmatrix}}{\left(\theta^2+4\theta+2\right)^2}$	$\beta_2 = 9$
$\gamma = \dfrac{\sigma^2}{\mu_1'} = \dfrac{\theta^2+4\theta+2}{\theta(\theta+1)(\theta+2)}$	$\gamma = \dfrac{\sigma^2}{\mu_1'} = \dfrac{1}{\theta}$

Table 3: Values of μ_1', μ_2, C.V, $\sqrt{\beta_1}$, β_2 and θ of Aradhana, Sujatha and Lindley distributions for varying values of the parameter θ.

Values of θ for Aradhana Distribution								
	0.01	0.05	0.1	0.3	0.5	1	1.5	2
μ_1'	299.000	59.001	29.005	9.033	5.077	2.200	1.310	0.900
μ_2	29999.990	1199.954	299.914	33.143	11.763	2.760	1.134	0.590
CV	0.579	0.587	0.597	0.637	0.676	0.755	0.813	0.853
$\sqrt{\beta_1}$	1.155	1.155	1.155	1.167	1.193	1.295	1.402	1.496
β_2	5.000	5.000	5.001	5.024	5.087	5.381	5.758	6.135
γ	100.334	20.338	10.340	3.669	2.317	1.255	0.865	0.656

Values of θ for Aradhana Distribution								
	0.01	0.05	0.1	0.3	0.5	1	1.5	2
μ_1'	299.493	59.464	29.431	9.331	5.273	2.250	1.304	0.875
μ_2	30000.737	1200.69	300.624	33.722	12.198	2.938	1.197	0.609
CV	0.578	0.583	0.589	0.622	0.662	0.762	0.839	0.892
$\sqrt{\beta_1}$	1.155	1.154	1.151	1.140	1.146	1.248	1.397	1.536
β_2	5.000	4.998	4.992	4.955	4.945	5.170	5.656	6.215
γ	100.172	20.192	10.214	3.614	2.313	1.306	0.918	0.696

Values of θ for Aradhana Distribution								
	0.01	0.05	0.1	0.3	0.5	1	1.5	2
μ_1'	199.010	39.048	19.091	5.897	3.333	1.500	0.933	0.667
μ_2	19999.020	799.093	199.174	21.631	7.556	1.750	0.729	0.389
CV	0.711	0.724	0.739	0.789	0.825	0.882	0.915	0.935
$\sqrt{\beta_1}$	1.414	1.417	1.422	1.464	1.512	1.620	1.699	1.756
β_2	6.000	6.007	6.025	6.162	6.343	6.796	7.173	7.469
γ	100.493	20.465	10.433	3.668	2.267	1.167	0.781	0.583

Table 4: Over-dispersion, equi-dispersion and under-dispersion of Aradhana, Sujatha, Lindley and exponential distributions for varying values of their parameter θ.

Distribution	Over-Dispersion $\left(\mu < \sigma^2\right)$	Equi-Dispersion $\left(\mu = \sigma^2\right)$	Under-Dispersion $\left(\mu > \sigma^2\right)$
Aradhana	$\theta < 1.283826505$	$\theta = 1.283826505$	$\theta > 1.283826505$
Sujatha	$\theta < 1.364271174$	$\theta = 1.364271174$	$\theta > 1.364271174$
Lindley	$\theta < 1.170086487$	$\theta = 1.170086487$	$\theta > 1.170086487$
Exponential	$\theta < 1$	$\theta = 1$	$\theta > 1$

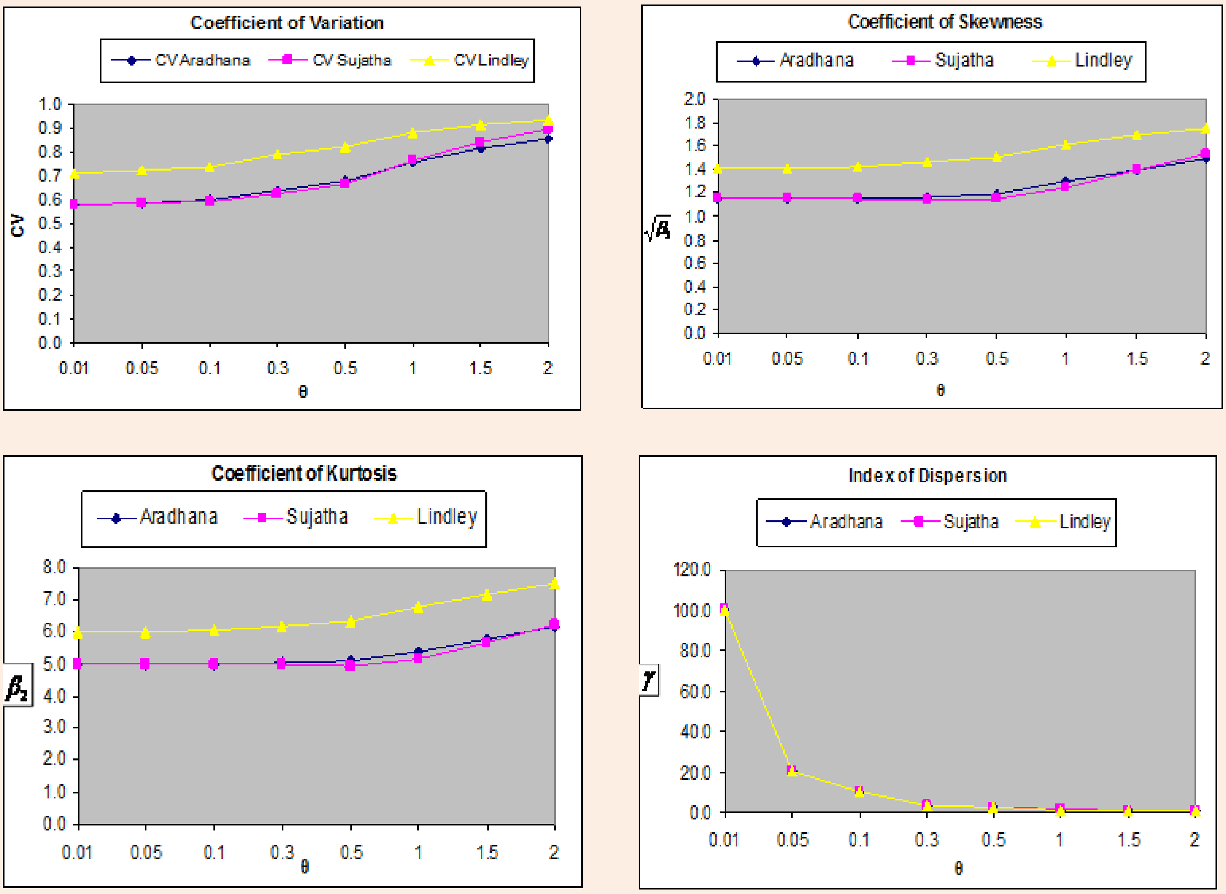

Figure 1: Graphs of coefficient of variation (C.V), coefficient of skewness ($\sqrt{\beta_1}$) coefficient of kurtosis (β_2) and index of dispersion (γ) for Aradhana, Sujatha, and Lindley distributions are for varying values of their parameter θ.

Table 5: MLE's, -2ln L, AIC, AICC, BIC, K-S Statistics of the fitted distributions of Data sets 1-16.

	Model	Parameter Estimate	-2ln L	AIC	AICC	BIC	K-S Statistic
Data 1	Aradhana	1.346393	149.88	151.88	151.94	154.02	0.345
	Sujatha	1.350050	154.81	156.81	156.87	158.95	0.349
	Lindley	0.996116	162.56	164.56	164.62	166.70	0.371
	Exponential	0.663647	177.66	179.66	179.73	181.80	0.402
Data 2	Aradhana	0.043272	952.58	954.58	954.62	957.18	0.186
	Sujatha	0.043566	951.78	953.78	953.97	954.91	0.185
	Lindley	0.028859	983.11	985.11	985.15	987.71	0.242
	Exponential	0.014635	1044.87	1046.87	1046.91	1049.48	0.357
Data 3	Aradhana	0.040968	227.28	229.28	229.47	230.41	0.108
	Sujatha	0.041232	227.17	229.17	229.36	230.30	0.107
	Lindley	0.027321	231.47	233.47	233.66	234.61	0.149
	Exponential	0.013845	242.87	244.87	245.06	246.01	0.263
Data 4	Aradhana	0.013454	1255.26	1257.26	1257.30	1259.86	0.069
	Sujatha	0.013484	1255.54	1257.54	1257.58	1260.14	0.070
	Lindley	0.00897	1251.34	1253.34	1253.38	1255.95	0.098
	Exponential	0.004505	1280.52	1282.52	1282.56	1285.12	0.190
Data 5	Aradhana	0.029756	794.28	796.28	796.34	798.56	0.182
	Sujatha	0.029898	794.48	796.48	796.54	798.77	0.183
	Lindley	0.019841	789.04	791.04	791.10	793.32	0.133
	Exponential	0.010018	806.88	808.88	808.94	811.16	0.198
Data 6	Aradhana	0.115577	989.49	991.49	991.52	994.39	0.399
	Sujatha	0.117453	985.69	987.69	987.72	990.59	0.396
	Lindley	0.077247	1041.64	1043.64	1043.68	1046.54	0.448
	Exponential	0.04006	1130.26	1132.26	1132.29	1135.16	0.525
Data 7	Aradhana	0.013206	801.83	803.83	803.90	805.89	0.297
	Sujatha	0.013234	802.84	804.84	804.91	806.90	0.298
	Lindley	0.008804	763.75	765.75	765.82	767.81	0.245
	Exponential	0.004421	744.87	746.87	746.94	748.93	0.166
Data 8	Aradhana	0.013364	608.87	610.87	610.96	612.65	0.278
	Sujatha	0.013394	609.39	611.39	611.48	613.17	0.279
	Lindley	0.008910	579.16	581.16	581.26	582.95	0.219
	Exponential	0.004475	564.02	566.02	566.11	567.80	0.145
Data 9	Aradhana	0.290304	874.71	876.71	876.74	879.56	0.179
	Sujatha	0.298963	879.82	881.82	881.85	884.67	0.187
	Lindley	0.196045	839.06	841.06	841.09	843.91	0.116
	Exponential	0.106773	828.68	830.68	830.72	833.54	0.077

Data 10	Aradhana	0.049506	350.55	352.55	352.69	353.95	0.415
	Sujatha	0.049887	352.47	354.47	354.61	355.87	0.418
	Lindley	0.033021	323.27	325.27	325.42	326.67	0.345
	Exponential	0.016779	305.26	307.26	307.40	308.66	0.213
Data 11	Aradhana	1.132874	116.06	118.06	118.18	119.59	0.169
	Sujatha	1.146073	115.54	117.54	117.66	119.07	0.164
	Lindley	0.823821	112.61	114.61	114.73	116.13	0.133
	Exponential	0.532081	110.91	112.91	113.03	114.43	0.089
Data 12	Aradhana	0.276551	638.34	640.34	640.38	642.94	0.080
	Sujatha	0.284621	639.64	641.64	641.68	644.24	0.088
	Lindley	0.186571	638.07	640.07	640.12	642.68	0.058
	Exponential	0.101245	658.04	660.04	660.08	662.65	0.163
Data 13	Aradhana	0.024537	193.60	195.60	195.91	196.31	0.453
	Sujatha	0.024634	193.94	195.94	196.25	196.65	0.454
	Lindley	0.01636	181.34	183.34	183.65	184.05	0.386
	Exponential	0.008246	173.94	175.94	176.25	176.65	0.277
Data 14	Aradhana	1.123193	56.37	58.37	58.59	59.36	0.302
	Sujatha	1.136745	57.50	59.50	59.72	60.49	0.309
	Lindley	0.816118	60.50	62.50	62.72	63.49	0.341
	Exponential	0.526316	65.67	67.67	67.90	68.67	0.389
Data 15	Aradhana	0.094318	242.23	244.23	244.37	245.66	0.274
	Sujatha	0.095610	241.50	243.50	243.64	244.93	0.270
	Lindley	0.062988	253.99	255.99	256.13	257.42	0.333
	Exponential	0.032455	274.53	276.53	276.67	277.96	0.426
Data 16	Aradhana	0.917023	219.90	221.90	221.96	224.13	0.350
	Sujatha	0.936119	221.61	223.61	223.67	225.84	0.362
	Lindley	0.659000	238.38	240.38	240.44	242.61	0.390
	Exponential	0.407941	261.74	263.74	263.80	265.97	0.434

Data Set 1: The data set represents the strength of 1.5cm glass fiber**s** measured at the National Physical Laboratory, England. Unfortunately, the units of measurements are not given in the paper, and they are taken from Smith & Naylor [22].

0.55	0.93	1.25	1.36	1.49	1.52	1.58	1.61	1.64	1.68	1.73	1.81	2.00
0.74	1.04	1.27	1.39	1.49	1.53	1.59	1.61	1.66	1.68	1.76	1.82	2.01
0.77	1.11	1.28	1.42	1.50	1.54	1.60	1.62	1.66	1.69	1.76	1.84	2.24
0.81	1.13	1.29	1.48	1.50	1.55	1.61	1.62	1.66	1.70	1.77	1.84	0.84
1.24	1.30	1.48	1.51	1.55	1.61	1.63	1.67	1.70	1.78	1.89		

Data Set 2: The data is given by Birnbaum & Saunders [23] on the fatigue life of 6061 – T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second. The data set consists of 101 observations with maximum stress per cycle 31,000 psi. The data ($\times 10^{-3}$) are presented below (after subtracting 65).

5	25	31	32	34	35	38	39	39	40	42	43	43
43	44	44	47	47	48	49	49	49	51	54	55	55
55	56	56	56	58	59	59	59	59	59	63	63	64
64	65	65	65	66	66	66	66	66	67	67	67	68
69	69	69	69	71	71	72	73	73	73	74	74	76
76	77	77	77	77	77	77	79	79	80	81	83	83
84	86	86	87	90	91	92	92	92	92	93	94	97
98	98	99	101	103	105	109	136	147				

Data Set 3: The data set is from Lawless [24]. The data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life tests.

17.88	28.92	33.00	41.52	42.12	45.60	48.80	51.84	51.96	54.12	55.56	67.80
68.44	68.64	68.88	84.12	93.12	98.64	105.12	105.84	127.92	128.04	173.40	

Data Set 4: The data is from Picciotto [25] and arose in test on the cycle at which the Yarn failed. The data are the number of cycles until failure of the yarn.

86	146	251	653	98	249	400	292	131	169	175	176	76
264	15	364	195	262	88	264	157	220	42	321	180	198
38	20	61	121	282	224	149	180	325	250	196	90	229
166	38	337	65	151	341	40	40	135	597	246	211	180
93	315	353	571	124	279	81	186	497	182	423	185	229
400	338	290	398	71	246	185	188	568	55	55	61	244
20	284	393	396	203	829	239	236	286	194	277	143	198
264	105	203	124	137	135	350	193	188				

Data Set 5: This data represents the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal [26].

12	15	22	24	24	32	32	33	34	38	38	43	44
48	52	53	54	54	55	56	57	58	58	59	60	60
60	60	61	62	63	65	65	67	68	70	70	72	73
75	76	76	81	83	84	85	87	91	95	96	98	99
109	110	121	127	129	131	143	146	146	175	175	211	233
258	258	263	297	341	341	376						

Data Set 6: This data is related with behavioral sciences, collected by Balakrishnan et al. [27]: The scale "General Rating of Affective Symptoms for Preschoolers (GRASP)" measures behavioral and emotional problems of children, which can be classified with depressive condition or not according to this scale. A study conducted by the authors in a city located at the south part of Chile has allowed collecting real data corresponding to the scores of the GRASP scale of children with frequency in parenthesis.

19(16),	20(15),	21(14),	22(9),	23(12),	24(10),	25(6),	26(9),	27(8),	28(5),	29(6),
30(4),	31(3),	32(4),	33,	34,	35(4),	36(2),	37(2),	39	42	44

Data Set 7: The data set reported by Efron [28] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using radiotherapy (RT).

6.53	7	10.42	14.48	16.1	22.7	34	41.55	42	45.28	49.4	53.62	63
64	83	84	91	108	112	129	133	133	139	140	140	146
149	154	157	160	160	165	146	149	154	157	160	160	165
173	176	218	225	241	248	273	277	297	405	417	420	440
523	583	594	1101	1146	1417							

Data Set 8: The data set reported by Efron [28] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using a combination of radiotherapy and chemotherapy (RT+CT).

12.20	23.56	23.74	25.87	31.98	37	41.35	47.38	55.46	58.36	63.47	68.46	78.26
74.47	81.43	84	92	94	110	112	119	127	130	133	140	146
155	159	173	179	194	195	209	249	281	319	339	432	469
519	633	725	817	1776								

Data set 9: This data set represents remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee & Wang [29].

0.08	2.09	3.48	4.87	6.94	8.66	13.11	23.63	0.20	2.23	3.52	4.98	6.97
9.02	13.29	0.40	2.26	3.57	5.06	7.09	9.22	13.80	25.74	0.50	2.46	3.64
5.09	7.26	9.47	14.24	25.82	0.51	2.54	3.70	5.17	7.28	9.74	14.76	6.31
0.81	2.62	3.82	5.32	7.32	10.06	14.77	32.15	2.64	3.88	5.32	7.39	10.34
14.83	34.26	0.90	2.69	4.18	5.34	7.59	10.66	15.96	36.66	1.05	2.69	4.23
5.41	7.62	10.75	16.62	43.01	1.19	2.75	4.26	5.41	7.63	17.12	46.12	1.26
2.83	4.33	5.49	7.66	11.25	17.14	79.05	1.35	2.87	5.62	7.87	11.64	17.36
1.40	3.02	4.34	5.71	7.93	11.79	18.1	1.46	4.40	5.85	8.26	11.98	19.13
1.76	3.25	4.50	6.25	8.37	12.02	2.02	3.31	4.51	6.54	8.53	12.03	
20.28	2.02	3.36	6.76	12.07	21.73	2.07	3.36	6.93	8.65	12.63	22.69	

Data Set 10: This data set is given by Linhart & Zucchini [30], which represents the failure times of the air conditioning system of an airplane.

23	261	87	7	120	14	62	47	225	71	246	21	42
20	5	12	120	11	3	14	71	11	14	11	16	90
1	16	52	95									

Data Set 11: This data set used by Bhaumik et al. [31], is vinyl chloride data obtained from clean upgradient monitoring wells in mg/l.

5.1	1.2	1.3	0.6	0.5	2.4	0.5	1.1	8	0.8	0.4	0.6	0.9
0.4	2	0.5	5.3	3.2	2.7	2.9	2.5	2.3	1	0.2	0.1	0.1
1.8	0.9	2	4	6.8	1.2	0.4	0.2					

Data set 12: This data set represents the waiting times (in minutes) before service of 100 Bank customers and examined and analyzed by Ghitany et al. [7] for fitting the Lindley [6] distribution.

0.8,	0.8,	1.3,	1.5,	1.8,	1.9,	1.9,	2.1,	2.6,	2.7,	2.9,	3.1,	3.2,
3.3,	3.5,	3.6,	4.0,	4.1,	4.2,	4.2,	4.3,	4.3,	4.4,	4.4,	4.6,	4.7,
4.7,	4.8,	4.9,	4.9,	5.0,	5.3,	5.5,	5.7,	5.7,	6.1,	6.2,	6.2,	6.2,
6.3,	6.7,	6.9,	7.1,	7.1,	7.1,	7.1,	7.4,	7.6,	7.7,	8.0,	8.2,	8.6,
8.6,	8.6,	8.8,	8.8,	8.9,	8.9,	9.5,	9.6,	9.7,	9.8,	10.7,	10.9,	11.0,
11.0,	11.1,	11.2,	11.2,	11.5,	11.9,	12.4,	12.5,	12.9,	13.0,	13.1,	13.3,	13.6,
13.7,	13.9,	14.1,	15.4,	15.4,	17.3,	17.3,	18.1,	18.2,	18.4,	18.9,	19.0,	19.9,
20.6,	21.3,	21.4,	21.9,	23.0,	27.0,	31.6,	33.1,	38.5				

Data Set 13: This data is for the times between successive failures of air conditioning equipment in a Boeing 720 airplane, Proschan [32].

74	57	48	29	502	12	70	21	29	386	59	27	153
26	326											

Data set 14: This data set represents the lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by Gross & Clark [33].

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2	1.7	2.7	4.1	1.8	1.5
1.2	1.4	3	1.7	2.3	1.6	2						

Data Set 15: This data set is the strength data of glass of the aircraft window reported by Fuller et al. [34].

18.83	20.8	21.657	23.03	23.23	24.05	24.321	25.5	25.52	25.8	26.69	26.77	26.78
27.05	27.67	29.9	31.11	33.2	33.73	33.76	33.89	34.76	35.75	35.91	36.98	37.08
37.09	39.58	44.045	45.29	45.381								

Data Set 16: The following data represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20mm [35].

1.312	1.314	1.479	1.552	1.700	1.803	1.861	1.865	1.944	1.958	1.966	1.997
2.006	2.021	2.027	2.055	2.063	2.098	2.140	2.179	2.224	2.240	2.253	2.270
2.272	2.274	2.301	2.301	2.359	2.382	2.382	2.426	2.434	2.435	2.478	2.490
2.511	2.514	2.535	2.554	2.566	2.570	2.586	2.629	2.633	2.642	2.648	2.684
2.697	2.726	2.770	2.773	2.800	2.809	2.818	2.821	2.848	2.880	2.954	3.012
3.067	3.084	3.090	3.096	3.128	3.233	3.433	3.585	3.858			

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Conflict of Interest

None.

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