On Modeling of Lifetime Data Using Akash, Shanker, Lindley and Exponential Distributions

Abstract

The statistical analysis and modeling of lifetime data are crucial for statisticians and research workers in almost all applied sciences including engineering, biomedical science, insurance, and finance, amongst others. The two important and popular one parameter distributions for modeling lifetime data are exponential and Lindley distributions. Shanker et al. [1] observed that there are many lifetime data where these distributions are not suitable from theoretical and applied point of view. Recently Shanker [2,3] has introduced two one parameter Lifetime distribution namely "Akash distribution" and "Shanker distribution" for modeling lifetime data.

In the present paper the relationships and comparative studies of Akash, Shanker, Lindley and exponential distributions, their distributional properties and estimation of parameter have been discussed. The applications, goodness of fit and theoretical justifications of these distributions for modeling life time data through various examples from engineering, medical science and other fields have been discussed and explained.

Keywords: Akash distribution; Shanker distribution; Lindley distribution; Exponential distribution; Statistical properties; Estimation of parameter; Goodness of fit

Research Article

Volume 3 Issue 6 - 2016

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Received: May 03, 2016 | **Published:** June 24, 2016

Introduction

In reliability analysis the time to the occurrence of event of interest is known as lifetime or survival time or failure time. The event may be failure of a piece of equipment, death of a person, development (or remission) of symptoms of disease, health code violation (or compliance). The modeling and statistical analysis of lifetime data are crucial for statisticians, research workers and policy makers in almost all applied sciences including engineering, medical science/biological science, insurance and finance, amongst others.

In statistics literature a number of lifetime distributions for modeling lifetime data-sets have been proposed. In this paper, the main objective is to have a critical and comparative study on one parameter lifetime distributions namely, Akash, Shanker, Lindley and exponential and their applications for modeling lifetime dats-sets from engineering, medical sciences, and other fields of knowledge.

Akash, Shanker, Lindley and Exponential Distributions

Akash distribution introduced by Shanker [2] for modeling lifetime data from engineering and medical science is a two-component mixture of an exponential (θ) distribution and a gamma $(3,\theta)$ distribution with their mixing proportions $\frac{\theta^2}{\theta^2+2}$ and $\frac{2}{\theta^2+2}$ respectively. Shanker [2] has discussed its various mathematical and statistical properties including its shape, moment generating function, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic orderings, mean deviations, distribution of order statistics, Bonferroni and Lorenz curves, Renyi entropy measure, stress-strength reliability , amongst others. Shanker et al. [3] has detailed study about modeling of various lifetime data from different fields using Akash, Lindley and exponential distributions and concluded that Akash distribution gives better fit in most of the lifetime data. Shanker [5] has also obtained a Poisson mixture of Akash distribution named, "Poisson-Akash (PAD)" for modeling count data.

Shanker distribution introduced by Shanker [2] for modeling lifetime data from engineering and medical science is a two-component mixture of an exponential (θ) distribution and a gamma $(2,\theta)$ distribution with their mixing proportions $\frac{\theta^2}{\theta^2+1}$ and $\frac{1}{\theta^2+1}$ respectively. Shanker [3] has discussed its various mathematical and statistical properties including its shape, moment generating function, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic orderings, mean deviations, distribution of order statistics, Bonferroni and Lorenz curves, Renyi entropy measure, stress-strength reliability , amongst others. Shanker [6] has also obtained a Poisson mixture of Shanker distribution named, "Poisson-Shanker (PSD)" for modeling count data.

Lindley [7] distribution is a two-component mixture of an exponential (θ) distribution and a gamma $(2,\theta)$ distribution with their mixing proportions $\frac{\theta}{\theta+1}$ and $\frac{1}{\theta+1}$ respectively. A detailed study about its various mathematical properties, estimation of parameter and application showing the superiority of Lindley distribution over exponential distribution for the waiting times before service of the bank customers has been done by Ghitany et al. [8]. A number of researchers have studied in detail the generalized, extended, mixtures and modified forms of Lindley distribution including Sankaran [9], Zakerzadeh & Dolati [10], Nadarajah et al. [11], Bakouch et al. [12], Shanker & Mishra [13,14], Shanker & Amanuel [15], Shanker et al. [16,17], Ghitany et al. [18], are some among others.

In statistical literature, exponential distribution was the first widely used lifetime model in areas ranging from studies on the lifetimes of manufactured to research involving survival or remission times in chronic diseases. The main reason for its wide usefulness and applicability as lifetime model is partly because of the availability of simple statistical methods for it and partly because it appeared to be suitable for representing the lifetimes of many things such as various types of manufactured items.

Let T be a continuous random variable representing the lifetimes of individuals in some population The expressions for probability density function, $f(t)$, cumulative distribution function, $F(t)$, hazard rate function, $h(t)$, mean residual life function, $m(t)$, mean μ_1', variance μ_2, coefficient of variation (C.V.), coefficient of Skewness $(\sqrt{\beta_1})$, coefficient of Kurtosis (β_2), and index of dispersion (γ) of Akash and Shanker distributions introduced by Shanker [2,3] are summarized in Table 1 and that of Lindley and exponential distributions are in Table 2.

A table of values for coefficient of variation (C.V.), coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, coefficient of Kurtosis (β_2), and index of dispersion (γ) for Akash , Shanker and Lindley distributions for varying values of their parameter are summarized in the Table 3.

The conditions under which Akash, Shanker and Lindley distributions are over-dispersed $\left(\mu<\sigma^2\right)$, equi-dispersed $\left(\mu=\sigma^2\right)$, and under-dispersed $\left(\mu>\sigma^2\right)$ are summarized in Table 4.

The graphs of C.V, $\sqrt{\beta_1}$, β_2 and γ of Akash, Shanker and Lindley distributions for varying values of the parameter θ are shown in Figure 1.

Parameter Estimation

Estimation of the parameter of Akash distribution

Assuming $(t_1, t_2, t_3, \dots, t_n)$ be a random sample of size n from Akash distribution, the maximum likelihood estimate (MLE) $\hat{\theta}$ and the method moment estimate (MOME) $\tilde{\theta}$ of θ is the soluti on of following cubic equation.

$$\bar{t}\,\theta^3-\theta^2+2\bar{t}\,\theta-6=0$$, where $\bar{t}$ is the sample mean

Estimation of the parameter of Shanker distribution

Let $(t_1, t_2, t_3, \dots, t_n)$ be a random sample of size n from Shanker distribution. The maximum likelihood estimate (MLE) $\hat{\theta}$ of θ is the solution of the following non-linear equation.

$$\frac{2n}{\theta\left(\theta^2+1\right)}+\sum_{i=1}^{n}\frac{1}{\theta+t_i}-n\,\bar{t}=0$$

The method of moment estimate (MOME) $\tilde{\theta}$ of θ is the solution of the following cubic equation

$$\bar{t}\,\theta^3-\theta^2+\bar{t}\,\theta-2=0$$, where $\bar{t}$ is the sample mean.

Estimation of the parameter of Lindley distribution

Assuming $(t_1, t_2, \dots, t_n)$ be a random sample of size n from Lindley distribution, the maximum likelihood estimate (MLE) $\hat{\theta}$ and the method moment estimate (MOME) $\tilde{\theta}$ of θ is given by

$$\hat{\theta}=\frac{-(\bar{t}-1)+\sqrt{(\bar{t}-1)^2+8\bar{t}}}{2\bar{t}};\quad \bar{t}>0$$, where $\bar{t}$ is the sample mean.

Estimation of the parameter of Exponential distribution

Assuming $(t_1, t_2, \dots, t_n)$ be a random sample of size n from exponential distribution, the maximum likelihood estimate (MLE) $\tilde{\theta}$ and the method moment estimate (MOME) $\tilde{\theta}$ of θ is given by $\hat{\theta}=\frac{1}{\bar{t}}$, where $\bar{t}$ is the sample mean.

Applications and Goodness of Fit

In this section the goodness of fit test of Akash, Shanker, Lindley and exponential distributions for following sixteen real lifetime data- sets using maximum likelihood estimate have been discussed.

In order to compare the goodness of fit of Akash, Shanker, Lindley and exponential distributions, $-2\ln L$, AIC (Akaike Information Criterion), AICC (Akaike Information Criterion Corrected), BIC (Bayesian Information Criterion), K-S Statistics (Kolmogorov-Smirnov Statistics) for all sixteen real lifetime data-sets have been computed and presented in Table 5. The formulae for computing AIC, AICC, BIC, and K-S Statistics are as follows:

$$AIC=-2\ln L+2k,\quad AICC=AIC+\frac{2k(k+1)}{(n-k-1)},\quad BIC=-2\ln L+k\ln n$$

and $D=\sup_x\left|F_n(x)-F_0(x)\right|$, where k = the number of parameters, n = the sample size and $F_n(x)$ is the empirical distribution function.

The best distribution is the distribution which corresponds to lower values of $-2\ln L$, AIC, AICC, BIC, and K-S statistics.

The best fitting has been shown by making -2ln L, AIC, AICC, BIC, and K-S Statistics in bold.

Conclusions

In this paper an attempt has been made to find the suitability of Akash, Shanker, Lindley and exponential distributions for modeling real lifetime data from engineering, medical science and other fields of knowledge. A table for values of the various characteristics of Akash, Shanker, and Lindley distributions has been presented for varying values of their parameter which reflects their nature and behavior. The conditions under which Akash, shanker, Lindley and exponential distributions are over-dispersed, equi-dispersed, and under-dispersed have been given. The goodness of fit test of Akash, Shanker, Lindley and exponential distributions for sixteen real lifetime data-sets have been presented using Kolmogorov-Smirnov test to test their suitability for modeling lifetime data.

Table 1: Characteristics of Akash and Shanker Distributions.

Akash Distribution	Shanker Distribution
$f(t)=\frac{\theta^3}{\theta^2+2}\left(1+t^2\right)e^{-\theta t}$	$f(t)=\frac{\theta^2}{\theta^2+1}(\theta+t)e^{-\theta t}$
$F(t)=1-\left[1+\frac{\theta t(\theta t+2)}{\theta^2+2}\right]e^{-\theta t}$	$F(t)=1-\left[1+\frac{\theta t}{\theta^2+1}\right]e^{-\theta t}$
$h(t)=\frac{\theta^3\left(1+t^2\right)}{\theta t(\theta t+2)+\left(\theta^2+2\right)}$	$h(t)=\frac{\theta^2(\theta+t)}{\left(\theta^2+1\right)+\theta t}$
$m(t)=\frac{\theta^2 t^2+4\theta t+\left(\theta^2+6\right)}{\theta\left[\theta t(\theta t+2)+\left(\theta^2+2\right)\right]}$	$m(t)=\frac{\theta^2+\theta t+2}{\theta\left(\theta^2+\theta t+1\right)}$
$\mu_1'=\frac{\theta^2+6}{\theta\left(\theta^2+2\right)}$	$\mu_1'=\frac{\theta^2+2}{\theta\left(\theta^2+1\right)}$
$\mu_2=\frac{\theta^4+16\theta^2+12}{\theta^2\left(\theta^2+2\right)^2}$	$\mu_2=\frac{\theta^4+4\theta^2+2}{\theta^2\left(\theta^2+1\right)^2}$
$C.V=\frac{\sigma}{\mu_1'}=\frac{\sqrt{\theta^4+16\theta^2+12}}{\theta^2+6}$	$C.V=\frac{\sigma}{\mu_1'}=\frac{\sqrt{\theta^4+4\theta^2+2}}{\theta^2+2}$
$\sqrt{\beta_1}=\frac{2\left(\theta^6+30\theta^4+36\theta^2+24\right)}{\left(\theta^4+16\theta^2+12\right)^{3/2}}$	$\sqrt{\beta_1}=\frac{2\left(\theta^6+6\theta^4+6\theta^2+2\right)}{\left(\theta^4+4\theta^2+2\right)^{3/2}}$
$\beta_2=\frac{3\left(\begin{array}{l}3\theta^8+128\theta^6+408\theta^4\\+576\theta^2+240\end{array}\right)}{\left(\theta^4+16\theta^2+12\right)^2}$	$\beta_2=\frac{3\left(3\theta^8+24\theta^6+44\theta^4+32\theta^2+8\right)}{\left(\theta^4+4\theta^2+2\right)^2}$
$\gamma=\frac{\sigma^2}{\mu_1'}=\frac{\theta^4+16\theta^2+12}{\theta\left(\theta^2+2\right)\left(\theta^2+6\right)}$	$\gamma=\frac{\sigma^2}{\mu_1'}=\frac{\theta^4+4\theta^2+2}{\theta\left(\theta^2+1\right)\left(\theta^2+2\right)}$

Table 2: Characteristics of Lindley and Exponential Distributions.

Lindley Distribution	Exponential Distribution
$f(t)=\frac{\theta^2}{\theta+1}(1+t)e^{-\theta t}$	$f(t)=\theta e^{-\theta t}$
$F(t)=1-\frac{\theta+1+\theta t}{\theta+1}e^{-\theta t}$	$F(t)=1-e^{-\theta t}$
$h(t)=\frac{\theta^2(1+t)}{\theta+1+\theta t}$	$h(t)=\theta$
$m(t)=\frac{\theta+2+\theta t}{\theta(\theta+1+\theta t)}$	$m(t)=\frac{1}{\theta}$
$\mu_1'=\frac{\theta+2}{\theta(\theta+1)}$	$\mu_1'=\frac{1}{\theta}$
$\mu_2=\frac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}$	$\mu_2=\frac{1}{\theta^2}$
$C.V=\frac{\sigma}{\mu_1'}=\frac{\sqrt{\theta^2+4\theta+2}}{\theta+2}$	$C.V=\frac{\sigma}{\mu_1'}=1$
$\sqrt{\beta_1}=\frac{2\left(\theta^3+6\theta^2+6\theta+2\right)}{\left(\theta^2+4\theta+2\right)^{3/2}}$	$\sqrt{\beta_1}=2$
$\beta_2=\frac{3\left(\begin{matrix}3\theta^4+24\theta^3+44\theta^2\\+32\theta+8\end{matrix}\right)}{\left(\theta^2+4\theta+2\right)^2}$	$\beta_2=9$
$\gamma=\frac{\sigma^2}{\mu_1'}=\frac{\theta^2+4\theta+2}{\theta(\theta+1)(\theta+2)}$	$\gamma=\frac{\sigma^2}{\mu_1'}=\frac{1}{\theta}$

Table 3: Values of μ_1', $\sqrt{\beta_1}$, CV, $\sqrt{\beta_1}$, β_2 and θ of Akash, Shanker and Lindley distributions for varying values of the parameter θ.

Values of θ for Akash Distribution								
	0.01	0.05	0.1	0.3	0.5	1	1.5	2
μ_1'	299.990	59.950	29.900	9.713	5.556	2.333	1.294	0.833
μ_2	30001.000	1200.996	300.985	34.208	12.691	3.222	1.306	0.639
CV	0.577	0.578	0.580	0.602	0.641	0.769	0.883	0.959
$\sqrt{\beta_1}$	1.155	1.153	1.149	1.115	1.084	1.165	1.388	1.614
β_2	5.000	4.997	4.987	4.897	4.785	4.834	5.473	6.391
γ	100.007	20.033	10.066	3.522	2.284	1.381	1.009	0.767
Values of θ for Shanker Distribution								
	0.01	0.05	0.1	0.3	0.5	1	1.5	2
μ_1'	199.990	39.950	19.901	6.391	3.600	1.500	0.872	0.600
μ_2	20000.000	799.998	199.990	22.146	7.840	1.750	0.676	0.340
CV	0.707	0.708	0.711	0.736	0.778	0.882	0.943	0.972
$\sqrt{\beta_1}$	1.414	1.414	1.414	1.421	1.452	1.620	1.779	1.876
β_2	6.000	6.000	6.000	6.020	6.121	6.796	7.593	8.159
γ	100.005	20.025	10.049	3.465	2.178	1.167	0.775	0.567
Values of θ for Lindley Distribution								
	0.01	0.05	0.1	0.3	0.5	1	1.5	2
μ_1'	199.010	39.048	19.091	5.897	3.333	1.500	0.933	0.667
μ_2	19999.020	799.093	199.174	21.631	7.556	1.750	0.729	0.389
CV	0.711	0.724	0.739	0.789	0.825	0.882	0.915	0.935
$\sqrt{\beta_1}$	1.414	1.417	1.422	1.464	1.512	1.620	1.699	1.756
β_2	6.000	6.007	6.025	6.162	6.343	6.796	7.173	7.469
γ	100.493	20.465	10.433	3.668	2.267	1.167	0.781	0.583

Table 4: Over-dispersion, equi-dispersion and under-dispersion of Akash, Shanker , Lindley and exponential distributions for varying values of their parameter θ .

Distribution	Over-Dispersion $\left(\mu<\sigma^2\right)$	Equi-Dispersion $\left(\mu=\sigma^2\right)$	Under-Dispersion $\left(\mu>\sigma^2\right)$
Akash	$\theta<1.515400063$	$\theta=1.515400063$	$\theta>1.515400063$
Shanker	$\theta<1.171535555$	$\theta>1.171535555$	$\theta>1.171535555$
Lindley	$\theta>1.170086487$	$\theta>1.170086487$	$\theta>1.170086487$
Exponential	$\theta<1$	$\theta=1$	$\theta>1$

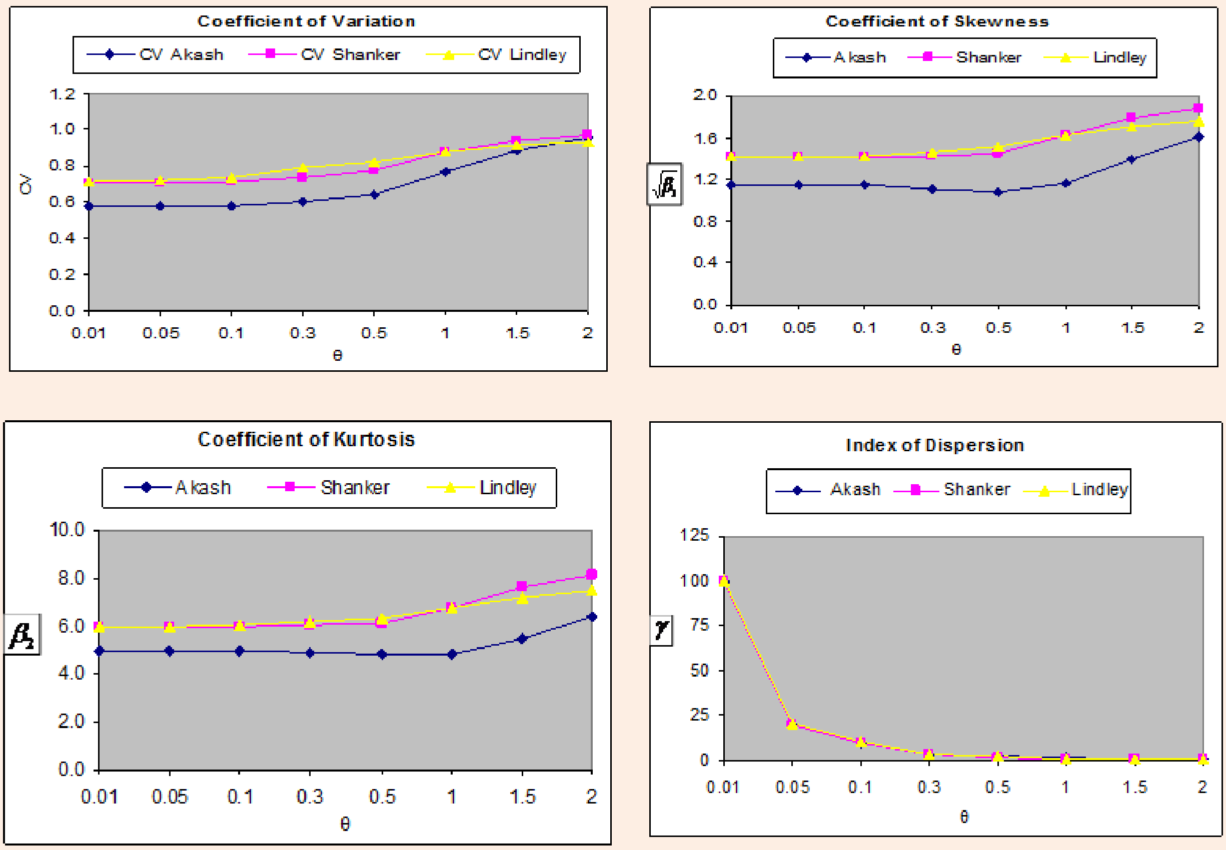

Figure 1: Graphs of C.V, $\sqrt{\beta_1}$, γ and γ of Akash, Shanker and Lindley distributions for varying values of the parameter θ .

Table 5: MLE's, -2ln L, AIC, AICC, BIC, K-S Statistics of the fitted distributions of data-sets 1-16.

	Model	Parameter Estimate	-2ln L	AIC	AICC	BIC	K-S Statistic
Data 1	Akash	1.355445	163.73	165.73	165.79	169.93	0.355
	Shanker	0.956264	162.28	164.28	164.34	166.42	0.346
	Lindley	0.996116	162.56	164.56	164.62	166.70	0.371
	Exponential	0.663647	177.66	179.66	179.73	181.80	0.402
Data 2	Akash	0.043876	950.97	952.97	953.01	955.58	0.184
	Shanker	0.029252	980.97	982.97	983.01	985.57	0.238
	Lindley	0.028859	983.11	985.11	985.15	987.71	0.242
	Exponential	0.014635	1044.87	1046.87	1046.91	1049.48	0.357
Data 3	Akash	0.041510	227.06	229.06	229.25	230.20	0.107
	Shanker	0.027675	231.06	233.06	233.25	234.19	0.145
	Lindley	0.027321	231.47	233.47	233.66	234.61	0.149
	Exponential	0.013845	242.87	244.87	245.06	246.01	0.263
Data 4	Akash	0.013514	1255.83	1257.83	1257.87	1260.43	0.110
	Shanker	0.009009	1251.19	1253.34	1253.38	1255.60	0.097
	Lindley	0.008970	1251.34	1253.34	1253.38	1255.95	0.098
	Exponential	0.004505	1280.52	1282.52	1282.56	1285.12	0.190
Data 5	Akash	0.030045	794.70	796.70	796.76	798.98	0.184
	Shanker	0.020031	788.57	790.57	790.63	792.28	0.133
	Lindley	0.019841	789.04	791.04	791.10	793.32	0.134
	Exponential	0.010018	806.88	808.88	808.94	811.16	0.198
Data 6	Akash	0.119610	981.28	983.28	983.31	986.18	0.393
	Shanker	0.079746	1033.10	1035.10	1035.13	1037.99	0.442
	Lindley	0.077247	1041.64	1043.64	1043.68	1046.54	0.448
	Exponential	0.040060	1130.26	1132.26	1132.29	1135.16	0.525
Data 7	Akash	0.013263	803.96	805.96	806.02	810.01	0.298
	Shanker	0.008843	764.62	766.62	766.69	768.06	0.246
	Lindley	0.008804	763.75	765.75	765.82	767.81	0.245
	Exponential	0.004421	744.87	746.87	746.94	748.93	0.166
Data 8	Akash	0.013423	609.93	611.93	612.02	613.71	0.280
	Shanker	0.008949	579.51	581.51	581.60	583.29	0.220
	Lindley	0.008910	579.16	581.16	581.26	582.95	0.219
	Exponential	0.004475	564.02	566.02	566.11	567.80	0.145
Data 9	Akash	0.310500	887.89	889.89	889.92	892.74	0.198
	Shanker	0.210732	847.37	849.37	849.40	852.22	0.132
	Lindley	0.196045	839.06	841.06	841.09	843.91	0.116
	Exponential	0.106773	828.68	830.68	830.72	833.54	0.077

Data 10	Akash	0.050293	354.88	356.88	357.02	358.28	0.421
	Shanker	0.033569	325.74	327.74	327.88	329.14	0.351
	Lindley	0.033021	323.27	325.27	325.42	326.67	0.345
	Exponential	0.016779	305.26	307.26	307.40	308.66	0.213
Data 11	Akash	1.165719	115.15	117.15	117.28	118.68	0.156
	Shanker	0.853374	112.91	114.91	115.03	116.44	0.131
	Lindley	0.823821	112.61	114.61	114.73	116.13	0.133
	Exponential	0.532081	110.91	112.91	113.03	114.43	0.089
Data 12	Akash	0.295277	641.93	643.93	643.95	646.51	0.100
	Shanker	0.198317	635.26	637.26	637.30	639.86	0.042
	Lindley	0.186571	638.07	640.07	640.12	642.68	0.058
	Exponential	0.101245	658.04	660.04	660.08	662.65	0.163
Data 13	Akash	0.024734	194.30	196.30	196.61	197.01	0.456
	Shanker	0.016492	181.58	183.58	183.89	184.29	0.388
	Lindley	0.016360	181.34	183.34	183.65	184.05	0.386
	Exponential	0.008246	173.94	175.94	176.25	176.65	0.277
Data 14	Akash	1.156923	59.52	61.52	61.74	62.51	0.320
	Shanker	0.803867	59.78	61.78	61.22	62.77	0.325
	Lindley	0.816118	60.50	62.50	62.72	63.49	0.341
	Exponential	0.526316	65.67	67.67	67.90	68.67	0.389
Data 15	Akash	0.097062	240.68	242.68	242.82	244.11	0.266
	Shanker	0.064712	252.35	254.35	254.49	255.78	0.326
	Lindley	0.062988	253.99	255.99	256.13	257.42	0.333
	Exponential	0.032455	274.53	276.53	276.67	277.96	0.426
Data 16	Akash	0.964726	224.28	226.28	226.34	228.51	0.348
	Shanker	0.658029	233.01	235.01	235.06	237.24	0.355
	Lindley	0.659000	238.38	240.38	240.44	242.61	0.390
	Exponential	0.407941	261.74	263.74	263.80	265.97	0.434

Data Set 1: The data set represents the strength of 1.5cm glass fibers measured at the National Physical Laboratory, England. Unfortunately, the units of measurements are not given in the paper, and they are taken from Smith & Naylor [19].

0.55	0.93	1.25	1.36	1.49	1.52	1.58	1.61	1.64	1.68	1.73	1.81	2.00
0.74	1.04	1.27	1.39	1.49	1.53	1.59	1.61	1.66	1.68	1.76	1.82	2.01
0.77	1.11	1.28	1.42	1.50	1.54	1.60	1.62	1.66	1.69	1.76	1.84	2.24
0.81	1.13	1.29	1.48	1.50	1.55	1.61	1.62	1.66	1.70	1.77	1.84	0.84
1.24	1.30	1.48	1.51	1.55	1.61	1.63	1.67	1.70	1.78	1.89		

Data Set 2: The data is given by Birnbaum & Saunders [20] on the fatigue life of 6061 – T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second. The data set consists of 101 observations with maximum stress per cycle 31,000 psi. The data ($\times 10^{-3}$) are presented below (after subtracting 65).

5	25	31	32	34	35	38	39	39	40	42	43	43
43	44	44	47	48	48	49	49	49	51	54	55	55
55	56	56	56	58	59	59	59	59	59	63	63	64
64	65	65	65	66	66	66	66	66	67	67	67	68
69	69	69	69	71	71	72	73	73	73	74	74	76
76	77	77	77	77	77	77	79	79	80	81	83	83
84	86	86	87	90	91	92	92	92	92	93	94	97
98	98	99	101	103	105	109	136	147				

Data Set 3: The data set is from Lawless (1982, p-228). The data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life tests.

17.88	28.92	33.00	41.52	42.12	45.60	48.80	51.84	51.96	54.12	55.56	67.80
68.44	68.64	68.88	84.12	93.12	98.64	105.12	105.84	127.92	128.04	173.40	

Data Set 4: The data is from Picciotto [21] and arose in test on the cycle at which the Yarn failed. The data are the number of cycles until failure of the yarn.

86	146	251	653	98	249	400	292	131	169	175	176	76
264	15	364	195	262	88	264	157	220	42	321	180	198
38	20	61	121	282	224	149	180	325	250	196	90	229
166	38	337	65	151	341	40	40	135	597	246	211	180
93	315	353	571	124	279	81	186	497	182	423	185	229
400	338	290	398	71	246	185	188	568	55	55	61	244
20	284	393	396	203	829	239	236	286	194	277	143	198
264	105	203	124	137	135	350	193	188				

Data Set 5: This data represents the survival times (in days) of 72 guinna pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal [22].

12	15	22	24	24	32	32	33	34	38	38	43	44
48	52	53	54	54	55	56	57	58	58	59	60	60
60	60	61	62	63	65	65	67	68	70	70	72	73
75	76	76	81	83	84	85	87	91	95	96	98	99
109	110	121	127	129	131	143	146	146	175	175	211	233
258	258	263	297	341	341	376						

Data Set 6: This data is related with behavioral sciences, collected by Balakrishnan N et al. [23]: The scale "General Rating of Affective Symptoms for Preschoolers (GRASP)" measures behavioral and emotional problems of children, which can be classified with depressive condition or not according to this scale. A study conducted by the authors in a city located at the south part of Chile has allowed collecting real data corresponding to the scores of the GRASP scale of children with frequency in parenthesis.

19(16)	20(15)	21(14)	22(9)	23(12)	24(10)	25(6)	26(9)			
27(8)	28(5)	29(6)	30(4)	31(3)	32(4)	33	34	35(4)	36(2)	37(2)
39	42	44								

Data Set 7: The data set reported by Efron [24] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using radiotherapy (RT).

6.53	7	10.42	14.48	16.10	22.70	34	41.55	42	45.28	49.40	53.62	63
64	83	84	91	108	112	129	133	133	139	140	140	146
149	154	157	160	160	165	146	149	154	157	160	160	165
173	176	218	225	241	248	273	277	297	405	417	420	440
523	583	594	1101	1146	1417							

Data Set 8: The data set reported by Efron [24] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using a combination of radiotherapy and chemotherapy (RT+CT).

12.20	23.56	23.7	25.9	31.98	37	41.35	47.38	55.46	58.36	63.47	68.46	78.3
74.5	81.43	84	92	94	110	112	119	127	130	133	140	146
155	159	173	179	194	195	209	249	281	319	339	432	469
519	633	725	817	1776								

Data set 9: This data set represents remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee & Wang [25].

0.08	2.09	3.48	4.87	6.94	8.66	13.11	23.63	0.20	2.23	3.52	4.98	6.97
9.02	13.29	0.40	2.26	3.57	5.06	7.09	9.22	13.80	25.74	0.50	2.46	3.64
5.09	7.26	9.47	14.24	25.82	0.51	2.54	3.70	5.17	7.28	9.74	14.76	6.31
0.81	2.62	3.82	5.32	7.32	10.06	14.77	32.15	2.64	3.88	5.32	7.39	10.34
14.83	34.26	0.90	2.69	4.18	5.34	7.59	10.66	15.96	36.66	1.05	2.69	4.23
5.41	7.62	10.75	16.62	43.01	1.19	2.75	4.26	5.41	7.63	17.12	46.12	1.26
2.83	4.33	5.49	7.66	11.25	17.14	79.05	1.35	2.87	5.62	7.87	11.64	17.36
1.40	3.02	4.34	5.71	7.93	11.79	18.10	1.46	4.40	5.85	8.26	11.98	19.13
1.76	3.25	4.50	6.25	8.37	12.02	2.02	3.31	4.51	6.54	8.53	12.03	
20.28	2.02	3.36	6.76	12.07	21.73	2.07	3.36	6.93	8.65	12.63	22.69	

Data Set 10: This data set is given by Linhart & Zucchini [26], which represents the failure times of the air conditioning system of an airplane.

23	261	87	7	120	14	62	47	225	71	246	21	42	20
	5	12	120	11	3	14	71	11	14	11	16	90	1
	16	52	95										

Data Set 11: This data set used by Bhaumik et al. [27], is vinyl chloride data obtained from clean up gradient monitoring wells in mg/l.

5.1	1.2	1.3	0.6	0.5	2.4	0.5	1.1	8	0.8	0.4	0.6	0.9	0.4
	2	0.5	5.3	3.2	2.7	2.9	2.5	2.3	1	0.2	0.1	0.1	1.8
	0.9	2	4	6.8	1.2	0.4	0.2						

Data set 12: This data set represents the waiting times (in minutes) before service of 100 Bank customers and examined and analyzed by Ghitany et al. [8] for fitting the Lindley [7] distribution.

0.8	0.8	1.3	1.5	1.8	1.9	1.9	2.1	2.6	2.7	2.9	3.1	3.2
3.3	3.5	3.6	4.0	4.1	4.2	4.2	4.3	4.3	4.4	4.4	4.6	4.7
4.7	4.8	4.9	4.9	5.0	5.3	5.5	5.7	5.7	6.1	6.2	6.2	6.2
6.3	6.7	6.9	7.1	7.1	7.1	7.1	7.4	7.6	7.7	8.0	8.2	8.6
8.6	8.6	8.8	8.8	8.9	8.9	9.5	9.6	9.7	9.8	10.7	10.9	11.0
11.0	11.1	11.2	11.2	11.5	11.9	12.4	12.5	12.9	13.0	13.1	13.3	13.6
13.7	13.9	14.1	15.4	15.4	17.3	17.3	18.1	18.2	18.4	18.9	19.0	19.9
20.6	21.3	21.4	21.9	23.0	27.0	31.6	33.1	38.5				

Data Set 13: This data is for the times between successive failures of air conditioning equipment in a Boeing 720 airplane, Proschan [28].

74	57	48	29	502	12	70	21	29	386	59	27	153	26
	326												

Data set 14: This data set represents the lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by Gross & Clark [29].

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2	1.7	2.7	4.1	1.8	1.5	1.2
	1.4	3	1.7	2.3	1.6	2							

Data Set 15: This data set is the strength data of glass of the aircraft window reported by Fuller et al. [30].

18.83	20.8	21.657	23.03	23.23	24.05	24.321	25.5	25.52	25.8	26.69	26.77	26.78
27.05	27.67	29.9	31.11	33.2	33.73	33.76	33.89	34.76	35.75	35.91	36.98	37.08
37.09	39.58	44.045	45.29	45.381								

Data Set 16: The following data represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20mm Bader & Priest [31,32].

1.312	1.314	1.479	1.552	1.700	1.803	1.861	1.865	1.944	1.958	1.966	1.997	2.006
2.021	2.027	2.055	2.063	2.098	2.140	2.179	2.224	2.240	2.253	2.270	2.272	2.274
2.301	2.301	2.359	2.382	2.382	2.426	2.434	2.435	2.478	2.490	2.511	2.514	2.535
2.554	2.566	2.570	2.586	2.629	2.633	2.642	2.648	2.684	2.697	2.726	2.770	2.773
2.800	2.809	2.818	2.821	2.848	2.880	2.954	3.012	3.067	3.084	3.090	3.096	3.128
3.233	3.433	3.585	3.858									

Acknowledgement

None.

Conflict of Interest

None.

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Citation: Shanker R, Fesshaye H (2016) On Modeling of Lifetime Data Using Akash, Shanker, Lindley and Exponential Distributions. Biom Biostat Int J 3(6): 00084. DOI: 10.15406/bbij.2016.03.00084