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On poisson-akash distribution and its applications

Abstract

A simple and interesting method for finding moments of 'Poisson-Akash distribution (PAD)' of Shanker,[1] a Poisson mixture of Akash distribution introduced by Shanker[2] has been suggested. The first two moments about origin and the variance of PAD has been obtained and presented. The applications and the goodness of fit of PAD has been discussed using data-sets relating to ecology genetics, and thunderstorms and the fit has been compared with Poisson and Poisson-Lindley distribution, a Poisson mixture of Lindley[3] distribution, introduced by Sankaran[4] and the goodness of fit of PAD shows satisfactory fit in most of data-sets.

Keywords: Akash distribution; Poisson-Akash distribution; Lindley distribution; Poisson-Lindley distribution; Compounding; Moments; Estimation of parameter; Goodness of fit

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Introduction

The probability mass function of Poisson-Akash distribution (PAD) having parameter θ given by

$$P(X=x)=\frac{\theta^3}{\theta^2+2}\cdot\frac{x^2+3x+\left(\theta^2+2\theta+3\right)}{(\theta+1)^{x+3}}\;;x=0,1,2,...,\theta>0 \qquad (1.1)$$

has been introduced by Shanker[1] for modeling various count data-sets. The PAD arises from Poisson distribution when its parameter λ follows one parameter Akash distribution introduced by Shanker[2] having probability density function

$$f(\lambda,\theta)=\frac{\theta^3}{\theta^2+2}\left(1+\lambda^2\right)e^{-\theta\lambda}\;;\lambda>0,\theta>0 \qquad (1.2)$$

We have

$$P(X=x)=\int_0^\infty\frac{e^{-\lambda}\lambda^x}{x!}\cdot\frac{\theta^3}{\theta^2+2}\left(1+\lambda^2\right)e^{-\theta\lambda}d\lambda \qquad (1.3)$$

$$=\frac{\theta^3}{\left(\theta^2+2\right)x!}\int_0^\infty e^{-(\theta+1)\lambda}\left[\lambda^x+\lambda^{x+2}\right]d\lambda$$

$$=\frac{\theta^3}{\theta^2+2}\cdot\frac{x^2+3x+\left(\theta^2+2\theta+3\right)}{(\theta+1)^{x+3}}\;;x=0,1,2,...,\theta>0 \qquad (1.4)$$

This is the probability mass function of Poisson-Akash distribution (PAD).

It has been shown by Shanker[2] that the Akash distribution (1.2) is a two component mixture of an exponential (θ) distribution, and a gamma (3, θ) distribution with their mixing proportions $\frac{\theta^2}{\theta^2+2}$ and $\frac{2}{\theta^2+2}$ respectively. Shanker[2] has discussed its mathematical and statistical properties including its shape, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic orderings, mean deviations, distribution of order statistics, Bonferroni and Lorenz curves, Renyi entropy measure, stress-strength reliability, amongst others along with the estimation of parameter and applications for modeling lifetime data from engineering and biomedical science. Shanker et al.,[3] has detailed and critical study about the applications of one parameter Akash, Lindley and exponential distributions for modeling lifetime data from biomedical science and engineering.

Sankaran[4] obtained Poisson-Lindley distribution (PLD) having probability mass function (p.m.f)

$$P(X=x)=\frac{\theta^2(x+\theta+2)}{(\theta+1)^{x+3}}\;;x=0,1,2,...,\theta>0 \qquad (1.5)$$

by compounding Poisson distribution with Lindley distribution when the parameter λ of Poisson distribution follows Lindley distribution, introduced by Lindley[5] having probability density function (p.d.f)

$$f(\lambda,\theta)=\frac{\theta^2}{\theta+1}(1+\lambda)e^{-\theta\lambda}\;;\lambda>0,\theta>0 \qquad (1.6)$$

In this paper a simple and interesting method for finding moments of Poisson-Akash distribution (PAD) introduced by Shanker[1] has been suggested and hence the first two moments about origin and the variance has been presented. It seems that not much work has been done on the applications of PAD so far for count data arising in various fields of knowledge. The applications and goodness of fit of PAD have been discussed with various count data from ecology, genetics and thunderstorms and the goodness of fit of PAD has been compared with Poisson distribution and Poisson-Lindley distribution (PLD). The goodness of fit of PAD shows satisfactory fit in most of the data-sets.

Moments of PAD

Using (1.3), the r th moment about origin of PAD (1.1) can be obtained as

$$\mu_r'=E\left[E\left(X^r|\lambda\right)\right]=\frac{\theta^3}{\theta^2+2}\int_0^\infty\left[\sum_{x=0}^\infty x^r\frac{e^{-\lambda}\lambda^x}{x!}\right]\left(1+\lambda^2\right)e^{-\theta\lambda}d\lambda \qquad (2.1)$$

It is obvious that the expression under the bracket in (2.1) is the r th moment about origin of the Poisson distribution. Taking $r=1$ in (2.1) and using the first moment about origin of the Poisson distribution, the first moment about origin of the PAD (1.1) can be obtained as

$$\mu_1'=\frac{\theta^3}{\theta^2+2}\int_0^\infty \lambda\left(1+\lambda^2\right)e^{-\theta\lambda}d\lambda=\frac{\theta^2+6}{\theta\left(\theta^2+2\right)} \quad (2.2)$$

Again taking $r=2$ in (2.1) and using the second moment about origin of the Poisson distribution, the second moment about origin of the PAD (1.1) can be obtained as

$$\mu_2'=\frac{\theta^3}{\theta^2+2}\int_0^\infty \left(\lambda^2+\lambda\right)\left(1+\lambda^2\right)e^{-\theta\lambda}d\lambda=\frac{\theta^3+2\theta^2+6\theta+24}{\theta^2\left(\theta^2+2\right)} \quad (2.3)$$

Similarly, taking $r=3$ and 4 in (2.1) and using the third and the fourth moments about origin of the Poisson distribution, the third and the fourth moments about origin of the PAD (1.1) can thus be obtained as

$$\mu_3'=\frac{\theta^4+6\theta^3+12\theta^2+72\theta+120}{\theta^3\left(\theta^2+2\right)} \quad (2.4)$$

$$\mu_4'=\frac{\theta^5+14\theta^4+42\theta^3+192\theta^2+720\theta+720}{\theta^4\left(\theta^2+2\right)} \quad (2.5)$$

The variance of the PAD (1.1) can thus be obtained as

$$\mu_2=\frac{\theta^5+\theta^4+8\theta^3+16\theta^2+12\theta+12}{\theta^2\left(\theta^2+2\right)^2} \quad (2.6)$$

It has been shown by Shanker[1] that PAD (1.1) has increasing hazard rate, unimodal and always over-dispersed, and thus is a suitable model for count data which are over-dispersed.

Parameter estimation of PAD

Maximum likelihood estimate (MLE) of the parameter: Let $(x_1,x_2,...,x_n)$ be a random sample of size n from the PAD (1.1) and let f_x be the observed frequency in the sample corresponding to $X=x$ $(x=1,2,3,...,k)$ such that $\sum_{x=1}^{k} f_x=n$, where k is the largest observed value having non-zero frequency.

The likelihood function L of the PAD (1.1) can be given by

$$L=\left(\frac{\theta^3}{\theta^2+2}\right)^n \frac{1}{(\theta+1)^{\sum_{x=1}^{k} f_x(x+3)}}\prod_{x=1}^{k}\left[x^3+3x+\left(\theta^2+2\theta+3\right)\right]^{f_x}$$

The log likelihood function is thus obtained as

$$\log L=n\log\left(\frac{\theta^3}{\theta^2+2}\right)-\sum_{x=1}^{k} f_x(x+3)\log(\theta+1)+\sum_{x=1}^{k} f_x\log\left[x^2+x+\left(\theta^2+2\theta+3\right)\right]$$

The first derivative of the log likelihood function is given by

$$\frac{d\log L}{d\theta}=\frac{3n}{\theta}-\frac{2n\theta}{\theta^2+2}-\frac{n(\bar{x}+3)}{\theta+1}+\sum_{x=1}^{k}\frac{2(\theta+1)f_x}{\left[x^2+x+\left(\theta^2+2\theta+3\right)\right]}$$

where $\bar{x}$ is the sample mean.

The maximum likelihood estimate (MLE), $\hat{\theta}$ of θ of PAD (1.1) is the solution of the equation $\frac{d\log L}{d\theta}=0$ and is thus given by the solution of the non-linear equation

$$\frac{3n}{\theta}-\frac{2n\theta}{\theta^2+2}-\frac{n(\bar{x}+3)}{\theta+1}+\sum_{x=1}^{k}\frac{2(\theta+1)f_x}{\left[x^2+x+\left(\theta^2+2\theta+3\right)\right]}=0$$

This non-linear equation can be solved by any numerical iteration methods such as Newton-Raphson method, Bisection method, Regula-Falsi method etc. In this paper Newton-Raphson method has been used to solve above non-linear equation to get maximum likelihood estimate of the parameter.

Method of moment estimate (MOME) of the Parameter: Let $(x_1,x_2,...,x_n)$ be a random sample of size n from the PAD (1.1). Equating the population mean to the corresponding sample mean, the MOME $\tilde{\theta}$ of θ of PAD (1.1) is the solution of the following cubic equation

$$\bar{x}\theta^3-\theta^2+2\bar{x}\theta-6=0$$

where $\bar{x}$ is the sample mean.

Applications and goodness of fit of PAD

When events seem to occur at random, Poisson distribution is a suitable statistical model. Examples of events where Poisson distribution is a suitable model includes the number of customers arriving at a service point, the number of telephone calls arriving at an exchange , the number of fatal traffic accidents per week in a given state, the number of radioactive particle emissions per unit of time, the number of meteorites that collide with a test satellite during a single orbit, the number of organisms per unit volume of some fluid, the number of defects per unit of some materials, the number of flaws per unit length of some wire, are some amongst others. Further, the conditions for using Poisson distribution are the independence of events and equality of mean and variance, which are rarely satisfied completely in biomedical science and thunderstorms due to the fact that the occurrences of successive events in biomedical science and thunderstorms are dependent. Negative binomial distribution is the appropriate choice for the situation where successive events are dependent but negative binomial distribution requires higher degree of over-dispersion Johnson et al.,[6] In biomedical science and thunderstorms, these conditions are not fully satisfied. Generally, the count data in biomedical science and thunderstorms are either over-dispersed or under-dispersed. The main reason for selecting PLD and PAD to fit count data from biomedical science and thunderstorms are that these two distributions are always over-dispersed and PAD has some flexibility over PLD.

Applications in ecology

Ecology is the branch of biology which deals with the relations and interactions between organisms and their environment, including their organisms. Since the organisms and their environment in the nature are complex, dynamic, interdependent, mutually reactive and interrelated, ecology deals with the various principles which govern such relationship between organisms and their environment. Firstly Fisher et al.,[7] discussed the applications of Logarithmic series distribution (LSD) to model count data in the science of ecology. Later, Kempton[8] who fitted the generalized form of Fisher's Logarithmic series

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distribution (LSD) to model insect data and concluded that it gives a superior fit as compared to ordinary Logarithmic series distribution (LSD). He also concluded that it gives better explanation for the data having exceptionally long tail. Tripathi & Gupta[9] proposed another generalization of the Logarithmic series distribution (LSD) which is flexible to describe short-tailed as well as long-tailed data and fitted it to insect data and found that it gives better fit as compared to ordinary Logarithmic series distribution. Shanker,[10] Mishra & Shanker[11] have discussed applications of generalized logarithmic series distributions (GLSD) to models data in ecology. Shanker & Hagos[12] have tried to fit PLD for data relating to ecology and observed that PLD gives satisfactory fit.

In this section we have tried to fit Poisson distribution (PD), Poisson -Lindley distribution (PLD) and Poisson-Akash distribution (PAD) to many count data from biological sciences using maximum likelihood estimates. The data were on haemocytometer yeast cell counts per square, on European red mites on apple leaves and European corn borers per plant.

It is obvious from above tables that in Table 4.1.1, PD gives better fit than PLD and PSD; in Table 4.1.2 PAD gives better fit than PD and PLD while in Table 4.1.3, PLD gives better fit than PD and PAD.

Table 4.1.1 Observed and expected number of Haemocytometer yeast cell counts per square observed by Gosset[13]

Number of yeast cells Per square	Observed frequency	Expected frequency		
		PD	PLD	PAD
0	213	202.1	234.0	236.8
1	128	138.0	99.4	95.6
2	37	47.1	40.5	39.9
3	18	10.7	16.0	16.6
4	3	1.8	6.2	6.7
5	1	0.2	2.4	2.7
6	0	0.1	1.5	1.7
Total		400.0	400.0	400.0
Estimate of parameter		$\hat{\theta}$=0.6825	$\hat{\theta}$=1.950236	$\hat{\theta}$=2.260342
χ^2		10.08	11.04	14.68
d.f.		2	2	2
p-value		0.0065	0.0040	0.0006

Table 4.1.2 Observed and expected number of red mites on Apple leaves

Number of red mites per leaf	Observed frequency	Expected frequency		
		PD	PLD	PAD
0	38	25.3	35.8	36.3
1	17	29.1	20.7	20.1
2	10	16.7	11.4	11.2
3	9	6.4	6.0	6.1
4	3	1.8	3.1	3.2
5	2	0.4	1.6	1.6
6	1	0.2	0.8	0.8
7+	0	0.1	0.6	0.7
Total	80	80.0	80.0	80.0
Estimate of parameter		$\hat{\theta} = 1.15$	$\hat{\theta} = 1.255891$	$\hat{\theta}$=1.620588
χ^2		18.27	2.47	2.07
d.f.		2	3	3
p-value		0.0001	0.4807	0.558

Table 4.1.3 Observed and expected number of European corn-borer of Mc Guire et al.,[14]

Number of corn-borer per plant	Observed frequency	Expected frequency		
		PD	PLD	PAD
0	188	169.4	194.0	196.3
1	83	109.8	79.5	76.5
2	36	35.6	31.3	30.8
3	14	7.8	12.0	12.4
4	2	1.2	4.5	4.9
5	1	0.2	2.7	3.1
Total	324	324.0	324.0	324.0
Estimate of parameter		$\hat{\theta}$=0.648148	$\hat{\theta}$=2.043252	$\hat{\theta}$=2.345109
χ^2		15.19	1.29	2.33
d.f.		2	2	2
p-value		0.0005	0.5247	0.3119

Applications in genetics

Genetics is the branch of biological science which deals with heredity and variation. Heredity includes those traits or characteristics which are transmitted from generation to generation, and is therefore fixed for a particular individual. Variation, on the other hand, is mainly of two types, namely hereditary and environmental. Hereditary variation refers to differences in inherited traits whereas environmental variations are those which are mainly due to environment. Much quantitative works seem to be done in genetics but so far no works has been done on fitting of PAD for count data in genetics. The segregation of chromosomes has been studied using statistical tool, mainly chi-square (χ^2). In the analysis of data observed on chemically induced chromosome aberrations in cultures of human leukocytes, Loeschke & Kohler[15] suggested the negative binomial distribution while Janardan & Schaeffer[16] suggested modified Poisson distribution. Shanker,[10] Mishra & Shanker[11] have discussed applications of generalized Logarithmic series distributions (GLSD) to model data in mortality, ecology and genetics. Shanker & Hagos[12] have detailed study on the applications of PLD to model data from genetics. In this section an attempt has been made to fit to data relating to genetics using PAD, PLD and PD using maximum likelihood estimate. Also an attempt has been made to fit PAD, PLD, and PD to the data of Catcheside et al.,[17,18] in Tables 4.2.2, 4.2.3, and 4.2.4.

Table 4.2.1 Distribution of number of Chromatid aberrations (0.2 g chinon 1, 24 hours)

Number of Aberrations	Observed Frequency	Expected Frequency		
		PD	PLD	PAD
0	268	231.3	257.0	260.4
1	87	126.7	93.4	89.7
2	26	34.7	32.8	32.1
3	9	6.3	11.2	11.5
4	4	0.8	3.8	4.1
5	2	0.1	1.2	1.4
6	1	0.1	0.4	0.5
7+	3	0.1	0.2	0.3
Total	400	400.0	400.0	400.0
Estimate of Parameter		$\hat{\theta}$=0.5475	$\hat{\theta}$=2.380442	$\hat{\theta}$=2.659408
χ^2		38.21	6.21	4.17
d.f.		2	3	3
p-value		0.0000	0.1018	0.2437

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Table 4.2.2 Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-60 $\mu g|kg$

Class/Exposure $(\mu g\|kg)$	Observed frequency	Expected frequency PD	PLD	PAD
0	413	374.0	405.7	409.5
1	124	177.4	133.6	128.7
2	42	42.1	42.6	42.1
3	15	6.6 }	13.3	13.9
4	5	0.8 }	4.1 }	4.6 }
5	0	0.1 }	1.2 }	1.5 }
6	2	0.0 }	0.5 }	0.7 }
Total	601	601.0	601.0	601.0
Estimate of parameter		$\hat{\theta}$=0.47421	$\hat{\theta}$=2.685373	$\hat{\theta}$=2.915059
χ^2		48.17	1.34	0.29
d.f.		2	3	3
p-value		0.0000	0.7196	0.9619

Table 4.2.3 Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-70 $\mu g|kg$

Class/Exposure $(\mu g\|kg)$	Observed frequency	Expected frequency PD	PLD	PAD
0	200	172.5	191.8	194.1
1	57	95.4	70.3	67.6
2	30	26.4	24.9	24.5
3	7	4.9 }	8.6 }	8.9 }
4	4	0.7 }	2.9 }	3.2 }
5	0	0.1 }	1.0 }	1.1 }
6	2	0.0 }	0.5 }	0.6 }
Total	300	300.0	300.0	300.0
Estimate of parameter		$\hat{\theta}$=0.55333	$\hat{\theta}$=2.353339	$\hat{\theta}$=2.626739
χ^2		29.68	3.91	3.12
d.f.		2	2	2
p-value		0.0000	0.1415	0.2101

Table 4.2.4 Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure-90 $\mu g|kg$

Class/exposure $(\mu g\|kg)$	Observed frequency	Expected frequency PD	PLD	PAD
0	155	127.8	158.3	160.7
1	83	109.0	77.2	74.3
2	33	46.5	35.9	35.3

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Table Continued

Class/exposure ($\mu g|kg$)	Observed frequency	Expected frequency		
		PD	PLD	PAD
3	14	13.2	16.1	16.5
		2.8	7.1	7.5
4	11	0.5	3.1	3.3
5	3	0.2	2.3	2.4
6	1			
Total	300	300.0	300.0	300.0
Estimate of parameter		$\hat{\theta}=0.853333$	$\hat{\theta}$=1.617611	$\hat{\theta}$=1.963313
χ^2		24.97	1.51	1.98
d.f.		2	3	3
p-value		0.0000	0.6799	0.5766

It is obvious from the fitting of PAD, PLD, and PD that PAD gives much closer fit in Tables 4.2.1, 4.2.2 and 4.2.3 but in Table 4.2.4, PLD gives better fit than PD and PAD.

Applications in thunderstorms

In thunderstorm activity, the occurrence of successive thunderstorm events (THE's) is often dependent process which means that the occurrence of a THE indicates that the atmosphere is unstable and the conditions are favorable for the formation of further thunderstorm activity. The negative binomial distribution (NBD) is a possible alternative to the Poisson distribution when successive events are possibly dependent Johnson et al.,[6] The theoretical and empirical justification for using the NBD to describe THE activity has been fully explained and discussed by Falls et al.,[19] Further, for fitting Poisson distribution to the count data equality of mean and variance should be satisfied. Similarly, for fitting NBD to the count data, mean should be less than the variance. In THE, these conditions are not fully satisfied. As a model to describe the frequencies of thunderstorms (TH's), given an occurrence of THE, the PAD can be considered because it is always over-dispersed. In this section, the thunderstorms data have been considered in tables 4.3.1, 4.3.2, 4.3.3, and 4.3.4.

It is obvious from the fitting of PAD, PLD, and PD that PAD gives much closer fit than PLD and PD in all data-sets relating to thunderstorms and hence PAD can be considered as an important model for modeling thunderstorms events.

Table 4.3.1 Observed and expected number of days that experienced X thunderstorms events at Cape Kennedy, Florida for the 11-year period of record for the month of June, January 1957 to December 1967, Falls et al.,[19]

No. of thunderstorms	Observed frequency	Expected frequency		
		PD	PLD	PAD
0	187	155.6	185.3	187.9
1	77	117.0	83.5	80.2
2	40	43.9	35.9	35.3
3	17	11.0	15.0	15.4
4	6	2.1	6.1	6.6
5	2	0.3	2.5	2.7
6	1	0.1	1.7	1.9
Total	330	330.0	330.0	330.0
ML estimate		$\hat{\theta}$=0.751515	$\hat{\theta}$=1.804268	$\hat{\theta}$=2.139736
χ^2		31.93	1.43	1.35
d.f.		2	3	3
p-value		0.0000	0.6985	0.7173

Table 4.3.2 Observed and expected number of days that experienced X thunderstorms events at Cape Kennedy, Florida for the 11-year period of record for the month of July, January 1957 to December 1967, Falls et al.,[19]

No. of thunderstorms	Observed frequency	Expected frequency		
		PD	PLD	PAD
0	177	142.3	177.7	180.0
1	80	124.4	88.0	84.7
2	47	54.3	41.5	40.9
3	26	15.8	18.9	19.4
4	9	3.5	8.4	8.9
5	2	0.7	6.5	7.1
Total	341	341.0	341.0	341.0
ML estimate		$\hat{\theta}=0.873900$	$\hat{\theta}=1.583536$	$\hat{\theta}=1.938989$
χ^2		39.74	5.15	5.02
d.f.		2	3	3
p-value		0.0000	0.1611	0.1703

Table 4.3.3 Observed and expected number of days that experienced X thunderstorms events at Cape Kennedy, Florida for the 11-year period of record for the month of August, January 1957 to December 1967, Falls et al.,[19]

No. of thunderstorms	Observed frequency	Expected frequency		
		PD	PLD	PAD
0	185	151.8	184.8	187.5
1	89	122.9	87.2	83.9
2	30	49.7	39.3	38.6
3	24	13.4	17.1	17.5
4	10	2.7	7.3	7.6
5	3	0.5	5.3	5.9
Total	341	341.0	341.0	341.0
ML estimate		$\hat{\theta}=0.809384$	$\hat{\theta}=1.693425$	$\hat{\theta}=2.038417$
χ^2		49.49	5.03	4.69
d.f.		2	3	3
p-value		0.0000	0.1696	0.1960

Table 4.3.4 Observed and expected number of days that experienced X thunderstorms events at Cape Kennedy, Florida for the 11-year period of record for the summer, January 1957 to December 1967, Falls et al.,[19]

No. of thunderstorms	Observed frequency	Expected frequency		
		PD	PLD	PAD
0	549	449.0	547.5	555.1
1	246	364.8	259.0	249.2
2	117	148.2	116.9	114.9
3	67	40.1	51.2	52.3
4	25	8.1	21.9	23.2
5	7	1.3	9.2	10.0
6	1	0.5	6.3	7.3
Total	1012	1012.0	1012.0	1012.0
ML estimate		$\hat{\theta}=0.812253$	$\hat{\theta}=1.688990$	$\hat{\theta}=2.033715$

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Table Continued

No. of thunderstorms	Observed frequency	Expected frequency		
		PD	PLD	PAD
χ^2		119.45	9.60	9.40
d.f.		3	4	4
p-value		0.0000	0.0477	0.0518

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Conflict of Interest

None.

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