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On Modeling of Lifetime Data Using One Parameter Akash, Lindley and Exponential Distributions

Abstract

The analysis and modeling of lifetime data are crucial in almost all applied sciences including medicine, insurance, engineering, and finance, amongst others. In the present paper an attempt has been made to discuss applications of Akash distribution introduced by Shanker [1], Lindley distribution and exponential distributions for modeling lifetime data from various fields. Firstly a table for values of the various characteristics of Akash distribution and Lindley distribution has been presented for various values of their parameter which reflects their nature and behavior. The expressions for the index of dispersion of Akash, Lindley and exponential distributions have been obtained and the conditions under which Akash, Lindley and exponential distributions are over-dispersed, equi-dispersed, and under-dispersed has been given. Several lifetime data from medical science and engineering have been fitted using Akash distribution along with Lindley and exponential distributions to study the advantages and disadvantages of these distributions for modeling lifetime data.

Keywords: Akash distribution; Lindley distribution; Exponential distribution; Index of dispersion; Estimation of parameter; Goodness of fit

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Introduction

The time to the occurrence of event of interest is known as lifetime or survival time or failure time in reliability analysis. The event may be failure of a piece of equipment, death of a person, development (or remission) of symptoms of disease, health code violation (or compliance). The modeling and statistical analysis of lifetime data are crucial for statisticians and research workers in almost all applied sciences including engineering, medical science/biological science, insurance and finance, amongst others.

Recently Shanker [1] has introduced a one parameter continuous distribution named, "Akash distribution" for modeling lifetime data from engineering and medical science and studied its various mathematical properties, estimation of its parameter, and its applications. A number of continuous distributions for modeling lifetime data have been introduced in statistical literature including exponential, Lindley, gamma, lognormal and Weibull, amongst others. The exponential, Lindley and the Weibull distributions are more popular in practice than the gamma and the lognormal distributions because the survival functions of the gamma and the lognormal distributions cannot be expressed in closed forms and both require numerical integration. Though Akash, Lindley and exponential distributions are of one parameter, Akash and Lindley distributions have advantage over the exponential distribution that the exponential distribution has constant hazard rate and mean residual life function whereas the Akash and Lindley distributions have increasing hazard rate and decreasing mean residual life function. Further, Akash distribution of Shanker [1] has flexibility over both Lindley and exponential distributions.

Exponential, Lindley and Akash Distributions

Exponential distribution

In statistical literature, exponential distribution was the first widely used lifetime distribution model in areas ranging from studies on the lifetimes of manufactured items [Davis [2], Epstein & Sobel [3]], Epstein [4] to research involving survival or remission times in chronic diseases [Feigl & Zelen [5]]. The main reason for its wide usefulness and applicability as lifetime model is partly because of the availability of simple statistical methods for it [Epstein & Sobel [3]] and partly because it appeared suitable for representing the lifetimes of many phenomenons such as various types of manufactured items [Davis [2]].

Lindley distribution

The Lindley distribution is a two-component mixture of an exponential distribution having scale parameter θ and a gamma distribution having shape parameter 2 and scale parameter θ with mixing proportions $\frac{\theta}{\theta+1}$ and $\frac{1}{\theta+1}$ and is given by Lindley [6] in the context of Bayesian Statistics as a counter example of fiducial Statistics. A detailed study about its various mathematical properties, estimation of parameter and application showing the superiority of Lindley distribution over exponential distribution for the waiting times before service of the bank customers has been done by Ghitany et al. [7]. The Lindley distribution has been generalized, extended, mixed, modified and its detailed applications in reliability and other fields of knowledge by different researchers including Sankaran [8] Hussain [9], Zakerzadeh & Dolati [10], Nadarajah et al. [11], Deniz & Ojeda [12], Mazucheli & Achcar [13], Bakouch et al. [14], Shanker & Mishra [15,16], Shanker et al. [17], Shanker & Amanuel [18], Elbatal et al. [19], Ghitany et al. [20], Merovci [21], Liyanage & Pararai [22], Ashour & Eltehiwy [23], Oluyede & Yang [24], Singh et al. [25], Sharma et al. [26], Shanker et al. [27], Alkarni [28], Pararai et al. [29], Abouammoh et al. [30] are some among others.

Although the Lindley distribution has been used to model lifetime data by many researchers and Hussain [9] has shown that the Lindley distribution is important for studying stress-strength reliability modeling, it has been observed that there are many situations in the modeling of lifetime data where the Lindley distribution may not be suitable from a theoretical or applied point of view. In fact, Shanker et al. [27] has detailed comparative study about the applicability of Lindley and exponential distributions for modeling various types of lifetime data and observed that none is a suitable model in all cases.

Akash distribution

Shanker [1] introduced a new distribution named, 'Akash distribution' which is flexible than the Lindley distribution for modeling lifetime data in reliability and in terms of its hazard rate shapes. Akash distribution is a two- component mixture of an exponential distribution having scale parameter θ and a gamma θ distribution having shape parameter 3 and scale parameter with mixing proportions $\frac{\theta^2}{\theta^2+2}$ and $\frac{1}{\theta^2+2}$ and has been shown by Shanker [1] that Akash distribution gives better fit than Lindley and exponential distributions in modeling some lifetime data.

Let T be a continuous random variable representing the lifetimes of individuals in some population. The expressions for probability density function, $f(t)$, cumulative distribution function, $F(t)$, survival function, $S(t)$, hazard rate function, $h(t)$, mean residual life function, $m(t)$, mean μ_1', variance μ_2, third moment about mean μ_3, fourth moment about mean μ_4, coefficient of variation (C.V.), coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, coefficient of Kurtosis (β_2), and index of dispersion (γ) of exponential, Lindley and Akash distributions are summarized in Table 1.

Table 1: Characteristics of Exponential, Lindley and Akash Distributions.

Exponential Distribution	Lindley Distribution	Akash Distribution
$f(t)=\theta e^{-\theta t}$	$f(t)=\frac{\theta^2}{\theta+1}(1+t)e^{-\theta t}$	$f(t)=\frac{\theta^3}{\theta^2+2}\left(1+t^2\right)e^{-\theta t}$
$F(t)=1-e^{-\theta t}$	$F(t)=1-\frac{\theta+1+\theta t}{\theta+1}e^{-\theta t}$	$F(t)=1-\left[1+\frac{\theta t(\theta t+2)}{\theta^2+2}\right]e^{-\theta t}$
$S(t)=e^{-\theta t}$	$S(t)=\frac{\theta+1+\theta t}{\theta+1}e^{-\theta t}$	$S(t)=\left[1+\frac{\theta t(\theta t+2)}{\theta^2+2}\right]e^{-\theta t}$
$h(t)=\theta$	$h(t)=\frac{\theta^2(1+t)}{\theta+1+\theta t}$	$h(t)=\frac{\theta^3\left(1+t^2\right)}{\theta t(\theta t+2)+\left(\theta^2+2\right)}$
$m(t)=\frac{1}{\theta}$	$m(t)=\frac{\theta+2+\theta t}{\theta(\theta+1+\theta t)}$	$m(t)=\frac{\theta^2 t^2+4\theta t+\left(\theta^2+6\right)}{\theta\left[\theta t(\theta t+2)+\left(\theta^2+2\right)\right]}$

$\mu_1' = \dfrac{1}{\theta}$	$\mu_1' = \dfrac{\theta+2}{\theta(\theta+1)}$	$\mu_1' = \dfrac{\theta^2+6}{\theta(\theta^2+2)}$
$\mu_2 = \dfrac{1}{\theta^2}$	$\mu_2 = \dfrac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2}$	$\mu_2 = \dfrac{\theta^4+16\theta^2+12}{\theta^2(\theta^2+2)^2}$
$\mu_3 = \dfrac{2}{\theta^3}$	$\mu_3 = \dfrac{2(\theta^3+6\theta^2+6\theta+2)}{\theta^3(\theta+1)^3}$	$\mu_3 = \dfrac{2(\theta^6+30\theta^4+36\theta^2+24)}{\theta^3(\theta^2+2)^3}$
$\mu_4 = \dfrac{9}{\theta^4}$	$\mu_4 = \dfrac{3\begin{pmatrix}3\theta^4+24\theta^3+44\theta^2\\+32\theta+8\end{pmatrix}}{\theta^4(\theta+1)^4}$	$\mu_4 = \dfrac{3\begin{pmatrix}3\theta^8+128\theta^6+408\theta^4\\+576\theta^2+240\end{pmatrix}}{\theta^4(\theta^2+2)^4}$
$C.V = \dfrac{\sigma}{\mu_1'} = 1$	$C.V = \dfrac{\sigma}{\mu_1'} = \dfrac{\sqrt{\theta^2+4\theta+2}}{\theta+2}$	$C.V = \dfrac{\sigma}{\mu_1'} = \dfrac{\sqrt{\theta^4+16\theta^2+12}}{\theta^2+6}$
$\sqrt{\beta_1} = 2$	$\sqrt{\beta_1} = \dfrac{2(\theta^3+6\theta^2+6\theta+2)}{(\theta^2+4\theta+2)^{3/2}}$	$\sqrt{\beta_1} = \dfrac{2(\theta^6+30\theta^4+36\theta^2+24)}{(\theta^4+16\theta^2+12)^{3/2}}$
$\beta_2 = 9$	$\beta_2 = \dfrac{3\begin{pmatrix}3\theta^4+24\theta^3+44\theta^2\\+32\theta+8\end{pmatrix}}{(\theta^2+4\theta+2)^2}$	$\beta_2 = \dfrac{3\begin{pmatrix}3\theta^8+128\theta^6+408\theta^4\\+576\theta^2+240\end{pmatrix}}{(\theta^4+16\theta^2+12)^2}$
$\gamma = \dfrac{\sigma^2}{\mu_1'} = \dfrac{1}{\theta}$	$\gamma = \dfrac{\sigma^2}{\mu_1'} = \dfrac{\theta^2+4\theta+2}{\theta(\theta+1)(\theta+2)}$	$\gamma = \dfrac{\sigma^2}{\mu_1'} = \dfrac{\theta^4+16\theta^2+12}{\theta(\theta^2+2)(\theta^2+6)}$

It can be easily verified that the Akash distribution is over-dispersed $\left(\mu<\sigma^2\right)$, equi-dispersed $\left(\mu=\sigma^2\right)$ and under-dispersed $\left(\mu>\sigma^2\right)$ for $\theta<(=)>\theta^*=1.515400063$ respectively. Further, Lindley distribution is over- dispersed $\left(\mu<\sigma^2\right)$, equi-dispersed $\left(\mu=\sigma^2\right)$ and under-dispersed for $\theta<(=)>\theta^*=1.170086487$ respectively, whereas as exponential distribution is over-dispersed $\left(\mu>\sigma^2\right)$, equi-dispersed $\left(\mu=\sigma^2\right)$ and under-dispersed $\left(\mu<\sigma^2\right)$ for $\theta<(=)>\theta^*=1$ respectively.

A table of values for coefficient of variation (C.V.), coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, coefficient of Kurtosis $\left(\beta_2\right)$, and index of dispersion $\left(\gamma\right)$ for Akash and Lindley distributions for various values of their parameter for comparative study are summarized in Table 2.

Table 2: Values of C.V., $\sqrt{\beta_1}$, β_2 and γ of Akash and Lindley Distributions for varying values of the parameter θ.

Values of θ for Akash Distribution							
	0.01	**0.05**	**0.09**	**0.5**	**0.8**	**1.5**	**2**
C.V	0.577379	0.578071	0.579679	0.641249	0.716741	0.882958	0.959166
$\sqrt{\beta_1}$	1.154643	1.153268	1.150133	1.083974	1.10564	1.388077	1.61372
β_2	4.999867	4.996681	4.989352	4.784948	4.735717	5.472724	6.391304
γ	100.0067	20.03328	11.17079	2.284444	1.615097	1.008913	0.766667
Values of θ for Lindley Distribution							
	0.01	**0.05**	**0.09**	**0.5**	**0.8**	**1.5**	**2**
C.V	0.710607	0.723943	0.736298	0.824621	0.863075	0.914732	0.935414
$\sqrt{\beta_1}$	1.414317	1.416546	1.421076	1.512281	1.580387	1.698866	1.756288
β_2	6.000294	6.006807	6.020488	6.342561	6.621505	7.172516	7.469388
γ	100.4926	20.46458	11.55007	2.266667	1.448413	0.780952	0.583333

Applications

The Akash, Lindley and exponential distributions have been fitted to a number of real lifetime data - sets to test their goodness of fit. Goodness of fit tests for sixteen real lifetime data- sets have been presented here. In order to compare Akash, Lindley and exponential distributions, $-2\ln L$, AIC (Akaike Information Criterion), AICC (Akaike Information Criterion Corrected), BIC (Bayesian Information Criterion), K-S Statistics (Kolmogorov-Smirnov Statistics) for all sixteen real lifetime data- sets have been computed and presented in Table 3. The formulae for computing AIC, AICC, BIC, and K-S Statistics are as follows:

Table 3: MLE's, -2ln L, AIC, AICC, BIC, K-S Statistics of the fitted distributions of data sets 1-16.

	Model	Parameter Estimate	-2ln L	AIC	AICC	BIC	K-S Statistic
Data 1	Akash Lindley Exponential	1.355445 0.996116 0.663647	163.73 162.56 177.66	165.73 164.56 179.66	165.79 164.62 179.73	169.93 166.70 181.80	0.355 0.371 0.402
Data 2	Akash Lindley Exponential	0.043876 0.028859 0.014635	950.97 983.11 1044.87	952.97 985.11 1046.87	953.01 985.15 1046.91	955.58 987.71 1049.48	0.184 0.242 0.357
Data 3	Akash Lindley Exponential	0.04151 0.027321 0.013845	227.06 231.47 242.87	229.06 233.47 244.87	229.25 233.66 245.06	230.20 234.61 246.01	0.107 0.149 0.263
Data 4	Akash Lindley Exponential	0.013514 0.00897 0.004505	1255.83 1251.34 1280.52	1257.83 1253.34 1282.52	1257.87 1253.38 1282.56	1260.43 1255.95 1285.12	0.071 0.098 0.190
Data 5	Akash Lindley Exponential	0.030045 0.019841 0.010018	794.70 789.04 806.88	796.70 791.04 808.88	796.76 791.10 808.94	798.98 793.32 811.16	0.184 0.133 0.198
Data 6	Akash Lindley Exponential	0.11961 0.077247 0.04006	981.28 1041.64 1130.26	983.28 1043.64 1132.26	983.31 1043.68 1132.29	986.18 1046.54 1135.16	0.393 0.448 0.525
Data 7	Akash Lindley Exponential	0.013263 0.008804 0.004421	803.96 763.75 744.87	805.96 765.75 746.87	806.02 765.82 746.94	810.01 767.81 748.93	0.298 0.245 0.166
Data 8	Akash Lindley Exponential	0.013423 0.00891 0.004475	609.93 579.16 564.02	611.93 581.16 566.02	612.02 581.26 566.11	613.71 582.95 567.80	0.280 0.219 0.145
Data 9	Akash Lindley Exponential	0.3105 0.196045 0.106773	887.89 839.06 828.68	889.89 841.06 830.68	889.92 841.09 830.72	892.74 843.91 833.54	0.198 0.116 0.077
Data 10	Akash Lindley Exponential	0.050293 0.033021 0.016779	354.88 323.27 305.26	356.88 325.27 307.26	357.02 325.42 307.40	358.28 326.67 308.66	0.421 0.345 0.213
Data 11	Akash Lindley Exponential	1.165719 0.823821 0.532081	115.15 112.61 110.91	117.15 114.61 112.91	117.28 114.73 113.03	118.68 116.13 114.43	0.156 0.133 0.089
Data 12	Akash Lindley Exponential	0.295277 0.186571 0.101245	641.93 638.07 658.04	643.93 640.07 660.04	643.95 640.12 660.08	646.51 642.68 662.65	0.100 0.058 0.163
Data 13	Akash Lindley Exponential	0.024734 0.01636 0.008246	194.30 181.34 173.94	196.30 183.34 175.94	196.61 183.65 176.25	197.01 184.05 176.65	0.456 0.386 0.277
Data 14	Akash Lindley Exponential	1.156923 0.816118 0.526316	59.52 60.50 65.67	61.52 62.50 67.67	61.74 62.72 67.90	62.51 63.49 68.67	0.320 0.341 0.389
Data 15	Akash Lindley Exponential	0.097062 0.062988 0.032455	240.68 253.99 274.53	242.68 255.99 276.53	242.82 256.13 276.67	244.11 257.42 277.96	0.266 0.333 0.426
Data 16	Akash Lindley Exponential	0.964726 0.659000 0.407941	224.28 238.38 261.74	226.28 240.38 263.74	226.34 240.44 263.80	228.51 242.61 265.97	0.348 0.390 0.434

$$AIC = -2\ln L + 2k \,,\; AICC = AIC + \frac{2k\left(k+1\right)}{\left(n-k-1\right)}$$
and
$$BIC = -2\ln L + k\ln n \qquad D = \sup_x \left| F_n\left(x\right) - F_0\left(x\right)\right|$$

where k = the number of parameters, n = the sample size and $F_n(x)$ is the empirical distribution function. The best distribution corresponds to lower values of $-2\ln L$, AIC, AICC, BIC, and K-S statistics. The fittings of Akash, Lindley and exponential distributions are based on maximum likelihood estimates (MLE).

Let $t_1, t_2, ..., t_n$ be a random sample of size n from exponential distribution. The likelihood function, L and the log likelihood function, $\ln L$ of exponential distribution are given by $L = \theta^n e^{-n\theta\bar{t}}$ and $\ln L = n\ln\theta - n\theta\bar{t}$.

The MLE $\hat{\theta}$ of the parameter θ of exponential distribution is the solution of the equation $\frac{d\ln L}{d\theta} = 0$ and is given by $\hat{\theta} = \frac{1}{\bar{t}}$, where $\bar{t}$ is the sample mean.

Let $t_1, t_2, ..., t_n$ be a random sample of size n from Lindley distribution. The likelihood function, L and the log likelihood function, $\ln L$ of Lindley distribution are given by

$$L = \left(\frac{\theta^2}{\theta+1}\right)^n \prod_{i=1}^{n}\left(1+t_i\right)e^{-n\theta\bar{t}} \qquad \text{and}$$

$$\ln L = n\ln\left(\frac{\theta^2}{\theta+1}\right) + \sum_{i=1}^{n}\ln\left(1+t_i\right) - n\theta\bar{t}$$

The MLE $\hat{\theta}$ of the parameter θ of Lindley distribution is the solution of the equation $\frac{d\ln L}{d\theta} = 0$ and is given by

$$\hat{\theta} = \frac{-\left(\bar{t}-1\right) + \sqrt{\left(\bar{t}-1\right)^2 + 8\bar{t}}}{2\bar{t}};$$ $\bar{t} > 0$, where $\bar{t}$ is the sample mean.

Let $t_1, t_2, ..., t_n$ be a random sample of size n from Akash distribution. The likelihood function, L and the log likelihood function, $\ln L$ of Akash distribution are given by $L = \left(\frac{\theta^3}{\theta^2+2}\right)^n \prod_{i=1}^{n}\left(1+t_i^2\right)e^{-n\theta\bar{t}}$

and $\ln L = n\ln\left(\frac{\theta^3}{\theta^2+2}\right) + \sum_{i=1}^{n}\ln\left(1+t_i^2\right) - n\theta\bar{t}$. The MLE $\hat{\theta}$ of the parameter $\hat{\theta}$ of Akash distribution is the solution of the equation $\frac{d\ln L}{d\theta} = 0$ and is the solution of following non-linear equation $\bar{t}\theta^3 - \theta^2 + 2\bar{t}\theta - 6 = 0$, where $\bar{t}$ is the sample mean.

It is obvious from the goodness of fit of Akash, Lindley and exponential distributions that the Akash distribution provides better fit than the Lindley and exponential distributions in data-sets 2, 3, 6, 14, 15, and 16; the Lindley distribution gives better fit than the exponential and Akash distributions in data-sets 1, 4, 5 and 12; the exponential distribution gives better fit than the Lindley and the Akash distributions in data sets 7, 8, 9, 10, 11, and 13 (Data sets 1-16).

Data Set 1: The data set represents the strength of 1.5cm glass fibers measured at the National Physical Laboratory, England. Unfortunately, the units of measurements are not given in the paper, and they are taken from Smith & Naylor [31].

0.55	0.93	1.25	1.36	1.49	1.52	1.58	1.61	1.64	1.68	1.73	1.81	2
0.74	1.04	1.27	1.39	1.49	1.53	1.59	1.61	1.66	1.68	1.76	1.82	2.01
0.77	1.11	1.28	1.42	1.5	1.54	1.6	1.62	1.66	1.69	1.76	1.84	2.24
0.81	1.13	1.29	1.48	1.5	1.55	1.61	1.62	1.66	1.7	1.77	1.84	0.84
1.24	1.3	1.48	1.51	1.55	1.61	1.63	1.67	1.7	1.78	1.89		

Data Set 2: The data is given by Birnbaum & Saunders [32] on the fatigue life of 6061 – T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second. The data set consists of 101 observations with maximum stress per cycle 31,000 psi. The data ($\times\ 10^{-3}$) are presented below (after subtracting 65).

5	25	31	32	34	35	38	39	39	40	42	43	43
43	44	44	47	47	48	49	49	49	51	54	55	55
55	56	56	56	58	59	59	59	59	59	63	63	64
64	65	65	65	66	66	66	66	66	67	67	67	68
69	69	69	69	71	71	72	73	73	73	74	74	76
76	77	77	77	77	77	77	79	79	80	81	83	83
84	86	86	87	90	91	92	92	92	92	93	94	97
98	98	99	101	103	105	109	136	147				

Data Set 3: The data set is from Lawless [33]. The data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life tests and they are.

17.88	28.92	33	41.52	42.12	45.6	48.8	51.84	51.96	54.12	55.56	67.8
68.44	68.64	68.88	84.12	93.12	98.64	105.12	105.84	127.92	128.04	173.4	

Data Set 4: The data is from Picciotto [34] and arose in test on the cycle at which the Yarn failed. The data are the number of cycles until failure of the yarn and they are.

86	146	251	653	98	249	400	292	131	169	175	176	76
264	15	364	195	262	88	264	157	220	42	321	180	198
38	20	61	121	282	224	149	180	325	250	196	90	229
166	38	337	65	151	341	40	40	135	597	246	211	180
93	315	353	571	124	279	81	186	497	182	423	185	229
400	338	290	398	71	246	185	188	568	55	55	61	244
20	284	393	396	203	829	239	236	286	194	277	143	198
264	105	203	124	137	135	350	193	188				

Data Set 5: This data represents the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal [35].

12	15	22	24	24	32	32	33	34	38	38	43	44
48	52	53	54	54	55	56	57	58	58	59	60	60
60	60	61	62	63	65	65	67	68	70	70	72	73
75	76	76	81	83	84	85	87	91	95	96	98	99
109	110	121	127	129	131	143	146	146	175	175	211	233
258	258	263	297	341	341	376						

Data Set 6: This data is related with behavioral sciences, collected by Balakrishnan N et al. [36]: The scale "General Rating of Affective Symptoms for Preschoolers (GRASP)" measures behavioral and emotional problems of children, which can be classified with depressive condition or not according to this scale. A study conducted by the authors in a city located at the south part of Chile has allowed collecting real data corresponding to the scores of the GRASP scale of children with frequency in parenthesis, which are.

19(16)	20(15)	21(14)	22(9)	23(12)	24(10)	25(6)	
26(9)	27(8)	28(5)	29(6)	30(4)	31(3)	32(4)	
33	34	35(4)	36(2)	37(2)	39	42	44

Data Set 7: The data set reported by Efron [37] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using radiotherapy (RT).

6.53	7	10.42	14.48	16.1	22.7	34	41.55	42	45.28	49.4	53.62	63
64	83	84	91	108	112	129	133	133	139	140	140	146
149	154	157	160	160	165	146	149	154	157	160	160	165
173	176	218	225	241	248	273	277	297	405	417	420	440
523	583	594	1101	1146	1417							

Data Set 8: The data set reported by Efron [37] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using a combination of radiotherapy and chemotherapy (RT+CT).

12.2	23.56	23.74	25.87	31.98	37	41.35	47.38	55.46	58.36	63.47	68.46	78.26
74.47	81.43	84	92	94	110	112	119	127	130	133	140	146
155	159	173	179	194	195	209	249	281	319	339	432	469
519	633	725	817	1776								

Data Set 9: This data set represents remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee & Wang [38].

0.08	2.09	3.48	4.87	6.94	8.66	13.11	23.63	0.2	2.23	3.52	4.98	6.97
9.02	13.29	0.4	2.26	3.57	5.06	7.09	9.22	13.8	25.74	0.5	2.46	3.64
5.09	7.26	9.47	14.24	25.82	0.51	2.54	3.7	5.17	7.28	9.74	14.76	6.31
0.81	2.62	3.82	5.32	7.32	10.06	14.77	32.15	2.64	3.88	5.32	7.39	10.34
14.83	34.26	0.9	2.69	4.18	5.34	7.59	10.66	15.96	36.66	1.05	2.69	4.23
5.41	7.62	10.75	16.62	43.01	1.19	2.75	4.26	5.41	7.63	17.12	46.12	1.26
2.83	4.33	5.49	7.66	11.25	17.14	79.05	1.35	2.87	5.62	7.87	11.64	17.36
1.4	3.02	4.34	5.71	7.93	11.79	18.1	1.46	4.4	5.85	8.26	11.98	19.13
1.76	3.25	4.5	6.25	8.37	12.02	2.02	3.31	4.51	6.54	8.53	12.03	
20.28	2.02	3.36	6.76	12.07	21.73	2.07	3.36	6.93	8.65	12.63	22.69	

Data Set 10: This data set is given by Linhart & Zucchini [39], which represents the failure times of the air conditioning system of an airplane.

23	261	87	7	120	14	62	47	225	71	246	21	42
20	5	12	120	11	3	14	71	11	14	11	16	90
1	16	52	95									

Data Set 11: This data set used by Bhaumik et al. [40], is vinyl chloride data obtained from clean up gradient monitoring wells in mg/l.

5.1	1.2	1.3	0.6	0.5	2.4	0.5	1.1	8	0.8	0.4	0.6	0.9
0.4	2	0.5	5.3	3.2	2.7	2.9	2.5	2.3	1	0.2	0.1	0.1
1.8	0.9	2	4	6.8	1.2	0.4	0.2					

Data set 12: This data set represents the waiting times (in minutes) before service of 100 Bank customers and examined and analyzed by Ghitany et al. [7] for fitting the Lindley [6] distribution.

0.8	0.8	1.3	1.5	1.8	1.9	1.9	2.1	2.6	2.7	2.9	3.1	3.2
3.3	3.5	3.6	4.0	4.1	4.2	4.2	4.3	4.3,	4.4	4.4	4.6	4.7
4.7	4.8	4.9	4.9	5.0	5.3	5.5	5.7	5.7	6.1	6.2	6.2	6.2
6.3	6.7	6.9	7.1	7.1	7.1	7.1	7.4	7.6	7.7	8.0	8.2	8.6
8.6	8.6	8.8	8.8	8.9	8.9	9.5	9.6	9.7	9.8	10.7	10.9	11.0
11.0	11.1	11.2	11.2	11.5	11.9	12.4	12.5	12.9	13.0	13.1	13.3	13.6
13.7	13.9	14.1	15.4	15.4	17.3	17.3	18.1	18.2	18.4	18.9	19.0	19.9
20.6	21.3	21.4	21.9	23.0	27.0	31.6	33.1	38.5				

Data Set 13: This data is for the times between successive failures of air conditioning equipment in a Boeing 720 airplane, Proschan [41].

74	57	48	29	502	12	70	21	29	386	59	27	153
26	326											

Data Set 14: This data set represents the lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by Gross & Clark [42].

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2	1.7	2.7	4.1	1.8	1.5
1.2	1.4	3	1.7	2.3	1.6	2						

Data Set 15: This data set is the strength data of glass of the aircraft window reported by Fuller et al [43].

18.83	20.8	21.657	23.03	23.23	24.05	24.321	25.5	25.52	25.8	26.69	26.77
26.78	27.05	27.67	29.9	31.11	33.2	33.73	33.76	33.89	34.76	35.75	35.91
36.98	37.08	37.09	39.58	44.045	45.29	45.381					

Data Set 16: The following data represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20mm , Bader and Priest [44].

1.312	1.314	1.479	1.552	1.7	1.803	1.861	1.865	1.944	1.958	1.966	1.997
2.006	2.021	2.027	2.055	2.063	2.098	2.14	2.179	2.224	2.24	2.253	2.27
2.272	2.274	2.301	2.301	2.359	2.382	2.382	2.426	2.434	2.435	2.478	2.49
2.511	2.514	2.535	2.554	2.566	2.57	2.586	2.629	2.633	2.642	2.648	2.684
2.697	2.726	2.77	2.773	2.8	2.809	2.818	2.821	2.848	2.88	2.954	3.012
3.067	3.084	3.09	3.096	3.128	3.233	3.433	3.585	3.858			

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Conflict of Interest

None.

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