Poisson Area-Biased Lindley Distribution and its Applications on Biological Data

Abstract

The purpose of this paper is to introduce a discrete distribution named Poisson-area-biased Lindley distribution and its applications on biological data. Poisson area-biased Lindley distribution is introduced with some of its basic properties including moments, coefficient of skewness and kurtosis are discussed. The method of moments and maximum likelihood estimation of the parameters of Poisson area-biased Lindley distribution are investigated. It is found that the parameter estimated by method of moments is positively biased, consistent and asymptotically normal. Application of the model to some biological data sets is compared with Poisson distribution.

Keywords: PABLD; PD; PLD; Area-biased; MOM; MLE; Factorial moments

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Shakila Bashir* and Mujahid Rasul

Department of Statistics, Forman Christian College, Pakistan

***Corresponding author:** Shakila Bashir, Assistant Professor, Department of Statistics, Forman Christian College (A Chartered University) Ferozepur Road Lahore (54600), Pakistan, Tel: +92 (42) 9923 1581; Email: shakilabashir@fccollege.edu.pk; shakilabashir15@gmail.com

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Introduction

Lindley [1] introduced a single parameter distribution named as Lindley distribution with probability distribution function (pdf)

$$f\left(x;\theta\right)=\frac{\theta^2}{\theta+1}\left(1+x\right)e^{-\theta x}, \qquad x>0, \theta>0. \quad (1.1)$$

The pdf (1.1) is the mixture of exponential (θ) and gamma $(2,\theta)$ distributions. The cumulative distribution function (cdf) of the Lindley distribution is

$$F\left(x\right)=1-\frac{\theta+1+\theta x}{\theta+1}e^{-\theta x}, \qquad x>0, \theta>0. \quad (1.2)$$

The first two moments of the Lindley distribution are

$$\mu_1'=\frac{\theta+2}{\theta\left(\theta+1\right)}, \quad \mu_2'=\frac{2\left(\theta+3\right)}{\theta^2\left(\theta+1\right)}.$$

Sankaran [2] introduced the Lindley mixture of Poisson distribution named Poisson-Lindley distribution with the following pdf

$$f\left(x;\theta\right)=\frac{\theta^2\left(x+\theta+2\right)}{\left(\theta+1\right)}, \qquad x=0,1,2,......., \quad \theta>0. \quad (1.3)$$

The pdf (1.3) is applied to count data and arises from Poisson distribution when its parameter λ follows a Lindley distribution. Ghitany & Al-Mutairi [3] discussed various properties of the Lindley distribution. Ghitany & Al-Mutairi [3] introduced size-biased Poisson Lindley distribution with applications. They considered the size biased form of the Poisson-Lindley distribution. Ghitany & Al-Mutairi [4] discussed estimation methods for the discrete Poisson-Lindley distribution. Srivastava & Adhikari [5] introduced a size-biased Poisson-Lindley distribution which is obtained by considering the size-biased form of the Poisson distribution with Lindley distribution without its size-biased form. Adhikari & Srivastava [6] proposed a Poisson size-biased Lindley distribution which is obtained by computing Poisson distribution without its size-biased form with size-biased Lindley distribution. Shanker & Fesshaye [7] discussed Poisson-Lindley distribution with several of its properties including factorial moments and parameter estimation. They applied the Poisson-Lindley distribution on ecology and genetics data sets and showed that it can be an important tool for modeling biological science data.

Rao [8] introduced the distributions that are used in situations when the recorded observations do not have an equal probability of selection and do not have the original distribution. The distributions used to handle such situations are called weighted distributions. Suppose that the original distribution comes from a distribution with pdf $f_0\left(x\right)$ and the observations is recorded to a probability re-weighted by a weight function $w\left(x\right)>0$, then the weighted distribution is defined as

$$w\left(x\right)=x \qquad (1.4)$$

The weighted distribution with $w\left(x\right)=x$ is called size-biased/length-biased distributions and $w(x)=x^2$ is called area-biased distribution. Patil & Ord [9] discussed size-biased sampling and related form-invariant weighted distributions. Patil & Rao [10] discussed some models leading to weighted distributions and showed applications of weighted distributions in many real sampling problems. Mir & Ahmad [11] introduced size-biased form of some discrete distributions with their applications.

In this paper we consider the Poisson area-biased Lindley distribution (PABLD) which is obtained by considering Poisson distribution without its area-biased form with area-biased Lindley distribution (ABLD).

Poisson Area-Biased Lindley Distribution

The Poisson area-biased Lindley distribution (PABLD) arises from the Poisson distribution with pdf

$$f(x;\lambda)=\frac{e^{-\lambda}\lambda^{x}}{x!},\qquad x=0,1,2,\ldots\ldots\quad \lambda>0,\ (2.1)$$

when its parameter λ follows the area-biased Lindley distribution (ABLD) in (2.1) with pdf

$$f(x;\theta)=\frac{\theta^{4}}{2(\theta+3)}x^{2}(1+x)e^{-\theta x},\qquad x>0,\theta>0.\ (2.3)$$

So

$$\int_{0}^{\infty} f(x;\lambda)f(\lambda;\theta)d\lambda=\frac{\theta^{4}}{2(\theta+3)x!}\int_{0}^{\infty} e^{-\lambda(\theta+1)}\left(\lambda^{x+2}+\lambda^{x+3}\right)d\lambda.$$

After simplifying it the pdf of PABLD is obtained

$$f(x;\theta)=\left(\frac{\theta}{\theta+1}\right)^{4}\frac{(x+1)(x+2)(\theta+x+4)}{2(\theta+3)(\theta+1)^{x}},\qquad x=0,1,2,\ldots..\quad \theta>0.\quad (2.4)$$

Properties of the poisson-area-biased-lindley distribution

The factorial moments of the PABLD in (2.1)

$$\mu'_{(r)}=\frac{(\theta+r+3)(r+2)!}{2(\theta+3)\theta^{r}}.$$

$$\mu'_{(r)}=\frac{(\theta+r+3)(r+2)!}{2(\theta+3)\theta^{r}}.\quad (2.5)$$

For $r=1,2,3\&4$ in (2.5), the first four factorial moments of the PABLD are

$$\mu'_{(1)}=\frac{3(\theta+4)}{\theta(\theta+3)}\quad \mu'_{(2)}=\frac{12(\theta+5)}{\theta^{2}(\theta+3)}\quad \mu'_{(3)}=\frac{60(\theta+6)}{\theta^{3}(\theta+3)},\quad \mu'_{(4)}=\frac{360(\theta+7)}{\theta^{4}(\theta+3)}\quad (2.6)$$

Since the first four raw moments of the PABLD are

$$\mu'_{1}=\frac{3(\theta+4)}{\theta(\theta+3)},\quad \mu'_{2}=\frac{3\left(\theta^{2}+8\theta+30\right)}{\theta^{2}(\theta+3)}\quad (2.7)$$

$$\mu'_{3}=\frac{3\left(\theta^{3}+16\theta^{2}+80\theta+120\right)}{\theta^{3}(\theta+3)},\quad \mu'_{4}=\frac{3\left(\theta^{4}+32\theta^{3}+260\theta^{2}+840\theta+840\right)}{\theta^{4}(\theta+3)}\quad (2.8)$$

The mean moments of PABLD are

$$\mu_{2}=\sigma^{2}=\frac{3\left(\theta^{3}+8\theta^{2}+30\theta+42\right)}{\theta^{2}(\theta+3)^{2}}.\quad (2.9)$$

$$\mu_{3}=\frac{3\left(\theta^{5}+10\theta^{4}+14\theta^{3}+36\theta^{2}-2160\theta+2664\right)}{\theta^{3}(\theta+3)^{3}}.\quad (2.10)$$

$$\mu_{4}=\frac{3\left(\theta^{7}+20\theta^{6}+2\theta^{5}+61122\theta^{4}-3662760\theta^{3}-5482800\theta^{2}+192240\theta+41688\right)}{\theta^{4}(\theta+3)^{4}}.\quad (2.11)$$

The coefficient of skewness and kurtosis of the PABLD are

$$\gamma_{1}=\sqrt{\beta_{1}}=\frac{\left(\theta^{5}+10\theta^{4}+14\theta^{3}+36\theta^{2}-2160\theta-2664\right)}{\sqrt{3\left(\theta^{3}+8\theta^{2}+30\theta+42\right)^{3}}}.\quad (2.12)$$

$$\beta_{2}=\frac{\left(\theta^{7}+20\theta^{6}+2\theta^{5}+61122\theta^{4}-3662760\theta^{3}-5482800\theta^{2}+192240\theta+41688\right)}{3\left(\theta^{3}+8\theta^{2}+30\theta+42\right)^{2}}.\quad (2.13)$$

For the PABLD, from (2.12) and (2.13) it can be seen that $(\gamma_{1},\beta_{2})\rightarrow(-5.65,7.88)$ as $\theta\rightarrow 0$, the model is negatively skewed and leptokurtic.

Some more properties of the PABLD are

$$\frac{f(x+1;\theta)}{f(x;\theta)}=\frac{(x+3)(\theta+x+5)}{(\theta+1)(x+1)(\theta+x+4)}.\quad (2.14)$$

$$\frac{f(x+1;\theta)}{f(x;\theta)}=\frac{\left(1+\frac{3}{x}\right)\left(\theta+\frac{1}{x}+5\right)}{(\theta+1)\left(1+\frac{1}{x}\right)\left(\theta+\frac{1}{x}+4\right)}.\quad (2.15)$$

The dispersion of the PABLD is defined to be

From equation (2.14) and Table 1, it can be observed that the PABLD is over-dispersed but as $\theta\rightarrow\infty$ then $\mu=\sigma^{2}$ and the

PABLD is equi-dispersed. Therefore for large θ the PABLD is equi-dispersed.

Method of Moments

If $x_1, x_2, \ldots, x_n$ be the random sample from PABLD with pdf (2.4), the method of moments (MOM) estimate $\tilde{\theta}$ of the parameter θ is given by

$$\tilde{\theta}=\frac{-3(\bar{x}-1)+\sqrt{9(\bar{x}-1)^2+48\bar{x}}}{2\bar{x}} \quad (3.1)$$

Theorem 1: The MOM estimator $\tilde{\theta}$ of θ is positively biased.

Proof: Let $\tilde{\theta}=\psi(\bar{x})$, where $\Psi(z)=\frac{-3(z-1)+\sqrt{9(z-1)^2+48z}}{2z}$.

So,

$$\psi''(z)=\frac{78z+69z^2+297z^3+108z^4+\left(108z+405z^2+135z^3\right)\sqrt{9(z-1)^2+48z}}{4z^4\left[9(z-1)^2+48z\right]^{3/2}}>0, \quad (3.2)$$

Then $\Psi(z)$ is strictly convex. By using the Jensen's inequality we have

$$E\left\{\psi\left(\bar{X}\right)\right\}>\psi\left\{E\left(\bar{X}\right)\right\}.$$

Since $\psi\left\{E\left(\bar{X}\right)\right\}=\psi(\mu)=\psi\left(\frac{3(\theta+4)}{\theta(\theta+3)}\right)=\theta$, therefore $E\left(\tilde{\theta}\right)>\theta$.

Theorem 2: The MOM estimator $\tilde{\theta}$ of θ is consistent and asymptotically normal: $\sqrt{n}\left(\tilde{\theta}-\theta\right)\xrightarrow{d}N\left(0,\nu^2(\theta)\right)$.

Where

$$\nu^2(\theta)=\frac{\theta^2(\theta+3)^2\left(\theta^3+8\theta^2+30\theta+42\right)}{3\left(\theta^2+8\theta+12\right)}. \quad (3.3)$$

Proof: -

Consistency: Since $\mu<\infty$, then $\bar{X}\xrightarrow{P}\mu$. And $z=\mu$ is a continuous function at $z=\mu$, then $\psi\left(\bar{X}\right)\xrightarrow{P}\psi(\mu)$, i-e. $\tilde{\theta}\xrightarrow{P}\theta$.

Asymptotic normality: as $\sigma^2<\infty$ then by using the central limit theorem we have

$$\sqrt{n}\left(\bar{X}-\mu\right)\xrightarrow{d}N\left(0,\sigma^2\right).$$

$\psi(\mu)$ is a differentiable function and $\psi'(\mu)\neq 0$, then by using the delta-method we have

$$\sqrt{n}\left(\psi\left(\bar{X}\right)-\psi(\mu)\right)\xrightarrow{d}N\left(0,\left[\psi'(\mu)\right]^2\sigma^2\right).$$

Finally we have $\psi\left(\bar{X}\right)=\tilde{\theta}$, $\psi(\mu)=\theta$ and

$$\psi'(\mu)=\frac{-1-6\mu-6\sqrt{9(\mu-1)^2+48\mu}}{4\mu^2\sqrt{9(\mu-1)^2+48\mu}}=-\frac{\theta^2(\theta+3)^2}{3\left(\theta^2+8\theta+12\right)}. \quad (3.4)$$

The theorem 2 follow the asymptotic $100(1-\alpha)\%$ confidence interval for θ is

$$\tilde{\theta}\pm z\frac{\alpha}{2}\frac{\nu\left(\tilde{\theta}\right)}{\sqrt{n}}. \quad (3.5)$$

Maximum Likelihood Estimation

Let $x_1, x_2, \ldots, x_n$ be the random sample on size n from PABLD with pdf (2.4), the maximum likelihood estimate (MLE) $\hat{\theta}$ of the parameter θ is the solution of the non-linear equation:

$$\frac{4n}{\theta}-\frac{n(4-\bar{x})}{(\theta+1)}-\frac{n}{(\theta+3)}+\sum_{i=1}^{n}\frac{1}{\theta+x_i+4}=0 \quad (4.1)$$

Applications

In this section the PABLD is applied to some biological data sets and compared with PD.

a. Guire, et al. [12] gave data on European corn borers per plant with 0, 1, 2, 3 and 4 and counts 83, 36, 14, 2, and 1.

Form Table 2, it can be seen that the PABLD gives much closer fit than the PD and PLD to the data set of number of bores per plant . Thus PABLD provides a better alternative to PD and PLD for modeling count data sets.

b. Beall [13] gave the distribution of Pyrausta nublilalis in 1937, no of insects 0, 1, 2, 3, 4 and 5 with counts 33, 12, 6, 3, 1 and 1.

Form Table 3, it can be seen that the PABLD gives better fit than the PD to the data set of number of insects. Thus PABLD provides a better alternative to PD for modeling count data sets.

c. Juday [14] and Thomas [15] gave data on macroscopic fresh-water fauna in dredge samples from the bottom of Weber Lake.

Form Table 4 it can be seen that the PABLD gives better fit than PD and PLD to the animal distribution of microcalanus nauplii. Thus PABLD provides a better alternative to PD and PLD for modeling count data sets.

d. Archibald [16] gave data on plant populations. The distribution of representing salicornia stricta.

Form Table 5, it can be seen that the PABLD gives better fit than the PD and PLD. Thus PABLD provides a better alternative to PD and PLD for modeling count data sets.

e. Archibald [16-18] gave data on plant populations. The distribution of representing Plantago maritime.

From Table 6 it is concluded that the PABLD gives better fit than the PD and almost equally good fit as PLD distribution to the distribution of Plantago maritime. Therefore the PABLD is better alternative to PD and PLD to model discrete data sets.

Note: The highlighted expected frequencies from Table 2-6 are the pooled frequencies that are less than 5, so the degrees of freedom are calculated according to them.

From Table 2-7, it is observed that the PABLD gives better fit than PD and PLD to the some biological count data sets. PD is a discrete distribution with parameter λ. Lindley distribution is a continuous life time distribution and PLD is the mixture of Poisson and Lindley distributions with parameter θ. The proposed model named PABLD is obtained by the mixture of the Poisson distribution and the area biased form of the Lindley distribution. The area biased distribution is a type of the weighted distribution with weight $w(x)=x^2$, due to mixture of PD and LD with this weight, the proposed model is showing applications better than PD and PLD to biological data sets. Mostly the applications of the weighted distributions to the data relating biology can be found in Patil & Rao [10].

Table 1: The dispersion of PABLD for different values of θ.

θ	$\mu=\sigma^2-\frac{3\left(\theta^2+18\theta+42\right)}{\theta^2\left(\theta+3\right)^2}$	θ	$\mu=\sigma^2-\frac{3\left(\theta^2+18\theta+42\right)}{\theta^2\left(\theta+3\right)^2}$
0.5	$\sigma^2-50.20408$	19	$\sigma^2-0.012792$
1	$\sigma^2-11.4375$	20	$\sigma^2-0.011371$
2	$\sigma^2-2.46$	21	$\sigma^2-0.010169$
3	$\sigma^2-0.972222$	22	$\sigma^2-0.009144$
4	$\sigma^2-0.497449$	23	$\sigma^2-0.008263$
5	$\sigma^2-0.294375$	24	$\sigma^2-0.007502$
6	$\sigma^2-0.191358$	25	$\sigma^2-0.006839$
7	$\sigma^2-0.132857$	26	$\sigma^2-0.006258$
8	$\sigma^2-0.096849$	27	$\sigma^2-0.005748$
9	$\sigma^2-0.073302$	28	$\sigma^2-0.005296$
10	$\sigma^2-0.05716$	29	$\sigma^2-0.004894$
11	$\sigma^2-0.045665$	30	$\sigma^2-0.004536$
12	$\sigma^2-0.037222$	31	$\sigma^2-0.004215$
13	$\sigma^2-0.030857$	32	$\sigma^2-0.003927$
14	$\sigma^2-0.025952$	50	$\sigma^2-0.00147$
15	$\sigma^2-0.022099$	100	$\sigma^2-0.000335$
16	$\sigma^2-0.019023$	500	$\sigma^2-1.23E-05$
17	$\sigma^2-0.016531$	1000	$\sigma^2-3.04E-06$
18	$\sigma^2-0.014487$	∞	σ^2

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Table 2: Chi-square goodness of fit test for PD, PLD and PABLD to European corn-borer data.

Number of Bores Per Plant X	Observed Frequency (Oi)	Expected Frequency (Ei)		
		Poisson Distribution	Poisson-Lindley Distribution	Poisson- Area-Biased Lindley Distribution
0	83	78.9	87.2	82.4
1	36	42.9	31.8	38.1
2	14	11.7	11.2	11.7
3	2	2.01	3.8	2
4	1	0.4	2	0.67
Total	136	136	136	135.87
Estimation of Parameters		$\hat{\theta} = 0.544118$	$\hat{\theta} = 2.372252$	$\hat{\theta} = 6.119427$
χ^2		1.885	0.757	0.312
d.f		1	1	1
p-value		0.1698	0.3843	0.576455

Table 3: Chi-square goodness of fit test for PD, PLD and PABLD to distribution of Pyrausta nublilalis in 1937.

Number of Insects x	Observed Frequency (Oi)	Expected Frequency (E_i)		
		Poisson Distribution	Poisson Lindley Distribution	Poisson Area-Biased Lindley Distribution
0	33	26.45	31.48	33.18
1	12	19.84	14.16	15.98
2	6	7.44	6.09	5.09
3	3	1.86	2.5	1.34
4	1	0.35	1.04	0.32
5	1	0.05	0.42	0.07
Total	56	55.99	55.73	55.98
Estimation of Parameters		$\tilde{\theta}$=1.808	$\tilde{\theta}$=1.808	$\tilde{\theta}$=5.859
χ^2		4.89	0.484	3.56
d.f		1	1	1
p-value		0.026977	0.00001	0.059131

Table 4: Chi-square goodness of fit test for PD, PLD and PABLD to animal distribution of microcalanus nauplii.

Individuals Per Unit	Microcalanus			
	Observed Frequency (Oi)	Expected Frequency (Ei)		
		Poisson Distribution	Poisson Lindley Distribution	Poisson Area-Biased Lindley Distribution
0	0	0.01	7.156	1.294
1	2	0.098	8.743	3.402
2	4	0.468	9.632	5.76
3	3	1.498	10.009	7.928
4	5	3.595	10.014	9.643
5	8	6.903	9.757	10.791
6	16	11.045	9.324	11.37
7	13	15.147	8.777	11.446
8	12	18.177	8.164	11.116
9	13	19.388	7.521	10.487
10	15	18.613	6.873	9.66
11	15	16.244	6.239	8.721
12	9	12.995	5.631	7.739
13	9	9.596	5.057	6.767
14	7	6.58	4.522	5.842
15	4	4.211	4.028	4.986
16	4	2.527	3.575	4.213
17	6	1.427	3.164	3.528
18	2	0.761	2.793	2.931
19	0	0.385	2.459	2.417
20	2	0.185	2.16	1.981
21	1	0.084	1.894	1.613
22	0	0.037	1.658	1.306
Total	150	149.97	149.7	150
Estimation of Parameters		$\tilde{\theta}$=9.6	$\tilde{\theta}$=0.192	$\tilde{\theta}$=0.404296
χ^2		30.39206	62.992	20.02153
d.f		10	13	12
p-value		0.000739	0.00001	0.06669

Table 5: Chi-square goodness of fit test for PD, PLD and PABLD to distribution of quadrant, representing salicornia stricta.

Plants Per Quadrant	Salicornia			
	Observed Frequency	Expected Frequency (Ei)		
	(Oi)	Poisson Distribution	Poisson Lindley Distribution	Poisson Area-Biased Lindley Distribution
0	4	0.127	7.874	2.277
1	3	0.843	8.939	5.267
2	8	2.804	9.199	7.861
3	13	6.216	8.947	9.553
4	11	10.333	8.389	10.265
5	9	13.743	7.665	10.156
6	8	15.232	6.871	9.465
7	10	14.471	6.069	8.43
8	3	12.029	5.299	7.245
9	3	8.888	4.582	6.05
10	8	5.91	3.931	4.934
11	3	3.573	3.35	3.943
12	4	1.98	2.839	3.099
13	4	1.013	2.394	2.399
14	0	0.481	2.01	1.834
15	3	0.213	1.681	1.387
16	0	0.089	1.402	1.038
17	0	0.035	1.165	0.77
18	1	0.013	0.966	0.566
19	0	0.004	0.799	0.414
20	3	0.001	0.659	0.3
Total	98	97.99	98	97.25275
Estimation of Parameters		$\tilde{\theta}$=0.269	$\tilde{\theta}$=0.269	$\tilde{\theta}$=0.577238
χ^2		65.55225	13.01986	7.381047
d.f		7	8	8
p-value		0.00001	0.111198	0.496138

Table 6: Chi-square goodness of fit test for PD, PLD and PABLD to distribution of quadrant, representing Plantago maritima.

Plants per Quadrant	Plantago			
	Observed Frequency	Expected Frequency (Ei)		
		Poisson Distribution	Poisson Lindley Distribution	Poisson Area-Biased Lindley Distribution
0	12	0.6409	11.471	4.273
1	8	3.2367	12.166	8.868
2	9	8.1727	11.749	11.897
3	13	13.7574	10.746	13.009
4	6	17.3687	9.484	12.59
5	8	17.5424	8.163	11.223
6	11	14.7648	6.895	9.428
7	7	10.652	5.741	7.571
8	8	6.7239	4.725	5.868
9	7	3.7729	3.853	4.42
10	3	1.9053	3.117	3.251
11	4	0.8747	2.505	2.344
12	1	0.3681	2.002	1.662
13	1	0.143	1.592	1.161
14	0	0.0516	1.261	0.801
15	0	0.0174	0.995	0.547
16	1	0.0055	0.782	0.369
17	0	0.0016	0.613	0.247
18	0	0.0005	0.48	0.164
19	1	0.0001	0.374	0.108
20	0	0.00003	0.291	0.071
Total	100	99.999	99.89	99.8709
Estimation of Parameters		$\tilde{\theta}$=5.05	$\tilde{\theta}$=0.345	$\tilde{\theta}$=0.752375
χ^2		55.48343	7.084	10.2781
d.f		6	7	7
p-value		0.00001	0.420187	0.173359

Table 7: The asymptotic 95% confidence intervals (C.I) for θ of PABLD.

Table	Data Sets	95 % C. I
II	Number of bores per plant	(5.989827, 6.249026)
III	Number of insects	(5.562813, 6.155574)
IV	Microcalanus	(0.39898, 0.40902)
V	Salicornia	(0.568854, 0.591146)
VI	Plantago	(0.738042, 0.766708)

Interval Estimation: By using equation (3.5) the parameter θ of PABLD is estimated by the interval estimation for the Biological data sets. The estimated interval for θ of PABLD by the interval estimation is closer to the estimated value by MOM.

Conclusion

The Poisson area-biased Lindley distribution (PABLD) is discrete distribution that is obtained by mixture of the Poisson distribution and area-biased Lindley distribution. Some important properties of the PABLD are derived. From Figure 1 it can be seen that the PABLD is positively skewed moreover it can be seen that as $\theta \to 0$, $(\gamma_1, \beta_2) \to (-5.65, 7.88)$ and the PABLD is negatively skewed and leptokurtic. Furthermore it is found that the PABLD is over-dispersed but as $\theta \to \infty$ the PABLD is equi-dispersed. The parameter of the PABLD is estimated by the method of moments (MOM) and it is proved that the $\tilde{\theta}$ of θ is positively biased, consistent and asymptotically normal. In section 4, the proposed model PABLD is applied to some biological data sets and compared with PD and PLD. It is observed that the PABLD gives better approach to the given data sets. Therefore it is concluded that PABLD is a better alternative to PD and PLD and it has useful applications in real life biological data sets. The asymptotic 95% confidence interval (C.I) for θ of PABLD is also found on these data sets and it is observed that the estimated interval for θ of PABLD by the interval estimation is closer to the estimated value obtained by MOM.

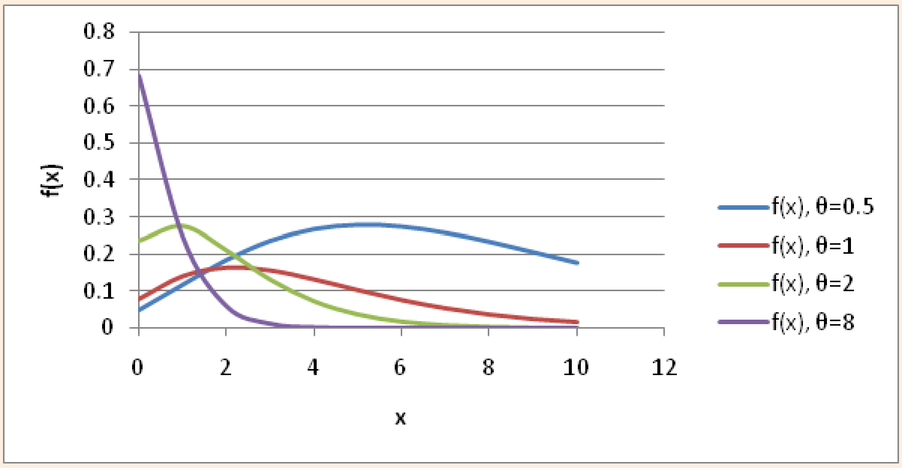

Figure 1: Plots of the pdf of PABLD for $\theta=0.5$, $\theta=1$, $\theta=2$ & $\theta=8$.

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Conflict of Interest

None.

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