

Origins of general relativity in Newton's laws of motion

Abstract

It is not common knowledge that the origins of General Relativity are embedded in Newton's laws of motion. The key concept is weightlessness: a property unique to inertial frames. We discuss common phenomena where effects of inertial and non-inertial frames can be distinguished. An apparatus is described that demonstrates the difference between inertial and non-inertial frames. Classical experiments of Galileo are analyzed and re-interpreted in a way that exposes the role of accelerating frames in his experiments. We cite experimental and theoretical evidence which distinguishes between gravity and acceleration. We emphasize the connection between gravity and acceleration, to the geometry of space time. The link between weightlessness and force is emphasized. An interpretation of force is presented that is consistent both in Newton's laws as well as General Relativity. We demonstrate the validity of gravitation, not as a force but as an outcome of geometry.

Keywords: space time, Newton's laws, gravity, spacecraft, acceleration

Volume 10 Issue 3 - 2026

Munawar Karim

 Department of Biology, Chemistry and Physics, Namibia
 University of Science and Technology, Windhoek, Namibia

Correspondence: Munawar Karim, Department of Biology, Chemistry and Physics, Faculty of Natural and Applied Sciences, Namibia University of Science and Technology, Windhoek, Namibia, Email mkarim@icloud.com

Received: August 18, 2026 | **Published:** August 31, 2026

Introduction

The contents of this paper address fundamental concepts of motion and inertia, concepts that students encounter in their first course in physics. Even at this early stage we show how these profound concepts can lead, in a natural way, to interpretations of gravity in terms of geometry. Understanding the contents will lead to an appreciation of the limitations of Newton's view of gravity not just as an approximation of the Einsteinian view. Results of experiments described in this paper show unequivocally that what we know as gravity is not a force but a manifestation of curvature in space time.

This paper is divided in two parts. In part I we examine familiar notions of Newton's laws as they pertain to reference frames. We uncover, with help from experiments on Earth, the subtle and all pervasive effect of non-inertial frames. We demonstrate results of common experiments which validate our conclusions. After a re-interpretation of Newton's equation, we show that the equation applies to all forces except that of gravity.

In part II we extend the discussion to a geometric origin of gravity that forms the basis of Einstein's General Theory of Relativity.

Inertial frames

A student's first exposure to physics is usually Newton's equation

$$F = \frac{dp}{dt} \approx ma \quad (1)$$

The equation says that a forced mass accelerates. If acceleration is determined by measuring, at regular intervals, the displacement of an object of mass m , there is an implicit requirement that measurements be made from an inertial, or non-accelerating reference frame, because the laws of physics retain their form only in inertial frames - stressing the importance of choice of coordinates. If the measuring platform itself is accelerating, then an object can appear to be accelerating even when it is in uniform motion. Think of an apple released from an accelerating rocket far from an gravitational source.

It is the net acceleration which appears in Newton's equation. In order to make this explicit, Newton's equation should be written with a qualifier:

$$F = ma_{net} \quad (2)$$

where a_{net} is the acceleration of the object, obtained after subtracting out (vectorially) any acceleration of the reference frame. There are some interesting consequences which we describe below.

Examples

Vertical motion

A simple spring-mass arrangement can be used to measure acceleration. In a model apparatus the acceleration is proportional to the extension of the spring. The acceleration is:

$$a = -\frac{kx}{m} \quad (3)$$

antiparallel to x . The static length of the spring is L . There are two ways to confirm that the spring length is extended to $L + x$. Align the apparatus in different directions. The length will change with direction; acquiring a maximum length of $L + x$ along some and just L along others. Alternatively, measure the spring length with different masses attached. If the length changes with m the frame is accelerating; if it does not, it is inertial.

Some examples: The acceleration is zero on the horizontal plane of Earth because $x = 0$ [Figure 1a]. However, $x \neq 0$ when the accelerometer is held stationary in the vertical position [Figure 1b] but $x = 0$ when the accelerometer is falling vertically [Figure 1] (see also the video¹). One can interpret the $x \neq 0$ case [Figure 1c] by appealing to the gravitational force of Earth acting on m (its weight). But then there is an inconsistency, because the same force of gravity is also acting on the freely falling mass in [Figure 1c], so why then is $x = 0$?

Another example: If the apparatus is used on a frame that is rotating with a uniform velocity, it will stretch along the radial direction but not along either of the tangential directions.

Conclusion: the frame is under acceleration opposite the radial direction, inertial along the tangents (Figure 1).

The first example shows that Figure 1b represents a frame accelerating vertically upwards because of the normal reaction from

the stationary surface. Thus a reference frame located on Earth's surface (from which most measurements are made) is a non-inertial frame in the vertical direction (even while ignoring Earth's rotation). Consequently its (upward) acceleration must be subtracted from all measurements in the vertical direction. A common example is the latitude-dependence of the apparent weight of an object. For an object on the equator, the vertical component of the centripetal acceleration is anti-parallel to the (locally vertical) acceleration of the ground [orientation of vectors for other locations are shown in Figure 2]. Thus subtracting the centripetal acceleration gives the true "weight" of the object due to ground acceleration alone.

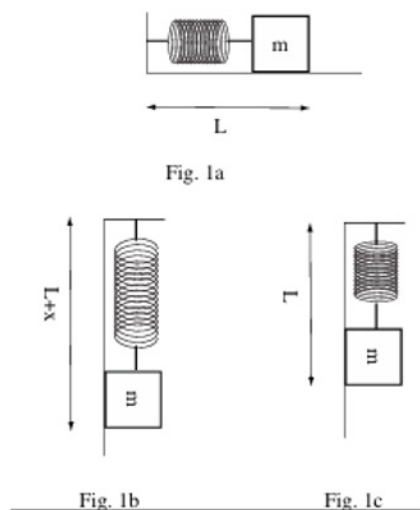


Figure 1 An ideal spring-mass accelerometer: (a) oriented horizontally (b) held vertically (c) in free fall. [Video 1](#).

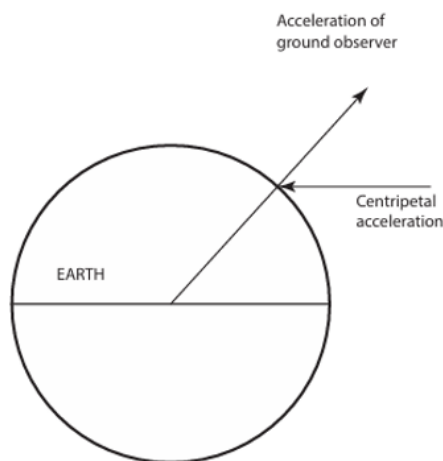


Figure 2 Acceleration vectors acting on an object on surface of rotating earth "weight" of object is the consequence of a non-inertial frame due both to ground acceleration and centripetal acceleration.

Parenthetically it may be noted that 150 years ago in Yonkers, NY, the famous elevator maker Elisha Otis invented a safety device that worked because of the difference between Figure 1b and Figure 1c. Were the elevator cable to snap, the spring would retract because the elevator would be in free fall; triggering a brake to prevent the cab from crashing to the ground. Otis demonstrated the system "by climbing aboard an elevator and severing the rope". This simple application was instrumental in promoting the use of elevators in tall buildings and subsequently, the dawning of the age of skyscrapers.

An example from our time is the protective mechanism in IBM ThinkPad laptop computers. A falling computer is detected by a solid-state accelerometer which promptly locks up the reading head, preventing its crash on the disk drive.

Some other interesting consequences: An object is dropped from rest and allowed to fall freely: its instantaneous displacement may be measured with respect to a reference frame on Earth. The displacements increase quadratically with time from which, taking second differences, acceleration $a = g$ may be inferred. But this reference frame, as shown above, itself is accelerating upwards @g. Using Newton's equation [Eq. (2)]:

$$F = ma_{net} = m(a - g) = 0 \quad (4)$$

implying that $a_{net} = 0$ [Figure 1c confirms this], or equivalently, it is in free fall ($F = 0$).

If the object were released from an inertial frame with zero initial momentum, its momentum and therefore its velocity remains zero. In other words the object should remain stationary in the inertial frame. Its apparent downwards acceleration is only an illusion caused by the upwardly accelerating observer. The object is in free fall because it is following a geodesic, and therefore weight-less [Figure 1c]. The observer by contrast is not following a geodesic because he is constrained by elastic forces to a stationary position on Earth's surface: he is forced to follow the geodesic motion of Earth's center. It is this force that accelerates the observer. If the falling object were observed by an observer at Earth's center, however, the observed acceleration would vanish, because in this case both observer and object are following geodesics. This experiment is easier than it appears. One need only to measure the acceleration of an object dropped from a fixed position, from locations at different altitudes. A graph of measured accelerations vs. distance from Earth's center would be a straight line passing through the origin - implying that the measured acceleration is zero for an observer at Earth's center.

An added proviso: the action of *any* force can compromise free fall e.g. a sky diver is not in free fall because of the force of air resistance. A penny and feather experiment by contrast is an accurate demonstration of free fall. An accelerating reference frame has other interesting consequences as measured in the Pound-Rebka experiment.

An orbiting spacecraft is also in free fall. Weightlessness is vividly demonstrated by observing the motion of astronauts in spacecraft. Although satellites in orbit execute circular motion about Earth's center, local frames do not measure any acceleration of the center of mass (centripetal acceleration is absent). Their trajectories are determined by the gravity field of Earth. A common explanation for weightlessness is to appeal to the centripetal acceleration to balance the force of gravity; if that were a valid explanation, it would apply equally to a satellite falling radially. There would still be a force of gravity, but because of radial motion, there is no centripetal acceleration to balance the force of gravity, which should cause the object to accelerate; but the satellite is in free fall towards Earth's center, and thus not accelerating. Yet another claim: calculate the acceleration of an object in circular orbit by measuring, from the center of the circle, its velocity difference at two instants and evaluating: $\lim_{\delta\tau \rightarrow 0} [V(\tau + \delta\tau) - V(\tau)] / \delta\tau$. Although this approach yields an acceleration, this is nothing but the acceleration of the rotating reference frame. The object itself is unaccelerated. So the purported explanation is invalid. The appearance of centripetal acceleration is a consequence of the rotating - hence accelerating - reference frame used to measure the displacement of the space-craft.

These common examples show, with a re-interpretation of acceleration, the intimate connection between weightlessness and inertial frames.

The concept of inertial frame is a common source of confusion in textbooks. From these examples it is evident that the center of mass, and only the center of mass of freely falling objects defines an inertial frame. Parts of the falling object that are away from the center of mass, are constrained by elastic forces that compel them to follow the geodesic of the center of mass. Such forces are identified as tidal and their origin is elastic.

However, not all circular motion is force-free i.e., in free fall. For example an object whirling at the end of a string. The string in this example, is under tension, and the object is under centripetal acceleration, as an accelerometer will confirm. Orbiting electrons in the field of nuclei or in magnetic fields are not in free fall either, nor are nucleons within nuclei. Their trajectories are governed by the Coulomb or Lorentz or the strong force, respectively, and they are accelerated; local frames in these cases do measure centripetal acceleration. Even if the motion is along a straight line e.g. two electric charges accelerate when they move towards or away from each other. However, neither electrons nor nucleons, accelerate when they move in a field of gravity.

It is sometimes claimed that, since electrons accelerate when falling in the field of gravity, they must radiate.²⁻⁴ We can now assert that they do not. A corollary is that a static free electron accelerates while on Earth's surface. The power radiated is:

$$P = \mu_0 \frac{g^2 e^2}{12\mu c} = \mu_0 \left(\frac{GM}{R^2} \right) \frac{e^2}{12\mu c} = 3 \times 10^{-52} J/s \quad (5)$$

Practically undetectable. What is the source of this energy? It is the energy stored in the form of stress of the earth due to its own gravity field.

In order to conserve the number of electrons it may be expected that free electrons will eventually escape too.

The gravitation field is unique; force-free trajectories occur in gravity alone.

Force

Newtonian gravity is a force; Einstein's gravity is not. So is gravity a force or not? Both cannot be true. This apparent conundrum can be resolved. It is a testable proposition. There is a clear test that identifies the presence or absence of force.

We can ask what *is* force? Newton's equation unequivocally identifies force as the agent of acceleration or a change in momentum; conversely, an object accelerates only under the action of force. We are referring again to net acceleration not the acceleration of reference frames.

Masses accelerate under the action of all known forces - electro-weak and strong. Gravity, by contrast, does not accelerate matter. By definition then, gravity does not appear to be a force.

How are we to interpret the force in Newton's other equation?

$$F = -G \frac{m_1 m_2}{r^2} \hat{r} \quad (6)$$

We can ask, do masses accelerate under this force? We have seen that masses fall freely in the field of gravity, they do not accelerate (more precisely, the masses follow extremum paths - geodesics). No

acceleration is measured by a local observer who is stationary with and falling freely with the mass. The velocity four-vector is purely time-like for both observer and test mass. The local, or proper acceleration is zero. The negative sign in Eq. (6) tells us that the geodesics of the two masses m_1 and m_2 converge. However, a purely time-like four-vector describes the worldline for both observers, so neither is accelerating. Under certain conditions the observed acceleration maybe the acceleration of the observer [see section II for details]. Consistent with the relation of acceleration and force, gravity does not qualify as a force.

The interpretation so far is also consistent with General Relativity. This brief discussion provides a compelling interpretation that gravity is not a force in Newtonian physics, not in General Relativity.

What then is the force in Eq. (6)? We know that a force is needed to compel a mass to follow a path that is not a geodesic. We also know that this force is proportional to m_1 and m_2 . We can identify the force in Eq. (6) as an external force required to force a mass away from its geodesic. Indeed the mass, either m_1 or m_2 accelerates when this force is applied.

The force in Eq. (6) is an external force that is required to compel a freely falling mass from following a geodesic, causing it to accelerate as expected. Arriving at this interpretation brings consistency to our interpretation of force to include the force in Eq. (7).

Some experiments⁵ have used the relation

$$F = m_a a = -G \frac{m M}{r^2} \hat{r} \quad (7)$$

to distinguish and measure the equality of inertial mass (m_a) with gravitational mass (m_g). An argument is made that while m_a is a measure of inertia, m_g is the mass that is "attracted" to the mass M - and since these appear to be two separate entities, there is no reason for them to be equal. Experiments were seen to be needed, and conducted, which established their equality.

When $\mathbf{a} = \mathbf{g}$, $\mathbf{F}$ is the weight of m_a . If m_a is not supported by $\mathbf{F}$ it would fall freely along a geodesic. $\mathbf{F}$ is therefore a force that prevents m_a from following a geodesic - an interpretation consistent with the previous conclusion about the nature of $\mathbf{F}$ in Eq. (6)

From the discussion so far it is not surprising that the results show them to be equal. If a force is applied to mass m_g it accelerates @ $\mathbf{a}$. There is no distinction between the two masses. m_g is also the inertial mass. There is no gravitational mass.

Cavendish's experiment in this light, appears all the more elegant. Test masses visibly, and exquisitely, alter their geodesics with a change in location of the large masses. The quantitative response yields the gravitational coupling constant. An experiment with spacetime before spacetime!

Compound motion

Next we examine the familiar motion of a projectile launched with an initial velocity v_0 at an angle θ with respect to the horizontal plane, often the first example in texts of motion in two dimensions. Conventional analysis tells us that, in the absence of friction, the trajectory is a parabola as observed from a reference frame on Earth. Using what we have established we can tell that the horizontal component of the velocity is unchanged because the reference frame is inertial in the horizontal plane (as in conventional analysis), however, contrary to conventional analysis, the vertical component is also constant because the projectile is in free fall. Thus the

trajectory should be a straight line as seen from an inertial frame. The apparent parabolic trajectory is merely an artifice of the non-inertial, stationary reference frame on Earth. (The parabolic trajectory is not to be confused with orbital motion, which for a typical experiment is a circle with a 6000km radius - practically a straight line for Earth-bound observers.) This amazing conclusion, so contrary to experience, is hard to imagine and must be seen to be believed. How does one see this?

We have designed a device to demonstrate this. A description follows: We used a 3ft. by 3ft. perforated board, painted black (for contrast), on it we marked off a 1ft. by 1ft. grid using white masking tape. Using a 2" diameter, 4" long mailing tube, we made a toy cannon. The cannon ball is a loose fitting white (for contrast against a black background) Styrofoam sphere. The ball is inserted in the mailing tube. There is a spring in the barrel, which is compressed by the ball. A wire across the ball keeps it in the barrel.

A trigger mechanism releases the wire and the ball is propelled out of the barrel. The cannon is attached to one of the lower corners of the perforated board, at an angle $\theta = 45^\circ$, pointing at the opposite corner (Figure 3).

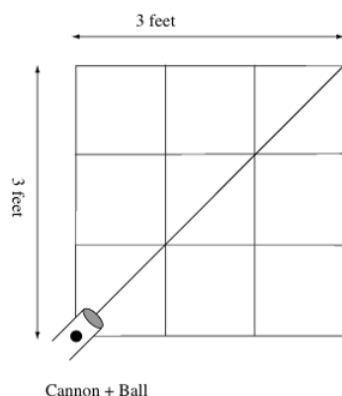


Figure 3 Toy cannon mounted on earth perforated board with 1 ft x 1 ft grid.

The device works in two modes: (i) the ball is launched and the usual parabolic trajectory is observed and recorded on a video camera [Figure 2]; (ii) the cannon is cocked; while keeping it vertical, the apparatus is lifted up to the ceiling (about 15 ft.). The apparatus is dropped and simultaneously a lanyard pulls the trigger to release the ball. The ball and grid are in free fall. Now it is possible to observe and record the trajectory against the backdrop of a freely falling frame (the grid). All this occurs in a second. It is astonishing to see, in the blink of an eye, that the trajectory is not parabolic; it is a straight line diagonally across the grid! A video recording is helpful for frame by frame viewing by students [Figure 3]. The straight line trajectory shows convincingly, that momentum is indeed conserved in force-free fields. A time-lapse video of the trajectory under two condition⁶ and [Figure 4] may be seen in these two videos.

There is no evidence of the "force of gravity" decelerating the ball.

A game of golf, basketball or baseball, looks a whole lot different when balls are moving in straight lines while Earth rises up to meet the balls (Figure 4) (Figure 5).

Pendulum accelerometer

In this example we show a pendulum on an inclined plane. The angle of the pendulum string is a measure of acceleration. Figure 6a may be interpreted, conventionally, as the effect of gravity on the

pendulum bob, resulting in an acceleration along the inclined plane of $a = g \sin \theta$.⁷ In our experimental set-up the pendulum assembly is mounted on an air-track. When the assembly is released and while it slides down the track, the pendulum reverts, after a few oscillations, to the orientation in Figure 6 ($\theta = 0$). Although the pendulum, like any object sliding down an inclined plane and observed by an observer on Earth, would appear to be accelerating, the angle of the pendulum string shows that $a = 0$, or that it is moving at a uniform velocity. Once again confirming that the perceived acceleration is the result of observations from a non-inertial frame (observer on ground). If the acceleration of the observer is removed, the system is seen to be moving with a constant momentum and therefore in a force-free environment (Figure 6).

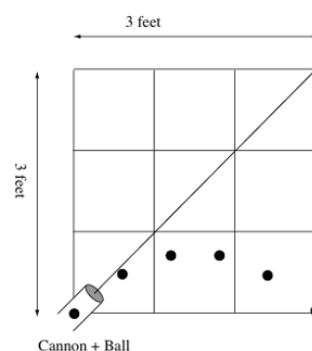


Figure 4 Trajectory in a non-inertial frame Video 2.

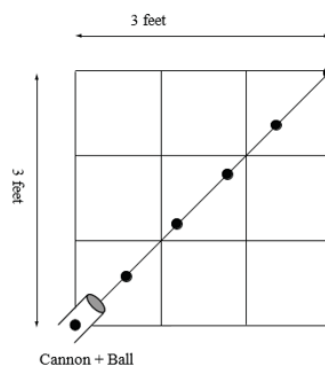


Figure 5 Trajectory in an inertial frame.

Object sliding down inclined plane (Galileo)

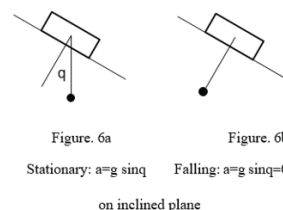


Figure 6 Pendulum accelerometer; (a) Stationary on inclined plane, (b) Sliding down inclined plane.

Galileo's experiments

What we have learned so far can be used to analyze Galileo's experiments with falling objects. Galileo slowed the fall of objects by using inclined planes, which had the effect of reducing the acceleration to $g \sin \theta$. Although the longer drop times facilitated measurements, what he was measuring was not the acceleration of bodies, of different materials, sliding down inclined planes but the constant acceleration of his laboratory. It was inevitable that all his measurements gave

the same acceleration because he was measuring nothing but the acceleration of his laboratory.

Imagine several objects at rest on a road. An observer on an accelerating car, approaches the objects. Analysis of the instantaneous positions of the objects, as measured by the observer on the car, will be consistent with acceleration.

Are the objects accelerating or the observer? An accelerometer mounted on the car will show that it is accelerating, whereas an accelerometer attached to the objects will show no acceleration in the horizontal plane. Thus the deduced acceleration is that of the reference frame and not the objects.

That Galileo's measured accelerations were identical was not accidental; the measurements merely proved that the laboratory was accelerating rather than the equality of the acceleration of bodies of different compositions.

Modern experiments that use freely falling objects to seek a composition-dependent force,^{8,9} or to confirm that subnuclear particles obey Newtonian gravity^{11,12} suffer from the Galilean defect. They are merely measuring the acceleration of the laboratory.

By contrast the experiments of Eotvos^{13,14} and its modern equivalents, which balance laboratory acceleration against centripetal acceleration (Dicke, Braginsky etc.,¹⁵ do show that acceleration is independent of material composition.

How should we interpret common sensations such as weight? Each pointmass in an object must follow its individual geodesic. If elastic forces tie point masses to the object, whether this be an object on Earth's surface, or any common object, these forces prevent the point masses from following natural geodesics, instead they are "forced" to follow the geodesic of the center of mass. An object on Earth's surface feels tied down by the force of gravity (sensed as weight), however this is nothing but the elastic force keeping the object from falling to Earth's center. At Earth's center the object loses its weight.

It is important, and as these examples show, that Newton's equation be interpreted with reference to inertial frames. As is evident from these demonstrations, objects in free fall are indeed weightless, that the "force of gravity" does not govern their motion. This observation leads quite naturally to the concept of gravity as a consequence of geometry.

Falling objects revisited

It is illustrative to examine the worldline of an object in free fall. For a ball released from rest, there is the observer and the ball Figure 7 (Earth's rotation is again ignored). The ball is in free fall so it traces a purely time-like geodesic (the vertical axis B, which is the time axis), while the world line (C) of the observer is not a geodesic because C is accelerating. The relative speed between the ball and observer is initially zero; their world lines touch at O - the moment the ball is released. After the ball is released the observer's velocity increases, shown as the increasing slope of the world line (C). Following Synge¹⁶ the trajectory of the ball can be calculated. Briefly the steps consist of choosing a coordinate frame on the world line of the cannon, then locating, with light signals, the ball in this frame [Figure 7]. The first curvature at the equator of the worldline of the terrestrial observer, is the acceleration:

$$g = 3.263 \times 10^{-8} s^{-1} = 978.05 \text{ cm/s}^2 \quad (8)$$

diagram in two dimensions (Figure 7).

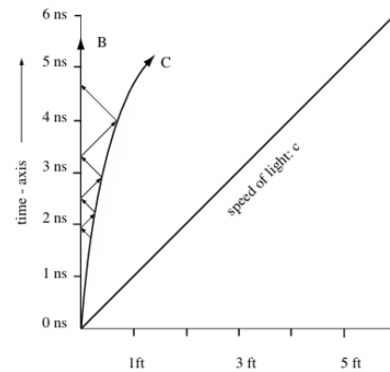


Figure 7 Worldline diagram in two dimensions speed of light is 45° on this scale. Acceleration is exaggerated. Arrows show bouncing light pulses between observer C and falling object B; these are recorded on video. Worldline of C follows the equation $z = 1/2 g \tau^2$.

The non-linearity of the observer's worldline shows up as the apparent acceleration of the ball. The ball is observed from an accelerating frame, the catenary worldline is a consequence. Seen by an inertial observer (following a geodesic), it is stationary. For a semi-technical but readable treatment see Wheeler.¹⁷

The equation of motion of the ball can be derived.¹⁸ There are two frames: the ground which is accelerating and the ball which is not. Let C be the observer on the ground and B the observer on the ball. Observer C measures an acceleration $a(\tau)$ at intervals of its proper time τ . The measured acceleration can be used to plot the world line of C in the inertial frame of B. During a small time interval the velocity u of C will change by velocity of C is $V(\tau)$; after an interval

$$V(\tau + \delta\tau) = \frac{V + \delta u}{1 + \frac{V\delta u}{c^2}} \quad (9)$$

using the relativistic law of addition of velocities. To lowest order in

$$V(\tau + \delta\tau) - V(\tau) = \delta u \left(1 - \frac{V^2}{c^2} \right) \quad (10)$$

or

$$\frac{dV}{d\tau} = \left(1 - \frac{V^2}{c^2} \right) a(\tau) \quad (11)$$

The solution to this equation is:

$$V(\tau) = c \tanh \phi(\tau) \quad (12)$$

where

$$c\phi(\tau) = \int_0^\tau a(\tau) d\tau \quad (13)$$

Initially $V(\tau)|_{\tau=0} = 0$. We now have the direction of the worldline of C in terms of the proper time τ ("intrinsic equation" of the curve). We can relate t and τ , the time intervals on B and C respectively, through invariant

$$d\tau^2 = dt^2 - \frac{dz^2}{c^2} = d\tau^2 - \frac{V^2 dt^2}{c^2} \quad (14)$$

$$\frac{dt}{d\tau} = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} = \cosh \phi(\tau) \quad (15)$$

and the displacement:

$$\frac{dz}{d\tau} = \frac{dz}{dt} \frac{dt}{d\tau} = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} = c \sinh \phi(\tau) \quad (16)$$

In our case the acceleration is constant ($a = g$). Integrating Eq. (15) gives

$$t = t_0 + \frac{c}{g} \sinh \frac{g\tau}{c} \quad (17)$$

and integration of Eq. (16) gives

$$z = z_0 + \frac{c^2}{g} (\cosh \frac{g\tau}{c} - 1) \quad (18)$$

the parametric representations $z(\tau)$ and $t(\tau)$, which trace out the worldline of C. From these relations it is evident that an observer on C can calculate its location on a worldline map at any instant τ without reference to the outside world - confirming that acceleration, unlike velocity, is absolute. For an elaborate discussion of the absolute nature of acceleration and its relation to Mach's principle see Ohanian.¹⁹

The worldline of C, as seen by an inertial observer, is a catenary. For time intervals the parametric $\tau \ll c/g \approx 10^7$ secs. (generally true for observations on Earth), one can approximate

$$\cosh \frac{g\tau}{c} \approx 1 + \left(\frac{g\tau}{c} \right)^2 \quad (19)$$

Substitution into Eq. (18) gives

$$z = z_0 + \frac{1}{2} g \tau^2 \quad (20)$$

Acceleration g is just the second derivative. The displacement increases quadratically with time, an expression that is familiar to any student. Thus the worldline of the (accelerated) observer is an approximate parabola, while that of the (stationary) ball is purely time-like.

Interpretation of gravity in terms of general relativity

How does one relate acceleration to gravity? We have seen that acceleration is the outcome of a force whereas there is no force of gravity. There is something called the "strong equivalence principle" which claims that in all freely falling, non-rotating laboratories the result of any local experiments are the same, independent of the gravitational field surrounding the laboratory.²⁰ In other words gravity is undetectable in local frames. This claim is disputed; for a thorough discussion see the book by Synge²¹ and the paper by Ohanian;²² for an explicit calculation using the Schwarzschild metric see Moreau, Neutze and Ross.²³ In a clear demonstration they compare line elements for accelerated frames and displaced Schwarzschild frames. A line element for accelerated frames is, in Cartesian coordinates²³

$$-c^2 d\tau^2 = -(1 - 2m/R + 2mz/R^2) c^2 dt^2 + dx^2 + dy^2 + (1 - 2m/R + 2mz/R^2) \quad (21)$$

Whereas the same line element in a Schwarzschild frame - in a gravitational field, is

$$-c^2 d\tau^2 = -(1 - 2m/R + 2mz/R^2) c^2 dt^2 + dx^2 + dy^2 + \left(1 - \frac{2m}{R} + \frac{2mz}{R^2} \right)^{-1} dz^2 \quad (22)$$

$$+ \frac{4m}{R} \left(1 - \frac{2m}{R} + \frac{2mz}{R^2} \right)^{-1} \left(\frac{x}{R} dx dz + \frac{y}{R} dy dz \right) \quad (23)$$

In these expressions it is *not* assumed that $m/R \ll 1$. The structure of the two line elements is radically different; note the off-diagonal terms in Eq. (22). One represents flat space while the other a curved geometry. The Riemann tensor is zero for the flat space line element (Eq. (21)) because it is just a coordinate transformation of a Minkowski line element. It is not zero for the curved geometry. There is a Riemann tensor because of the off-diagonal elements. No coordinate transformation can remove the Riemann tensor, in other words one cannot flatten curved space, not only not at a point but not even in a small region about the origin where $|x/R|$, $|y/R|$, and $|z/R| \ll 1$. This discussion demonstrates the intrinsic difference between flat and curved geometries.

The two metrics can be distinguished by comparing clock rates. Following Moreau, Neutze and Ross²³ clocks located at the origin $x_1 = y_1 = z_1 = 0$ and at $x_2 = y_2 = 0$, $z_2 > 0$ are compared. The accelerated clock runs at

$$d\tau_1^a = \left(1 - \frac{mz}{R^2} \right) d\tau_2 \quad (24)$$

At $z_2 = R$ the accelerated clock runs at

$$d\tau_1^a = \left(1 - \frac{m}{R} \right) d\tau_2$$

The clock in the Schwarzschild metric runs at

$$d\tau_1^s = \left(1 - \left(\frac{m}{R} - \frac{m}{R+z_2} \right) \right) d\tau_2 \quad (25)$$

Two different clock rates distinguishing two radically different geometries. Only when $z_2 \rightarrow \infty$ in the Schwarzschild metric do the two clocks (both now in flat space) run at the same rate.

Experimental refutation is provided by subjecting clocks (gamma-emitting nuclei) to accelerations of as high as $10^6 m/s^2$ or decaying muons to $10^{19} m/s^2$. Apart from special relativistic time dilation (transverse Doppler effect), no effect associated with general relativity is detected (for example red-shift),²⁴⁻²⁸ thus confirming that even huge accelerations in flat space cannot be mistaken for nor are they substitutes for a gravity field.

If acceleration were a substitute for gravity one may expect light, emitted from an accelerating source, to be red-shifted. Quantitatively, if a light source (gamma-emitting nuclei) and detector were mounted on the rim of a disk, separated by an angle α , of radius r rotating at ω radians/sec [??], we can calculate the expected red-shift (Figure 8).²⁹

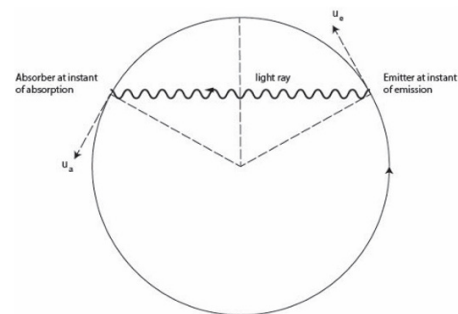


Figure 8 Light emission and absorption in accelerating frame. Source and absorber are mounted on centrifuge. Instantaneous velocity vectors are labeled as u_e and u_a . Arrow shows path of light ray.

Four velocities u_e and u_a represent the emitter and absorber at the instant of emission and absorption, p - the four-momentum of the light ray. E_e and E_a are the energies measured by emitter and absorber, respectively

In the frame of the emitter $u_e = (1, 0, 0, 0)$. Since space-time is *flat* we use the flat space metric with signature $(-1, +1, +1, +1)$. Thus $E_e = -p_0 = p^0$. Similarly $E_a = -p_\alpha u_a^\alpha$. The wave-lengths involved are λ_e and λ_a . The ratio

$$\frac{\lambda_a}{\lambda_e} = \frac{E_e}{E_a} = \frac{-p_\alpha u_e^\alpha}{-p_\alpha u_a^\alpha} \quad (26)$$

is frame independent. In an inertial frame the ratio is

$$\frac{\lambda_a}{\lambda_e} = \frac{p^0 u_e^0 - p^j u_e^j}{p^0 u_a^0 - p^j u_a^j} \quad (27)$$

Since magnitude of the rim velocity $v = r\omega$ is constant, the time-like components u_e^0 and u_a^0 are equal. Using the invariant Eq. (14):

$$d\tau^2 = dt^2 - \frac{dr^2}{c^2} \quad (28)$$

we can write for the time-like vector:

$$u_e^a = u_a^0 = \frac{dt}{d\tau} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (29)$$

The space components also have equal magnitudes, but not the directions.

$$|u_e| = |u_a| = \frac{dr}{d\tau} = \frac{dr}{dt} \frac{dt}{d\tau} = v \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (30)$$

Furthermore, Figure (8) shows that the angle $(\pi/2 - \beta)$ between u_e^j and p is the same as that between u_a^j and p . Thus $p^j u_e^j = p^j u_a^j$, implying

$$\frac{\lambda_a}{\lambda_e} = \frac{p^0 u_e^0 - p^j u_e^j}{p^0 u_a^0 - p^j u_a^j} = 1 \quad (31)$$

or that $\lambda_a = \lambda_e$. *There is no red-shift.* Motion in accelerated frames is adequately described by special relativity. The geometry of accelerated frames is the same as that of flat space; there is no hint of gravity in accelerated frames.

All the experimental results cited so far and the discussion following, tell us that no material property affects the trajectory of an object in the field of gravity. Inevitably one is led to a geometric interpretation of the gravity field. Geometry carves out specific trajectories which all objects must follow, even when the "object" is a light beam! Fundamentally, gravity is associated with curved space-time (non-zero Riemann tensor) whereas acceleration can occur in flat-space. Gravity and acceleration are as different as Hope and Crosby. There is a theorem that states that the Riemann tensor cannot be transformed away by any coordinate transformation i.e. a curved surface cannot be flattened. Conversely, the Riemann tensor is zero for flat-space and cannot be made non-zero through any transformation. Curved and flat surfaces differ in their geometry. Surfaces may possess "intrinsic" or "extrinsic" curvature. The sum of the interior angles of a triangle is π and the circumference of a circle is $2\pi r$ on a surface of extrinsic curvature, but not so on a surface with intrinsic curvature.

A cylinder, for example, is a surface with extrinsic curvature, because if cut along its length with a pair of scissors, it can be laid

out on a flat surface. A sphere by contrast, is a surface with intrinsic curvature. If a piece is cut out, no matter how small, it cannot be laid out on a flat surface. Thus a measurement of the Riemann tensor provides an unequivocal signature of a curved surface or equivalently the presence of gravity. Can one measure the Riemann tensor at a point (local experiment)? The answer is yes.

The apparatus is a meter stick in free fall (in near-Earth orbit). When oriented radially one end of the meter stick is moving along a different orbit than the other. This causes the meter stick to oscillate about its center of mass with a time period that depends on the local Riemann tensor of the Earth's field.³⁰ In terms of elements of the Riemann tensor $R_{\alpha\beta\gamma\delta}$ the time period is:

$$T = 2\pi(R_{\phi t \phi t} - R_{r t r t})^{-1/2} \quad (32)$$

$$T = 2\pi \left(\frac{GM}{R^3} + \frac{2GM}{R^3} \right)^{-1/2} \quad (33)$$

$$T = 2\pi \sqrt{\frac{R^3}{3GM}} = 50 \text{ min } s \quad (34)$$

in terms of Earth radius R and mass M , independent of the length of the meter stick. Although the experiment is performed with an extended object, a stick of *vanishing length* will oscillate with a period of 50 minutes in near Earth orbit. If on the other hand, the stick is merely accelerating, the Riemann tensor $\rightarrow 0$ and $T \rightarrow \infty$. The device measures the second derivative of the gravitational potential. The period is a direct measure of components of the Riemann tensor at the center of the meter stick. The oscillation period distinguishes, at a space-like point, between gravitation and acceleration.

Conclusion

Re-analyzing classical experiments and re-interpreting Newton's equation in the light of accelerating frames, we have uncovered within Newtonian dynamics some of the origins of the General Relativistic concept of gravity, not as a force, but as an outcome of geometry. Along the way we have shown that Newton's equation is valid only for forces such as the strong and electro-weak but *not* for gravity. Furthermore, the force in Newton's law of gravity is shown to be consistent with the geometrical origin of gravity. Although we have not introduced the Einstein equation, where the stress tensor on the right hand side describes all sources of energy *except* that of gravity, with our re-interpretation of Newton's equation we bring consistency between Newton's and Einstein's equations.

Acknowledgments

I am grateful to Jim Pellens (now deceased) for building the free fall apparatus.

Conflicts of interest

The author declares that there are no conflicts of interest.

Funding

None.

References

1. Munawar Karim. Dropbox. 2026
2. Martin JL. *General Relativity*. Prentice Hall; 1996:9.
3. Griffiths DJ. *Introduction to Electrodynamics*. 3rd ed. Prentice Hall; 1999.
4. de Almeida C, Saa A. The radiation of a uniformly accelerated charge is

- beyond the horizon: A simple derivation. *Am J Phys.* 2006;74(2):154–158.
5. Kreuzer LB. Experimental measurement of the equivalence of active and passive gravitational mass. *Phys Rev.* 1968;169:1007–1012.
 6. <https://dhhbt/km6RYq4V>
 7. <https://db.tt/8peYEPa>
 8. Klein W, Mittelstaedt P. A simple experimental demonstration of the principle of equivalence. *Am J Phys.* 1997;65(4):316.
 9. Niebauer TM, McHugh MP, Faller JE. Galilean test for the fifth force. *Phys Rev Lett.* 1987;59:609–612.
 10. Kuroda K, Mio N. Test of a composition-dependent force by a free-fall interferometer. *Phys Rev Lett.* 1989;62:1941–1944.
 11. Dabbs JWT, Harvey JA, Paya D, et al. Gravitational acceleration of free neutrons. *Phys Rev.* 1965;139.
 12. Witteborn FC, Fairbank WM. Experimental comparison of the gravitational force on freely falling electrons and metallic electrons. *Phys Rev Lett.* 1967;19:1049–1052.
 13. Eötvös RV. Mathematische und Naturwissenschaftliche Berichte aus Ungarn. *Journal of High Energy Physics, Gravitation and Cosmology.* 1889;8:65–68.
 14. Weinberg S. Gravitation and cosmology: principles and applications of the general theory of relativity. *John Wiley & Sons*; 1972:11.
 15. Dicke RH. The Sun's rotation and relativity. *Nature.* 1964;202:432–435.
 16. Synge JL. Relativity: the general theory. North-Holland; 1964:132.
 17. Wheeler JA. *A journey into gravity and spacetime.* WH Freeman and Company; 1990.
 18. Martin JL. *General Relativity.* Prentice Hall; 1996:25.
 19. Ohanian H, Ruffini R. *Gravitation and spacetime.* WW Norton; 1994.
 20. Dicke RH. In: DeWitt C, DeWitt B, eds. *Relativity, Groups, and Topology.* Gordon and Breach; 1969:168.
 21. Synge JL. *Relativity: The General Theory.* North-Holland; 1971.
 22. Ohanian H. What is the principle of equivalence? *Am J Phys.* 1977;45(10):903–909.
 23. Moreau W, Neutze R, Ross DK. The equivalence principle in the Schwarzschild geometry. *Am J Phys.* 1994;62(11):1037–1040. see also R.P Bernar, LCB Crispino, A Higuchi, et al. Lima Jr., *Am. J. Phys.* 2020;88(10):874 (corrected Riemann tensor).
 24. Hay HJ, Schiffer JP, Cranshaw TE, et al. Measurement of the red shift in an accelerated system using the Mössbauer effect in Fe57. *Phys Rev Lett.* 1960;4:165–166.
 25. Kobe DH. Generalization of coulomb's law to Maxwell's equations using special relativity. *Am J Phys.* 1986;54(7):631–636.
 26. Kündig W. Measurement of the transverse Doppler effect in an accelerated system. *Phys Rev.* 1963;129(6):2371–2375.
 27. Champeney DC, Isaak GR, Khan AM. An 'aether drift' experiment based on the Mössbauer effect. *Phys Lett.* 1963;7(4):241–243.
 28. Eisele AM. On the behaviour of an accelerated clock. *Helv Phys Acta.* 1987;60:1024.
 29. Misner CW, Thorne KS, Wheeler JA. *Gravitation.* WH Freeman and Company; 1970:63.
 30. Gron Ø. A tidal force pendulum. *Am J Phys.* 1983;51(5):429–431.