

# Time-frequency analyses applied to sound brought about by continual impacts for time-varying axial stress measurement of round bar

## Abstract

We developed the method to measure a time-varying stress of a round bar by time-frequency analyses. The larger tension we give to a guitar string, the higher tone the string emits, i.e. the string is changing its natural frequency to be higher. The frequency of the string is a function of the tension. When we look at the relation conversely, we can estimate tension by the frequency. We conceived the idea that if we could measure the variation of the frequency, we would be able to evaluate how the tension varied with time. The same condition holds for a round bar. We are able to estimate the time-varying axial stress of the bar by measuring the variation of the natural frequency of the bar. In order to verify of this idea, we conducted the experiment. We applied a sinusoidal stress to the bar. We gave continual impacts to the bar to bring about free vibrations. The vibrations produce sound. The sound provides data to follow the variation of the natural frequency. The procedure which consisted of a short-time Fourier transform application and signal processing programs we developed was applied to the sound. We could obtain the sequence of points which described the relation between the first mode natural frequency of the bar and time. We converted the sequence of the frequency to the sequence of the axial stress employing the relation between the natural frequency and the axial stress of the bar. We compared the stress sequence estimated by the procedure with the stress variation measured by the strain gauge attached to the bar. The sequences by the procedure and the variation by the strain gauge agreed well for various sinusoidal applied stresses for the loading cycle of less than 10 Hz.

**Keywords:** short-time fourier transform, time-varying stress, sound, natural frequency, round bar

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## Introduction

If we could evaluate easily the magnitude and cycle of a time-varying stress which was acting on members of a structure or a machine, it would offer us a fair evaluation of their safety. Strain gauges have been used to measure stress of structures and machines. In order to apply the method, we need many gauges. Measurement by strain gauges takes a lot of labor and accompanies troublesome work, and is unsuitable to measure working stress over a long period.

Other kinds of methods to estimate strain distribution over an area have been developed. Those are the digital image correlation method<sup>1</sup> or the acoustic-elastic method.<sup>2</sup> These methods need to compare a picture or a property before and after the deformation of a structure or a specimen. The procedure is still complicated and is not easily employed on site.

A natural frequency gives us various kinds of information. We tried to solve various problems employing a natural frequency such as for discriminating a genuine coin to a false coin,<sup>3</sup> for measuring Young's moduli of an anisotropic material or for measuring Young's modulus over a wide temperature range.<sup>4</sup> We investigated a relation between the first mode natural frequency and the axial stress of the bar with fixed-supported ends or with simply-supported ends. The theoretical relations and experimental results agreed. This agreement enables us to measure the axial stress of the bar supported by these ends using the natural frequency.<sup>5</sup>

The larger tension we give to a guitar string, the higher tone the string emits. The same circumstances are true for the relation between the axial stress of a round bar and the natural frequency of the bar. The larger tensile stress we give to the bar, the higher frequency the bar

vibrates with. When the axial stress of the bar changes over time, the first mode natural frequency of the bar also changes over time. This suggests us that if we could measure the variation of the frequency of the bar, we would be able to estimate how the tensile stress of the bar changed over time.

In this paper, we examined the possibility whether we could measure a time-varying axial stress of a round bar by the natural frequency of the bar. Applying a sinusoidal load to the bar, we measured the sound by free vibrations which were brought about by continual impacts to the bar. The procedure which consisted of a short-time Fourier transform method and our developed program was applied to the sound. We obtained the sequence of points which gave the variation between the first mode natural frequency of the bar and time. We converted it to the sequence between the axial stress and time. We compared the stress sequence obtained by the procedure with the stress variation measured by the strain gauge attached on the bar.

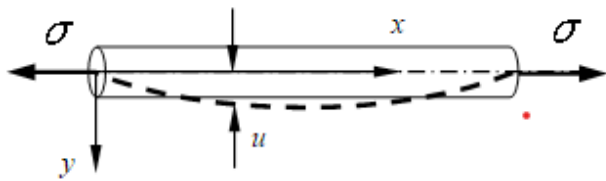
## Theory

### Natural frequency of beam subjected to axial stress

A vibration equation of a beam subjected to an axial stress  $\sigma$ , shown in Figure 1, is given by the expression (1).

$$\frac{\partial^2 u}{\partial t^2} + \frac{EI}{\rho A} \frac{\partial^4 u}{\partial x^4} + \frac{\sigma}{\rho} \frac{\partial^2 u}{\partial x^2} = 0 \quad (1)$$

Here  $u$  is a deflection of the beam.  $x$ ,  $y$  and  $t$  are the location and time variables respectively.  $I$  is a sectional secondary moment,  $\rho$  is a density,  $E$  is Young's modulus and  $A$  is a sectional area.



**Figure 1** Transverse vibration of beam.

Applying a fixed-supported end condition to both ends of the beam, we obtain simultaneous equations from the equation (1).

By solving the simultaneous equations, we can obtain the first mode natural frequency  $f$  of the beam. We can describe the solution as the expression (2). See reference<sup>5</sup> for details.

$$f = f(\sigma, E, \rho, A, L, I) \quad (2)$$

Here  $L$  is the length of the specimen. The natural frequency is a function of the applied axial stress.

### Short-time Fourier transform method

In order to extract natural frequencies out of a signal, we use a fast Fourier transform (FFT) method<sup>6</sup> given by the expression (3).

$$(\omega) = \int_{-\infty}^{\infty} \zeta(t) e^{-j\omega t} dt \quad (3)$$

Here  $\zeta(t)$  is a signal in time domain,  $\omega$  is the angular frequency and  $j$  is the imaginary unit. When we apply an FFT method to sound by free vibrations of a round bar, we can obtain natural frequencies of the bar. But the method does not give us any information on time.

Recently, time-frequency analyses such as a short time Fourier transform (STFT) method or a wavelet analysis have been developed.

They teach us how the frequency varied with time. The expression converted by a short-time Fourier transform,  $G$  is given by the expression (4).

$$(t, \omega) = \int_{-\infty}^{\infty} \zeta(\tau) e^{-j\omega\tau} w(\tau - t) d\tau \quad (4)$$

Here  $w(t)$  is a window function. Variables of  $G$  are time and an angular frequency. We can know how the frequency varies with time by  $G$ . These methods are now employed for various fields and explained in detail elsewhere.<sup>6,7</sup> We employed the STFT method provided in the software libraries of MATLAB.<sup>8</sup>

There is the uncertainty principle given by the expression (5) also in signal analysis.

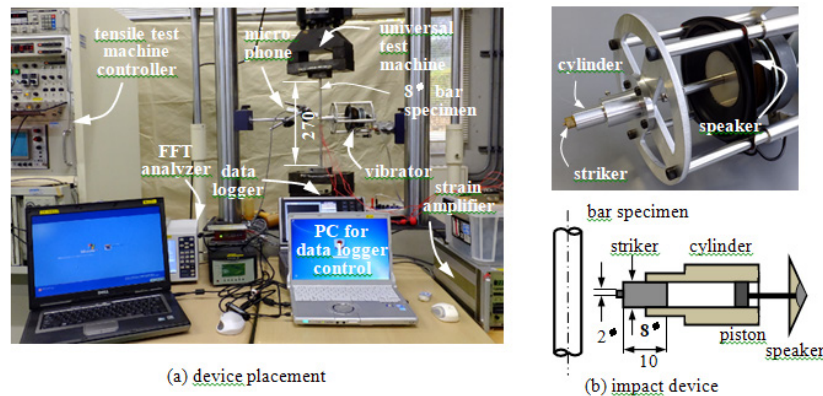
$$\Delta t \Delta \omega > \frac{1}{2} \quad (5)$$

Because of the principle, an STFT method provides a band spectrum, i.e. a diffused distribution of a power intensity with time and a frequency.

## Experiment and calculation

### Experiment system

We used a round bar specimen made out of steel. Young's modulus was 213 GPa which was obtained by another material test. The diameter is 8.0 mm and the length between fixed-supported ends is 270 mm. Figure 2 shows the photo of the placement for our experiment devices. The devices consist of a microphone to measure sound, a strain amplifier to measure applied stress, a data logger to record sound, a universal tensile test machine to apply a sinusoidal load, a vibrator to excite free vibrations and an FFT analyzer.



**Figure 2** Devices for experiment.

Applying static axial stress to the bar, we gave an impact to the bar. The impact brings about the free vibration. The vibration produces sound. Analyzing the sound by an FFT device, we obtain the first mode natural frequency of the bar. We can correlate the frequency with the axial stress of the bar. We investigated the relation between the frequency and the axial stress for various axial stresses. The relation is employed to convert the frequency to the axial stress.

When we give an impact to the bar, a free vibration occurs by the impact and produces sound. We are able to extract the first mode natural frequency of the bar from the sound by signal analyses. But, the duration of the free vibration is brief. We can only obtain the

frequency and therefore the axial stress of the bar at the moment of the impact. In order to know how the frequency changed over time, it is necessary to make the bar vibrate continually. For the continual vibration, we made the impact device shown in Figure 2(b). The device consists of a speaker and a vibrator. The vibrator consists of a cylinder, a piston and a striker. The piston and the striker are set into the cylinder. A motion of the speaker diaphragm makes the piston in the cylinder slide forwards and backwards. The piston was driven by the cycle of around 30 Hz. The air pressure generated by the motion of the piston moves the striker. The striker hits the bar repeatedly and the bar goes on vibrating.

It is preferable that the striker does not give influence on the vibration of the bar. The pneumatically-driven striker is expected to move away promptly from the bar after hitting it. Whole experiment system is shown in Figure 3. The left-hand side of the figure shows the impact system. The function generator transmits a triangular

waveform voltage to the amplifier. The amplifier intensifies the voltage to be a necessary power to drive a voice coil motor of the speaker. We attached strain gauges onto the bar to measure a variation of the applied axial stress.

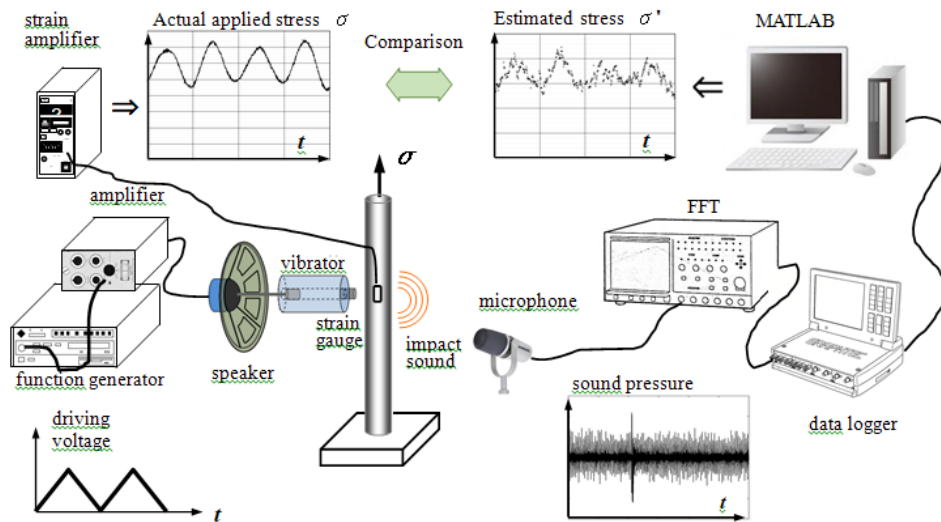


Figure 3 Whole experiment system.

The right-hand side of the figure shows measuring and analyzing system. We measured a sound variation by a microphone and stored the variation into a data logger. Sampling time for the sound was 5 kHz. The first mode natural frequency of the bar ranges from 450 to 530 Hz depending on the axial stress. The sampling time is large enough to measure the natural frequency of the bar. We analyzed the variation by the procedure, which consisted of the short-time Fourier method provided by MATLAB and the programs we developed. The detailed explanation of the procedure is taken up in the next chapter. We could obtain the variation of the first mode natural frequency of the bar over time. We converted the sequence of the frequency variation to the sequence of the axial stress  $\sigma'$ . We compared the sequence  $\sigma'$  obtained by the procedure with the stress variation  $\sigma$  measured by the strain gauge as shown in Figure 3.

### Calculation procedure

The procedure mentioned in the previous section is explained below. Typical example by the procedure are given in Figure 4. Figure 4(a) is the sound measured by a microphone. The sound is fluctuating violently. Unavoidable background noise from equipment in our experimental laboratory was mixed into the sound. Figure 4(b) shows the result by an FFT analysis for the original sound shown in Figure 4(a). The noise had higher power intensity and was obstructive for our analyses.

First, in recording the sound, we applied an analogue low-pass filter, allowing the sound only in the range of frequency below the cutoff frequency, 630 Hz to pass.

Secondly, we applied a software band-pass filter by MATLAB. We allowed the sound only in the range of frequencies between two cutoff frequencies,  $f_L$  and  $f_H$  to pass in order to focus on the first mode natural frequency range of the bar. We set  $f_L = 460$  Hz and  $f_H = 520$  Hz, which corresponded to 7 MPa and 145 MPa respectively. We supposed the applied stress would occur within the range between these stresses. Figure 4(c) shows a line spectrum obtained by the FFT

analysis for the band-passed sound signal. The spectrum reveals some characteristic frequencies of the sound, but gives no information on time.

Thirdly, we applied the STFT method provided in MATLAB to the band-passed sound data. We used hamming window for the STFT method. Figure 4(d), a picture like a sea of clouds, shows the spectrogram obtained by the method. Horizontal axes are time and a frequency. Vertical axis is a power intensity. The monotone colors encode frequency power levels which are scaled by the indicator in the right-hand side. The STFT method gives a diffused distribution of a power intensity with time and a frequency as shown in Figure 4(d).

Fourthly, we extracted the variation of the first mode natural frequency as a function of time from the diffused distribution by the way explained hereafter. Out of the vertical plane at  $t = t_m$  with the axes of the frequency and the power intensity as shown in Figure 4(e), we picked up frequencies between  $f_L$  and  $f_H$ . And we tried to find the frequency  $f_m$  which had the maximum power intensity among them. We regarded that the frequency  $f_m$  was the natural frequency of the bar at the time  $t = t_m$ . But, even the noise at  $t = t_p$  as shown in Figure 4(e) has the frequency  $f_p$  which has the maximum power intensity between  $f_L$  and  $f_H$ . It is not proper to regard the frequency  $f_p$  as the natural frequency. Thereupon, we put restriction on the qualification as the natural frequency. If the frequency  $f$  at  $t = t$  had the maximum power intensity which was larger than the threshold we specified, we regarded that the frequency  $f$  was the natural frequency at  $t = t$ . But, if maximum intensity were lower than the threshold, we regarded that we could not find the natural frequency of the bar.

We repeated these operations at each time in the spectrogram and obtained the sequence of points between the first mode natural frequency and time.

In the last place, we removed spikes or outliers from the sequence by a data smoothing. We adopted seven-point simple forward smoothing.

By these signal processing procedures, which we call hereafter the time-frequency analyses, we could obtain the sequence of points which gave the relation between the first mode natural frequency of the bar and time. The sequence is located on the ridgeline shown by the dotted line in Figure 4(d). We converted the frequency sequence to the stress sequence employing the expression in Figure 5. Figure 4(f) shows the comparison between the stress sequence estimated by the analyses and the stress variation measured by the strain gauge attached on the surface of the bar. Its horizontal axis is time and the vertical axis is stress.

We applied various sinusoidal stress variations to the bar. By the comparisons, we examined the availability and the effective range of our time-frequency analyses.

## Results

### Relation between 1<sup>st</sup> mode natural frequency and static axial stress

Applying various magnitudes of static axial tensile stress to the bar by a tensile testing machine, we measured the first mode natural frequency of the bar. Figure 5 shows the relation between the applied axial stress and the natural frequency. We entered the approximate expression of the relation into the figure. If we measure the first mode natural frequency of the bar, we can estimate the axial stress of the bar by the expression in Figure 5.

### Result for sinusoidal stress by 1 Hz

We applied a sinusoidal stress to the bar with the loading cycle by 1 Hz. Figure 6(a) shows the sound after the filterings by the analogue-low pass and software band-pass. Figure 6(b) shows the result by the FFT analysis for the sound after filterings. Only the frequencies around the first mode natural frequency are revealed. Figure 6(c) shows the three-dimensional spectrogram obtained by the STFT method for the sound. The figure shows the relation among time, a frequency and a power intensity. The colors encode frequency power levels which are scaled by the indicator in the right-hand side. Figure 6(d) shows the planar spectrogram which we looked down at Figure 6(c). We extracted the variation of the first mode natural frequency of the bar over the time from the spectrogram. We converted it to the relation between the axial stress and time employing the expression in Figure 5.

Figure 6(e) shows the comparison between the applied stress variation measured by the strain gauge and the estimated axial stress sequence obtained by the time-frequency analyses. The horizontal axis is time and the vertical axis is stress. The estimated stress is shown by the sequence of square points. The applied stress is shown by the solid line. Figure 6(f) zoomed in on the time range from 4 and 8 sec. The sequence of points by the analyses seems to be following the variation of the applied stress.

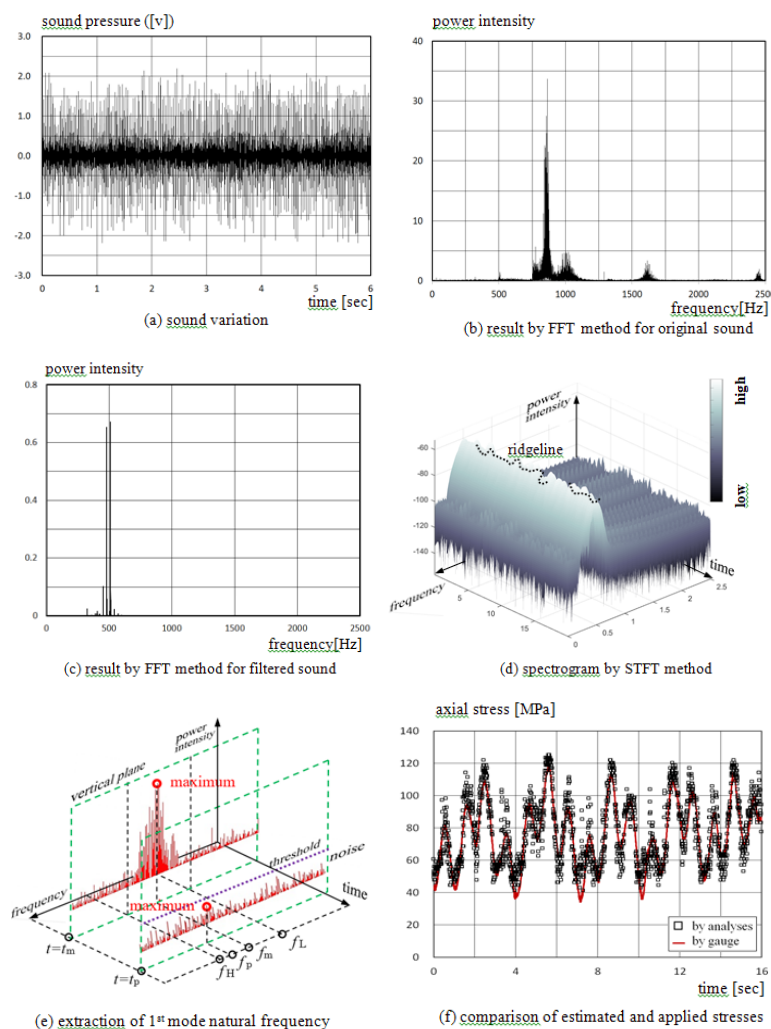


Figure 4 Example results by time-frequency analyses.



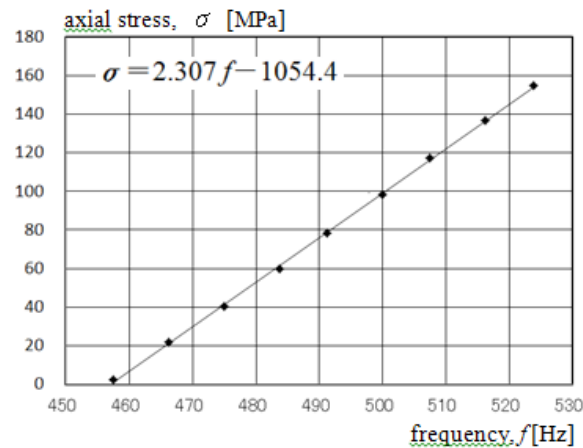


Figure 5 Relation between 1<sup>st</sup> mode natural frequency and axial stress.

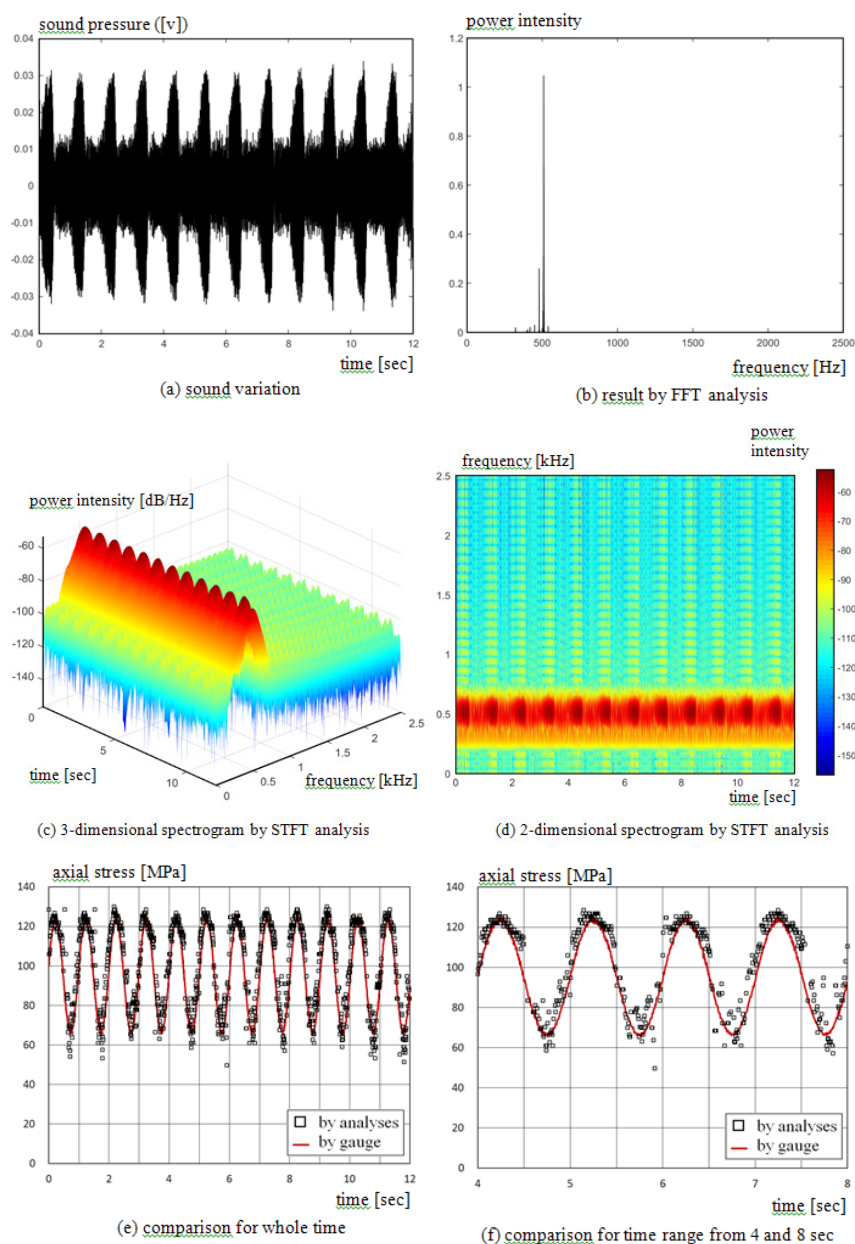
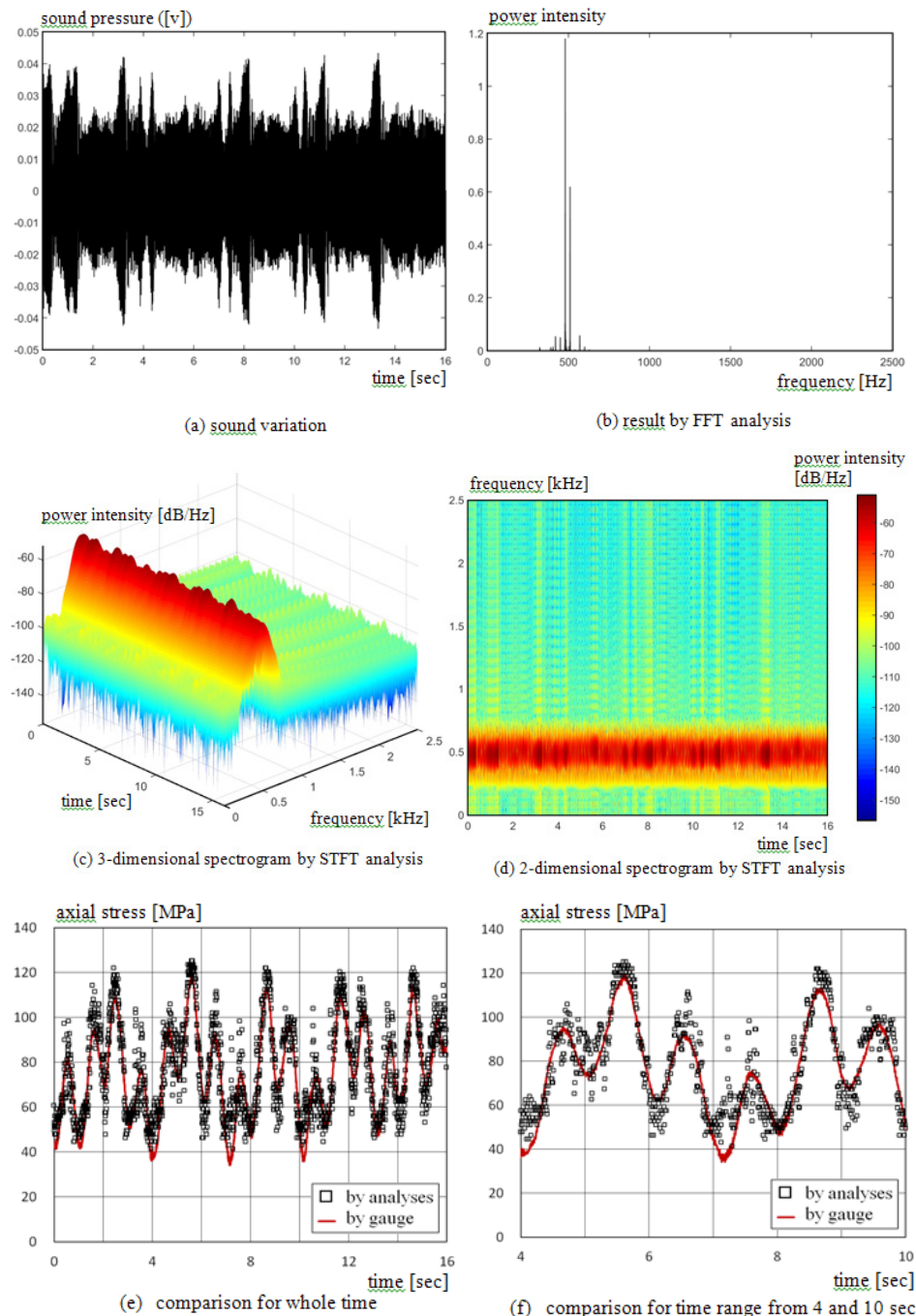


Figure 6 Sinusoidal loading by 1 Hz.

### Result for irregular sinusoidal stress by 1 Hz

We applied irregular sinusoidal stress to the bar with the loading cycle by 1 Hz. Figure 7(a) shows the sound after the analogue-low pass and software band-pass filterings. Figure 7(b) shows the result by the FFT analysis for the sound after the filterings. Figure 7(c) shows the three-dimensional spectrogram obtained by the STFT method

for the sound. Figure 7(d) shows the planar spectrogram. Figure 7(e) shows the comparison between the applied stress shown by the solid line and the estimated axial stress shown by square points. Figure 7(f) zoomed in on the time range from 4 and 10 sec. The sequence of square points obtained by the analyses seems to be capturing feature of the applied stress variation, though some points are off the line.



**Figure 7** Irregular sinusoidal loading by 1 Hz.

### Result for sinusoidal stress by 10 Hz

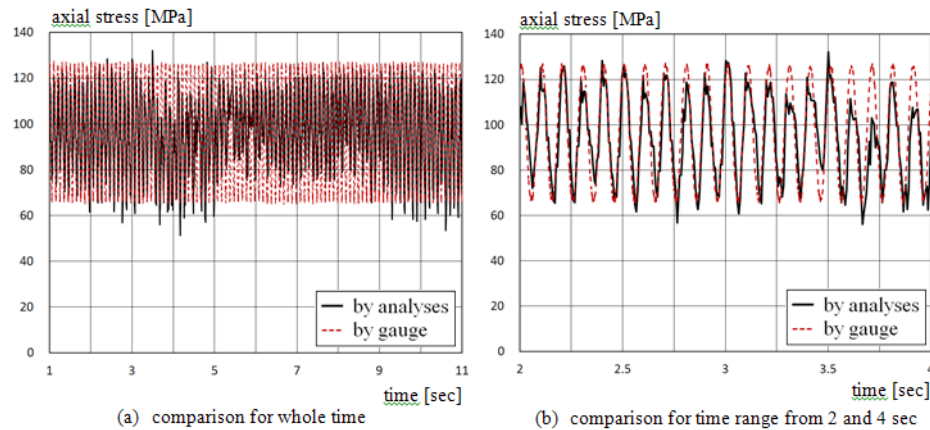
We applied sinusoidal stress with the loading cycle by 10 Hz to the bar. We have ever shown the figures of sound variation, results by the FFT analysis and by the STFT method. But they do not give us useful

information. We leave them out hereafter and give only the results of the comparison.

Figure 8 shows the comparisons between the applied stress variation and the estimated axial stress variation. The estimated stress

is shown by the solid line and the applied stress by the broken line. Figure 8 displays the time along the horizontal axis and the stress along the vertical axis. Figure 8(b) zoomed in on the time range from 2 and 4 sec.

The estimated axial stress by the analyses seems to be going along with the variation of the applied stress, though the amplitude by the analyses was sometimes smaller than the applied one.



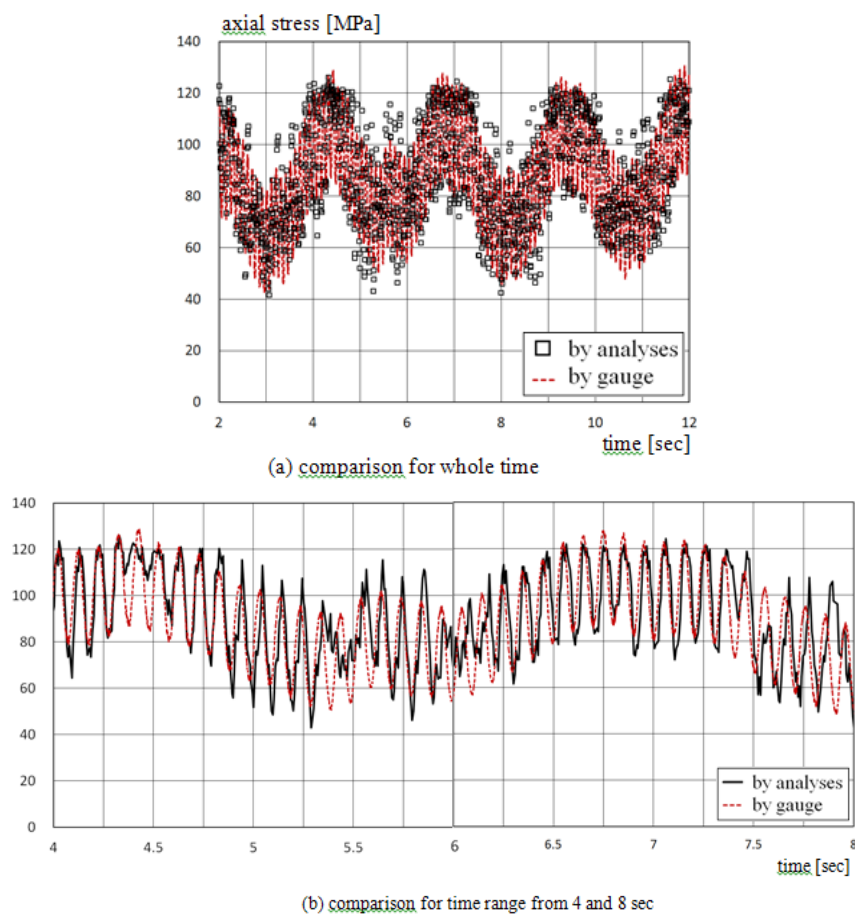
**Figure 8** Sinusoidal loading by 10 Hz.

### Result for irregular sinusoidal stress by 10 Hz

We applied irregular sinusoidal stress with the loading cycle by 10 Hz to the bar. Figure 9 shows the comparisons between the applied stress variation and the estimated axial stress variation. The estimated stress is shown by the sequence of square points in Figure 9(a) and the

solid line in Figure 9(b). The applied stress is shown by the broken line. Figure 9(b) zoomed in on the time range from 4 and 8 sec.

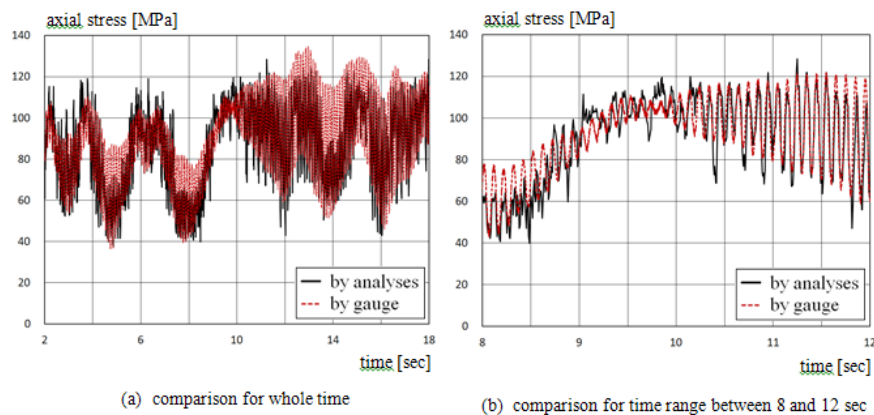
The estimated stress seems to be going along with the variation of the applied stress, though the amplitude by the analyses seems to be larger than one by the strain gauge.



**Figure 9** Irregular sinusoidal loading by 10 Hz.

We applied another irregular sinusoidal stress with the loading cycle by 10 Hz to the bar. Figure 10 shows the comparisons between the applied stress variation and the estimated axial stress variation.

The estimated stress is shown by the solid line. The applied stress is shown by the broken line. Figure 10(b) zoomed in on the time range from 8 and 12 sec.



**Figure 10** Irregular sinusoidal loading by 10 Hz.

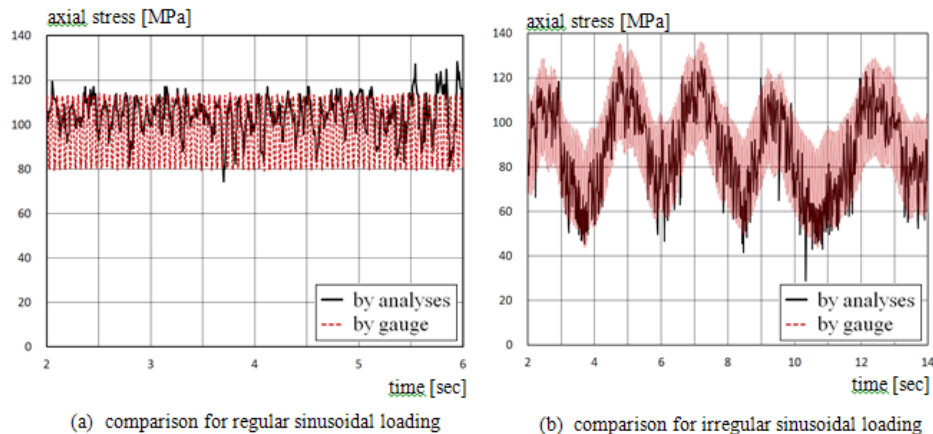
The estimated stress by the analyses seems to be going along with the variation of the applied stress, though the amplitude by the analyses seems to be occasionally smaller than one by the strain gauge.

### Result for sinusoidal stress by 20 Hz

We applied regular or irregular sinusoidal stresses with the loading cycle by 20 Hz to the bar. Figure 11 shows the comparisons between the applied stress variation measured by the strain gauge and the estimated axial stress variation by the time-frequency analyses.

The estimated stress is shown by the solid line. The applied stress is shown by the broken line. Figure 11(a) shows the result for the regular sinusoidal stress and Figure 11(b) for irregular sinusoidal stress.

The amplitude by the analyses is half of one by the strain gauge for both cases. The variation by the analyses does not follow one by the gauge adequately. It seems that by our current capability, it is impossible to evaluate the axial stress variation for the loading cycle higher than 20 Hz.



**Figure 11** Results by loading cycle of 20 Hz.

## Conclusion

We estimated by a natural frequency how an axial stress of a round bar varied with time. We gave continual impacts to the bar with the cycle of around 30 Hz. The impacts brought about free vibrations of the bar. The vibrations produce sound. We stored the sound into a data logger. We analysed the sound by the procedure which consisted of a short-time Fourier transform method provided by MATLAB and programs we developed. The procedure gave us the sequence of points between the first mode natural frequency of the bar and time. Employing the relation between the first mode natural frequency and an axial stress which we examined beforehand, we converted the sequence of the natural frequency to one between the axial stress and time. We compared the stress sequence obtained by the procedure

with the variation measured by the strain gauge attached on the bar. We could estimate various time-varying sinusoidal axial stresses of the bar for the loading cycle less than 10 Hz.

We employed a pneumatically-driven striker for continual impacts, but the impacts by the striker was unstable and inefficient. We could not obtain the adequate time-varying axial stress variation for the loading cycle higher than 20 Hz. We think that higher hitting cycle over 60 Hz and larger impact energy to the bar will enable us to measure the time-varying axial stress for the loading cycle higher than 20 Hz.

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## Conflicts of interest

Authors declare that there is no conflict of interest.

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