Appendix 1

The Q.C.D(SU(∞)) loop wave equation

It has been fully discussed in the literature that after formal manipulations on the objects involved, specially with the (ill defined) Yang-Mills path integral in the continuum, one arrives at the following (formal) loop wave equation for the Wilson (see eq(7-a) in the text) loop in $Q.C.D(SU(N_c))$:

$$\int_0^{2\pi} d\bar{\sigma}\{\frac{\delta^2}{\delta X_\mu(\bar{\sigma})\delta X^\mu(\bar{\sigma})} - \left(\int d^D x \langle\Omega| F_{\mu\nu}(x)F_{\mu\alpha}(x)|\Omega\rangle_{SU(N_c)}\right)\frac{dX^\nu(\bar{\sigma})}{d\bar{\sigma}}\frac{dX^\alpha(\bar{\sigma})}{d\bar{\sigma}}\} =$$

$$(g^2 N_c)\{\int_0^{2\pi} d\sigma \int_0^{2\pi} d\bar{\sigma}\delta^{(D)}(X^\mu(\sigma)-X^\mu(\bar{\sigma}))\left(\left|\frac{dX^\beta(\sigma)}{d\sigma}\frac{dX_\beta(\bar{\sigma})}{d\bar{\sigma}}\right|\right)$$

$$\left\langle \Phi_{N_c}[X_\mu(\tilde{\sigma});0\le\tilde{\sigma}\le\bar{\sigma}]\times\Phi_{N_c}[X_\mu(\tilde{\sigma});\bar{\sigma}\le\tilde{\sigma}\le 2\pi]\right\rangle_{SU(N_c)}\Bigg\}$$

where $\langle \quad \rangle_{SU(N_c)}$ denotes the average over the (ill defined) Yang-Mills euclidean path integral.

As it has been observed in the main text, the correct mathematical meaning of eq(1-1) should be searched on the well-define lattice formulation of Q.C.D.

However one defines a “zeroth-order" for the full stochastic-Dhyson Schwinger equation through the crude force approximation, called by us of $Q.C.D(SU(\infty))$, where the Yang-Mills path integral of the gauge invariant observables factorize, as a mathematical methods working calculational hypothesis.

It has argued in the literature that such approximation on the continuum formal Yang-Mills theory should be justified by the well-known t'Hooft diagrammatic framework.[2] However this suggestion has not been fully proved in our opinion. The loop equation eq(1-2) should be better regarded perhaps in the framework of Random Matrix Theory.

As a result one gets eq(7-b) written in the main text.

Note that we have made the simplifying assumption of the space-time isotropy of the non perturbative Yang-Mills strenght field condensate on eq(1-1) (see refs.[2,11] Otherwise one must consider the $Q.C.D(SU(\infty))$ free string moving in a sort of metric back ground, namely:[13]

$$G^{(0)}(C_{xx}) = \int_{\partial X^\mu = C_{xx}} D^F[X^\mu(\xi)]\exp\left\{-\frac{1}{2}\int_\xi [(\partial_t X^\mu)^2 + G_{\mu\nu}\partial_\sigma X^\mu \partial_\sigma X^\nu](\sigma,\zeta)\right\}$$

with

$$G_{\nu\alpha} = \int d^D x \langle\Omega|(F_{\mu\nu}F_{\mu\alpha})(x)|\Omega\rangle_{SU(\infty)}$$

Appendix 2

The correct free string propagator in loop space Q.C.D(SU(∞))

Let us sketchy the anomaly evaluation of the string path integral eq(9) in the bulk of this note.

The main point is by following closely the detailed proof as given by us in our previous work[9] to fix the dipheomorfism gauge to the conformal gauge both on the path integral over the string vector position and its Riemaniann structure also (both regarded a priori independent), but related by the covariant delta constraint integral in the path integral measure.[10,11] Namely

$$X^\mu(\xi) = X^{\mu,CL}(\xi) + \sqrt{\pi\alpha'}Y^\mu(\xi) \qquad (2\text{-}1)$$

$$\left(\Delta_{h(X^{\mu,CL})}\right)X^{\mu,CL} = 0 \qquad (2\text{-}2)$$

$$\partial X^{\mu,CL} = C^\mu_{xx} \qquad (2\text{-}3)$$

$$g_{ab}(\xi) = h_{ab}(X^{\mu,CL}(\xi))\beta^2(\xi) \equiv h^{CL}_{ab}(\xi)\beta^2(\xi) \qquad (2\text{-}4)$$

As a result one obtain the Liouville model in the sigma model like form with the usual Feynmann product measure:

$$G(C_{xx}) = \int\prod_{(\xi)}(\sqrt{h^{CL}(\xi)}\,d\beta(\xi))$$

$$\times\left\{\prod_{(\xi)}(\sqrt[4]{h^{CL}(\xi)}\,dY^{\mu}(\xi))\right.$$

$$\exp\left\{+\frac{(26-D)}{4\delta\pi}\int d^2\xi\left(\sqrt{h_{CL}}\,h_{CL}^{ab}\partial_a(\beta)\partial_a\left(\frac{1}{\beta}\right)\right)(\xi)\right\}$$

$$\times\exp\left\{-\frac{\mu_R^2}{2}\int d^2x(\sqrt{h_{CL}}.\beta^2)(\xi)\right.$$

$$\times\exp\left\{-\frac{1}{2\pi\alpha'}\int d^2\xi(\sqrt{h_{CL}}\,h_{CL}^{+-}\partial_+Y^{\mu}\partial_-Y_{\mu})(\xi)\right\}$$

$$\times\exp\left\{\int d^2\xi(\sqrt{h_{CL}}\,R(h_{ab}^{CL}).\beta)(\xi)\right\}$$

$$\times\delta_{cov}^{(F)}[(\beta^2(\xi)h_{+-}^{CL}-\partial_+Y^{\mu}\partial_-Y_{\mu})(\xi)]$$

Since the above two-dimensional model is non-renormalizable after the realization of the path integral over the $\beta(\xi)$ field (due to the delta functional constraint eq(2-5), one must vanishe the conformall anormaly factor term by choosing D=26. At this point it is worth to remark that $\mu_R^2=\lim_{\varepsilon\to 0}\frac{(2-D)}{\varepsilon}$ and should be considered as just a formal renormalization of the Regge constant $\mu^2=\frac{1}{\pi\alpha'}$.[14,15]

It is worth also to note that non trivial topology is straightforwardly taken into account through the classical minimal surface equation eq(2-2)-eq(2-3), now a Dirichlet problem (only at the surface conformal gauge[10,11] to be solved at a Riemann surface of arbitrary genus (see ref[16] – Chapter 6 – Section 5: canonical conformal mappings of multiply connected regions; and ref[17] – Chapter IV - § 31 – Dirichlet's problem for multi-connected regions).[16,17]

It may be also solved by the introduction of the Surface's Riemann Structure Techmilles parameter directly on the decomposition eq(2-4).[14]

The resulting anomaly free and in the euclidean light cone gauge fixed string path integral is the well-known Mandelstan string path integral in the "slites" string parameter domain ξ .

Appendix 3

The space-time supersymmetric self avoiding string

Let us call attention that in this case the fermions random surface oriented tensor has as candidate the following expression (non normalized to unity)

$$\Im_{\mu\nu}^{F}(X^{\alpha,F}(\xi,\theta))\equiv\left(\frac{\varepsilon^{ab}\partial_a X_{\mu}\partial_b X_{\nu}+\psi^{\mu}(\gamma_a\partial_b)\psi^{\nu}}{[\det[H_{ab}]]^{1/2}}\right)(\xi) \qquad (3\text{-}1)$$

where the fermionic surface Riemann matrix is explicitly given by $(\xi=(\sigma,\tau))$

$$H_{ab}(\xi)=(\partial_a X^{\mu}\partial_b X_{\mu}+\psi^{\mu}(\gamma_a\partial_b)\psi_{\mu})(\xi) \qquad (3\text{-}2)$$

The Nambu-Goto area functional is thus given by

$$S^{(0)}=\frac{1}{2\pi\alpha'}\int d^2\xi[\det[H_{ab}]]^{1/2}(\xi) \qquad (3\text{-}3)$$

The self suppressing term is expected to be given by

$$S^{(1)}=\int d^2\xi\int d^2\xi'\int d\theta\int d\theta'[(\bar{\psi}\psi)(\xi)(\bar{\psi}\psi)(\xi')]$$
$$\Im_{\mu\nu}^{F}(X^{\alpha,F}(\xi,\theta))\delta^{(D)}(X_{\mu}^{F}(\xi,\theta)-X_{\mu}^{F}(\xi',\theta')\times\Im_{\mu\nu}^{F}(X^{\alpha,F}(\xi',\theta')) \qquad (3\text{-}4)$$

Appendix 4

In this appendix we intend to write the classical string representation for the $Q.C.D(SU(\infty))$ euclidean quantum Wilson Loop a suggested by eq(7-a) for 26=D+N and frozen internal fermionic degrees of freedom.

By solely considering the classical light-cone gauge fixed minimal surface $X^{\mu,CL}(\xi)$ and defined for a domain string parameter of arbitrary topological genera (see remarks below eq(2-5) – Appendix 2). One has thus the representation below:

$$\Phi_{\infty}^{Semi-Classical}[C_{xx}] \sim \int_{M^{\mathrm{Teic}}(h)} d\nu(\tau_i) \exp\{-\frac{1}{2\pi\alpha'} \int_D d^2\xi (\sqrt{h}\, h^{+-})(\tau_i, \xi) \times \partial_+ X^{\mu,CL}(\xi,\tau_i) \partial_- X_{\mu}^{CL}(\xi,\tau_i)\} \quad (4\text{-}1)$$

Here $h_{ab}(\xi,\tau_i)$ are the metric representatives on the Teichmüller space of the possible topological structure on the metric space of the surface 2D manifold $\{X^{\mu,CL}(\xi)\}$ and $d\nu(\tau_i)$ denotes the measure the integration of such Teichmüller space of dimensionality 6g-6 where g is the string surface genus. The exact expression of such measure $d\nu(\tau_i)$ is explicitly.[15]

However a more suitable Q.C.D string effective representation path integral can be given by U(11) Gross-Neveu model defined over a classical Riemannian surface of genus g and summed up over all theses classical Riemann surfaces with the associated Nambu-Goto height area functional, but with the path integral defined only over the surface's Teichmüller parameters.

Note that this classical Riemann surface is explicitly given by eq(2-1)-eq(2-4) of Appendix 2. This result thus lends perhaps to the loop space-string integrability of $Q.C.D(SU(\infty))$.