

Two equivalent inverse eigenvalue problems about graded-index optical fibers

Abstract

In the two papers^{1,2} recently published, the authors Hayat Rezgui and Abdelaziz Choutri present deep studies, in both theoretic and numerical aspects, of two equivalent inverse eigenvalue problems.

The corresponding direct problem (solved by a finite element method) consists of computing guided modes that propagate, under weak guidance assumptions, in a graded-index optical fiber with a circular-shaped section and an homogenous cladding (I recommend the two articles^{3,4} for more details and extended studies of the direct problem).

In these two articles^{1,2} the authors use original algorithms and iterative basic schemes to tackle the two studied inverse eigenvalue problems:

I. For the first inverse eigenvalue problem (without use of regularization), which is studied in the article,² the authors concentrate their attention to use a special fast and rapidly convergent algorithm for solving a convex constraint optimization problem using Lagrange multipliers. Examples and several numerical illustrations, related to the considered inverse eigenvalue problem, show the robustness and the higher efficiency obtained by the suggested approach that converges geometrically and linearly to the exact refractive index.

II For the second inverse eigenvalue problem that is considered in the article,¹ an original iterative method (based on Tikhonov regularization and assisted by L-curve method, absolute and relative errors calculation) has been exhibited and developed for the computation of the regularized (unique) solution of a discrete convex unconstrained minimization problem (in the absence of any prior information). Furthermore, the results of numerical experiments confirm the reliability of the proposed technique.

Keywords: Optical fibers, graded-index profile, refractive-index, inverse eigenvalue problem, unconstrained optimization problem, Lagrange multipliers, Tikhonov regularization, L-curve method, absolute error, relative error, iterative algorithms

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Introduction

The science of optical fibers is a fascinating field. A great amount of research work is being carried out in all parts of the world to promote optical fiber technology. One of the most interesting developments in recent years in the field of telecommunication and data transmission systems is the use of optical fibers to carry information in a way similar to that employing radiowaves and microwaves.⁵

The idea of using optical fibers to transmit information appeared in the early 1960s with the advent of lasers, and the advantages of transmitting information by optical fibers are multiple compared to other communication media (optical fibers are much lighter and thinner compared to the conventional copper cables, and they are cheaper than copper wires. One pair of optical fibers carries a rate of 10 times stronger than 250 pairs of copper wires).⁶⁻⁹

Thanks to comfort and energy saving provided by optical fibers, these last are perfect for:

- Medical applications.

- The lighting fields.
- The road transport system.
- Various military applications requiring a high-quality equipment.

The interest in graded-index optical fibers is increasing owing to their extensive possible applications. Without these components, it is difficult to imagine any further development of fiber-optical systems for transmitting information, medical and industrial endoscopy, copying technology, fast computer input-output devices, and facsimile communication.¹⁰

Objective

The aim of the two articles^{1,2} is to study two equivalent inverse problems that consist (for a given frequency) in seeking for a suitable graded-index optical fiber, which is able to carry electromagnetic fields having known propagation constants.

The authors adopted original iterative basic schemes to tackle these two inverse eigenvalue problems, one by Tikhonov regularization method¹ and the other without regularization.²

Numerical experiments

The discrete obtained problems have been solved in a finite sequence of algebraic operations, efficiently implementable on a computer. Otherwise, in order to assess the benefits of an original and a very interesting estimation provided by the announced main theorem 4.1.¹

The effectiveness and the performance of the proposed approach have been illustrated by 5 computed examples for different grade profile parameters where the L-curve method was used for determining the optimal value of the regularization parameter. Moreover, other significative numerical examples (3 examples) were presented in Rezgui et al.²

Conclusion

In conclusion, the authors focused (in these two articles) on two fascinating equivalent inverse eigenvalue problems. The obtained results appeared numerically reasonable, reliable and precise. They were based on a scientifically valid methodology and encourage the hope that the adopted approaches in these articles may develop into a useful tool for studying another physical or mathematical problems.

The results of these works can be applied to practical investigations in several ways, and more long-term potential future works could focus on a number of areas, in particular areas that are relevant but have not been the focus of these articles.

The used algorithms are iterative in nature and the computation of each iteration is not complicated, which makes it possible to solve other general problems. It would be very interesting to see if other numerical applicable algorithms could be developed for the studied inverse eigenvalue problems. As observed from the previous computational results and some preliminary future work results, I notice that the algorithms and schemes used in these articles are new, very interesting and promising.

The two articles^{1,2} will make a significant contribution to the field and help to arouse and inspire some additional interest and to encourage and stimulate further research into this topic. For more details, you can also see.¹¹⁻¹³

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Conflict of interest

Author declares there is no conflict of interest.

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