Review Article

Kinetic theory of cosmic baryon distribution functions in expanding space times

Abstract

It is generally assumed in modern cosmology that matter and radiation before the phase of matter recombination was in a perfect thermodynamic equilibrium with baryons described by Maxwell distributions and photons described by a Planckian law. Looking here, however, a bit deeper into the kinetic theory of the physical processes close to and just after the recombination of electrons and protons, reveals that in a homologously expanding universe baryon distribution functions will not keep their Maxwellian profile and connected with it, that their most relevant velocity moments, i.e. their density and their temperature, vary in an unexpected manner, also evidencing that, in contrast to the classical view, the entropy of free baryons changes with cosmic time.

Keywords: cosmology, kinetic theory, hubble expansion

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Introduction

Let us start our considerations here from the cosmic era of baryon-electron (or proton-electron) recombination. In standard cosmology it is usually assumed that at the cosmic recombination era photons and matter, meaning electrons and protons, in this phase of the cosmic evolution, are dynamically and physically tightly bound to each other and undergo strong mutual interactions via Coulomb collisions and Compton collisions. With these conditions taken for granted, a pure thermodynamical equilibrium state appears to be guaranteed, implying that particles (protons and electrons) are Maxwell distributed in velocity-space and photons have a Planckian blackbody distribution in frequency. Looking a little more in detail on this relevant point, it is, however, by far not so evident that these assumptions really can be expected to be fulfilled during this period of cosmic evolution. This is because photons and particles are reacting very differently to the cosmological expansion; photons generally are considered to be cooling due to cosmologically being redshifted,[1,2] while particles in first order are not directly feeling the expansion, unless they feel it adiabatically by mediation through numerous Coulomb collisions like they do in a box with subsonic expansion of its walls. The expanding walls with an expansion velocity $\boldsymbol{V}$ should keep in touch with the particles; meaning that slow particles with particle velocities $\boldsymbol{v} < \boldsymbol{V}$ do not feel the expansion since not interacting with the moving walls, while those with velocities $\boldsymbol{v} > \boldsymbol{V}$ feel it, since their reflection velocities when coming back from the wall is reduced, i.e. $\boldsymbol{v}` < \boldsymbol{v}$.

In addition, Coulomb collisions redistributing velocities among particles have a specific property which makes things highly problematic in this context. This is because Coulomb collision cross sections are strongly dependent on the relative velocity $\boldsymbol{w}$ of the colliding particles, namely being proportional to $\boldsymbol{w}^{-4}$.[3] This evidently has the consequence that high-velocity particles are much less collision-dominated compared to low-velocity ones, they may even be considered as collision-free at supercritically large velocities $\boldsymbol{v} \geq \boldsymbol{v}_c$. So while the low-velocity branch of the distribution may still cool adiabatically as a collision-dominated gas and thus reflects cosmic expansion in an adiabatic form, the high-velocity branch in contrast behaves collision-free and hence changes in a different form. This violates the concept of a joint equilibrium temperature and of a resulting mono-Maxwellian velocity distribution function and means that there may be a critical evolutionary phase of the universe, due to different forms of cooling in the low- and high-velocity branches of the particle velocity distribution function, which do not permit the persistence of a Maxwellian equilibrium distribution at later cosmic times. We shall look into this interesting evolutionary phase trying to draw some first conclusions.

Theoretical approach

We start out from the generally accepted assumption in modern cosmology, that during the collision-dominated phase of the cosmic evolution, just before the time of matter recombination, matter and radiation are in a thermodynamic equilibrium state. In the following cosmic evolution this equilibrium, however, will experience perturbations as had already been emphasized by Fahr & Loch.[4] The following part of the paper rather shall demonstrate that, even though a Maxwellian distribution would actually prevail at the beginning of the collision-free expansion phase (i.e. just after the recombination phase when electrons and protons recombine to H-atoms and photons start propagating through cosmic space effectively without further interaction with matter), this distribution function would, nevertheless, not persist in the universe during the ongoing of the collision-free expansion. For that purpose let us first consider a collision-free particle population in an expanding Robertson-Walker universe. Hereby it is clear that due to the cosmological principle and, connected with it, the homogeneity requirement, also the velocity distribution function of the particles must be isotropic in $\boldsymbol{v}$ and independent on the local place. It thus must be of the following general form

$$f(v,t) = n(t) \cdot \overline{f}(v,t) \qquad (1)$$

Where $n(t)$ denotes the cosmologically varying density only depending on the worldtime t , and $\overline{f}(v,t)$ is the normalized, time-dependent, isotropic velocity distribution function connected with the property of a normalized function $\overline{f}$ of the magnitude $\boldsymbol{v}$ of the particle velocity according to: $\int \overline{f}(\boldsymbol{v},\boldsymbol{t}) \boldsymbol{d}^3\boldsymbol{v} = 1$.

If we take into account that particles, moving freely with their velocity $\boldsymbol{v}$ into the $\boldsymbol{v}$-associated direction over a distance $\boldsymbol{l}$, are restituting at this new place, despite the differential Hubble flow and the explicit time-dependence of $\boldsymbol{f}$, a locally prevailing co-variant, but perhaps form-invariant distribution function $\boldsymbol{f}\left(\boldsymbol{v}`,\boldsymbol{t}\right)$, then the associated functions $\boldsymbol{f}\left(\boldsymbol{v}`,\boldsymbol{t}\right)$ and $\boldsymbol{f}\left(\boldsymbol{v},\boldsymbol{t}\right)$ must be related to each other in a very specific, Liouville-like way.[5,6] Quantifying this required relation needs some special care, since particles that are moving freely in an homologously expanding Hubble universe, do in this specific case at their motions not conserve their associated phasespace volumes $\boldsymbol{d}^6\text{Ö} = \boldsymbol{d}^3\boldsymbol{v}\,\boldsymbol{d}^3\boldsymbol{x}$, since in a homologously expanding space, no particle Lagrangian $\boldsymbol{L}\left(\boldsymbol{v},\boldsymbol{x}\right)$ exists and thus no Hamiltonian canonical relations of their dynamical coordinates $\boldsymbol{v}$ and $\boldsymbol{x}$ are valid. Hence, Liouville´s theorem[7] requires that not the differential 6D-phase space volumes, but the conjugated differential phase space densities are identical, i.e.

$$\boldsymbol{f}\left(\boldsymbol{v}`,\boldsymbol{t}\right)\boldsymbol{d}^3\boldsymbol{v}`\boldsymbol{d}^3\boldsymbol{x}` = \boldsymbol{f}\left(\boldsymbol{v},\boldsymbol{t}\right)\boldsymbol{d}^3\boldsymbol{v}\,\boldsymbol{d}^3\boldsymbol{x} \qquad (2)$$

At the place where these particles arrive after passage over a distance $\boldsymbol{l}$, the particle population has a relative Hubble drift given by $\boldsymbol{v}_{\boldsymbol{H}} = \boldsymbol{l}\cdot\boldsymbol{H}$ co-aligned with $\boldsymbol{v}$, where $\boldsymbol{H} = \boldsymbol{H}(\boldsymbol{t})$ means the time-dependent, actual Hubble parameter. Thus, the original particle velocity $\boldsymbol{v}$ is locally tuned down to $\boldsymbol{v}` = \boldsymbol{v} - \boldsymbol{l}\cdot H$, since at the present place $\boldsymbol{x}´$ deplaced from the original place by the increment $\boldsymbol{l}$, all velocities have to be judged with respect to the new reference frame with a differential Hubble drift of $\boldsymbol{l}\cdot\boldsymbol{H}$. In addition, all dimensions of the space volume within a time $\text{Ä}\boldsymbol{t}$ are cosmologically expanded, so that $\boldsymbol{dx}` = \boldsymbol{dx}\left(1+\boldsymbol{H}\text{Ä}\boldsymbol{t}\right)$ holds. A complete re-incorporation into the locally valid distribution function then implies, with linearizably small quantities $\text{Ä}\boldsymbol{t} \cong \boldsymbol{l}/\boldsymbol{v}$ and $\Delta\boldsymbol{v} = -\boldsymbol{l}\cdot H$, that one can express the above requirement in the following form

$$\boldsymbol{f}\left(\boldsymbol{v}`,\boldsymbol{t}\right)\boldsymbol{d}^3\boldsymbol{v}`\boldsymbol{d}^3\boldsymbol{x} = \left(\boldsymbol{f}\left(\boldsymbol{v},\boldsymbol{t}\right) + \frac{\partial\boldsymbol{f}}{\partial\boldsymbol{t}}\Delta\boldsymbol{t} + \frac{\partial\boldsymbol{f}}{\partial\boldsymbol{v}}\Delta\boldsymbol{v}\right)\cdot(1+\left(\frac{\Delta\boldsymbol{v}}{\boldsymbol{v}}\right)^2)\left(1+\boldsymbol{H}\cdot\Delta\boldsymbol{t}\right)^3\boldsymbol{d}^3\boldsymbol{v}\,\boldsymbol{d}^3\boldsymbol{x} = \boldsymbol{f}\left(\boldsymbol{v},\boldsymbol{t}\right)\boldsymbol{d}^3\boldsymbol{v}\,\boldsymbol{d}^3\boldsymbol{x} \qquad (3)$$

This then means for terms of first order

$$\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{t}}\Delta\boldsymbol{t} + \frac{\partial\boldsymbol{f}}{\partial\boldsymbol{v}}\Delta\boldsymbol{v} + 2\cdot\frac{\Delta\boldsymbol{v}}{\boldsymbol{v}}\boldsymbol{f} + 3\boldsymbol{H}\cdot\Delta\boldsymbol{t}\cdot\boldsymbol{f} = 0 \qquad (4)$$

And consequently

$$\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{t}}\frac{\boldsymbol{l}}{\boldsymbol{v}} - \boldsymbol{l}\cdot\boldsymbol{H}\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{v}} - 2\frac{\boldsymbol{lH}}{\boldsymbol{v}}\boldsymbol{f} + 3\boldsymbol{H}\frac{\boldsymbol{l}}{\boldsymbol{v}}\boldsymbol{f} = 0 \qquad (5)$$

Finally leading to the following requirement (Requirement A)

$$\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{t}} = \boldsymbol{vH}\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{v}} - \boldsymbol{Hf} \qquad (6)$$

Starting from a Maxwell distribution $\boldsymbol{f}\left(\boldsymbol{v},\boldsymbol{t}_0\right) = \textbf{Max}\left(\boldsymbol{v},\boldsymbol{t}_0\right)$, one could try to solve the upper particle differential equation and find the solution for all later distribution functions. Here, however, we prefer to make use of a much simpler procedure: Namely looking now for the most interesting velocity moments, like density and pressure, of the function $\boldsymbol{f}$ fulfilling the above partial differential equation, - then multiplying this equation with a) $4\pi v^2 dv$ and b) $\left(4\text{ð}/3\right)\boldsymbol{mv}^4\boldsymbol{dv}$ and integrating over the whole velocity space leads to the following relations:

$$\boldsymbol{a)n} = \boldsymbol{n}_0\boldsymbol{e}^{-2\boldsymbol{H}(\boldsymbol{t}-\boldsymbol{t}_0)} \qquad (7)$$

And

$$\boldsymbol{b)P} = \boldsymbol{P}_0\boldsymbol{e}^{-4\boldsymbol{H}(\boldsymbol{t}-\boldsymbol{t}_0)} \qquad (8)$$

This then immediately makes evident that with the above solutions, one for instance finds that the particle entropy $\boldsymbol{S} = \boldsymbol{P}/\boldsymbol{n}^{\tilde{a}}$, given by

$$\frac{\boldsymbol{P}}{\boldsymbol{n}^{\tilde{a}}} = \frac{\boldsymbol{P}_0}{\boldsymbol{n}_0^{\tilde{a}}}\boldsymbol{e}^{-(4-2\tilde{a})\boldsymbol{H}(\boldsymbol{t}-\boldsymbol{t}_0)} = \frac{\boldsymbol{P}_0}{\boldsymbol{n}_0}\boldsymbol{e}^{-\frac{2}{3}\boldsymbol{H}(\boldsymbol{t}-\boldsymbol{t}_0)} \qquad (9)$$

Surprisingly enough is not constant with time $\boldsymbol{t}$, which means that in fact no adiabatic behavior of the expanding particle gas occurs, and that the gas entropy $\boldsymbol{S}$ for that reason also is not constant but is decreasing according to the following relation

$$\boldsymbol{S} = \boldsymbol{S}\left(\boldsymbol{t}\right) = \boldsymbol{S}_0\ln\left(\frac{\boldsymbol{P}}{\boldsymbol{n}^{\tilde{a}}}\right) = -\frac{2}{3}\boldsymbol{H}\left(\boldsymbol{t}-\boldsymbol{t}_0\right) \qquad (10)$$

At this point of our study, it is perhaps historically interesting to see that, when assuming the commonly used Hamilton canonical relations to be valid (i.e. $\frac{\boldsymbol{dL}}{\boldsymbol{dp}_i} = \frac{\boldsymbol{dx}_i}{\boldsymbol{dt}};-\frac{\boldsymbol{dL}}{\boldsymbol{dx}_i} = \frac{\boldsymbol{dp}_i}{\boldsymbol{dt}}$), the Liouville theorem would then instead of Requirement A simply require $\boldsymbol{f}\left(\boldsymbol{v}`,\boldsymbol{t}\right) = \boldsymbol{f}\left(\boldsymbol{v},\boldsymbol{t}\right)$ and hence would lead to the following form of a Vlasow equation (Requirement B)

$$\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{t}} - \boldsymbol{vH}\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{v}} = 0 \qquad (11)$$

In that case, the first velocity moment is found from the following relation

$$\frac{\partial\boldsymbol{n}}{\partial\boldsymbol{t}} = \int 4\text{ð}\boldsymbol{v}^3\boldsymbol{H}\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{v}}\boldsymbol{dv} = 4\text{ð}\boldsymbol{H}\int\frac{\partial(\boldsymbol{v}^3\boldsymbol{f})}{\partial\boldsymbol{v}}\boldsymbol{dv} - 12\text{ð}\boldsymbol{H}\int\boldsymbol{v}^2\boldsymbol{fdv} \qquad (12)$$

Yielding

$$\frac{\partial\boldsymbol{n}}{\partial\boldsymbol{t}} = -3\boldsymbol{nH} \qquad (13)$$

Which can easily be identified with the solution $\boldsymbol{n}\sim\boldsymbol{R}^{-3}$, i.e. a baryon density falling off inversely proportional to the cosmic volume. Looking furthermore also for the higher moment $\boldsymbol{P}$, then in this case one is lead to

$$\frac{\partial\boldsymbol{P}}{\partial\boldsymbol{t}} = \frac{4}{3}\int\boldsymbol{v}^5\boldsymbol{H}\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{v}}\boldsymbol{dv} = -5\boldsymbol{HP} \qquad (14)$$

Which now in this case shows that the actual entropy based on $\tilde{a} = 5/3$ evaluates to

$$\boldsymbol{S} = \frac{\boldsymbol{P}}{\boldsymbol{n}^{\tilde{a}}} = \frac{\boldsymbol{P}_0}{\boldsymbol{n}_0^{\tilde{a}}}\boldsymbol{e}^{-(5-3\tilde{a})\boldsymbol{H}(\boldsymbol{t}-\boldsymbol{t}_0)} = \textbf{const}.!!! \qquad (15)$$

That means in this”classic“case also an adiabatic expansion is retained, however, based on wrong assumptions!- Assumptions namely that the classical Hamilton canonical relations would be valid here which they are in fact not due to the non-conservative Hubble-forces acting, connected with the relation $\boldsymbol{dp}/\boldsymbol{dt} = \boldsymbol{pH}$.

In contrast going back to the cosmically correct Vlasow equation (Requirement A), one can then check whether or not this equation allows that an initial Maxwellian velocity distribution function persists during the ongoing collision-free expansion. Here, one finds for $\boldsymbol{f}\left(\boldsymbol{v},\boldsymbol{t}\right)\sim\textbf{Max}\left(\boldsymbol{v},\boldsymbol{t}\right)\sim\boldsymbol{n}\left(\boldsymbol{t}\right)\boldsymbol{T}\left(\boldsymbol{t}\right)^{-3/2}\boldsymbol{e}^{-\boldsymbol{mv}^2/2\boldsymbol{KT}(\boldsymbol{t})}$, $\boldsymbol{n}$ and $\boldsymbol{T}$ being time-dependent moments of $\boldsymbol{f}$, that one obtains the two relevant Vlasow derivatives in the following form:

$$\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{t}} = \boldsymbol{f}\left(\frac{\boldsymbol{d}\left(\ln\boldsymbol{n}\right)}{\boldsymbol{dt}} - \frac{3\dot{\boldsymbol{T}}}{2\boldsymbol{T}} + \frac{\boldsymbol{mv}^2}{2\boldsymbol{KT}}\frac{\dot{\boldsymbol{T}}}{\boldsymbol{T}}\right) \qquad (16)$$

And

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$$\frac{\partial f}{\partial v} = -f\frac{mv}{KT} \quad (17)$$

These two expressions then lead to the following Vlasow requirement (see Requirement A):

$$\frac{d(\ln n)}{dt} - \frac{3\dot{T}}{2T} + \frac{mv^2}{2KT}\frac{\dot{T}}{T} = -H\left(\frac{mv^2}{KT} + 1\right) \quad (18)$$

Where $\dot{T}$ denotes the time derivative of T , i.e. dT / dt .

In order to fulfill the above equation, obviously the terms with v^2 have to cancel each other, since n and T are velocity moments of f , hence independent on v . This is evidently only satisfied, if the change of the temperature with cosmic time is given by

$$T = T_0 e^{-2H(t-t_0)} \quad (19)$$

This dependence in fact is obtained when inspecting the earlier found solutions for the moment's n and P, because these solutions exactly give

$$T = \frac{P}{Kn} = \frac{P_0}{Kn_0}e^{-(4-2)H(t-t_0)} = T_0 e^{-2H(t-t_0)} \quad (20)$$

With that, the Vlasov requirement found above reduces to the following expression

$$\frac{d(\ln n)}{dt} - \frac{3\dot{T}}{2T} = -H \quad (21)$$

Which then finally leads to requirement

$$-2H - \frac{3}{2}(-2H) = -H \quad (22)$$

Making it mathematically evident that this requirement is not fulfilled, and thus meaning that consequently a Maxwellian particle distribution cannot be maintained, not even at a collision-free cosmic expansion.

Conclusion

With the above result, we are now finally lead to the statement that a correctly derived Vlasow equation for the cosmic gas particles in a Hubble universe leads to a collision-free expansion behavior that neither runs adiabatic for the cosmic gas, nor does it conserve the initially Maxwellian form of the distribution function f . Under these auspices, it can, however, also easily be demonstrated that under ongoing collisional interaction of cosmic photons with cosmic particles via Compton collisions in case of non-Maxwellian particle distributions unavoidably lead to deviations from the Planckian blackbody spectrum.[4] This makes it hard to be convinced of a pure Planck spectrum for the CMB photons at times around or just after the cosmic matter recombination. Our results now further raise the question whether or not matter and radiation as ingredients in the GRT energy-momentum tensor have to be carefully reanalyzed on the basis of their unexpected non-equilibrium behavior. This should perhaps be taken together with most recent results by Fahr & Heyl[8,9] showing that the energy density of cosmic radiation (i.e. the CMB photons) does not fall off with the reciproke of the fourth, but only with the reciproke of the third power of the scale of the universe. We shall look into that problem in an upcoming publication.

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Conflicts of interest

The authors declare that there is no conflict of interest.

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