Abstract

SQUID-magnetostrictor is the high sensitivity system capable to measure deviations of mechanical tension. This measuring system may find application in different fields but it seems to be rather promising as a gravitational waves detector. The reason is that the working magnetostrictive body serves as a receiving antenna sensitive to the deviations of the metric tensor $g_{ij}$ in the gravitational wave and converts it directly to the magnetic flux deviation detected by SQUID. Estimations of the sensitivity of such system give the value on the level of $\delta P / \delta P_{\text{grav}} = 10^{-13} P_{\text{grav}} / \sqrt{\Delta f_{\text{SQUID}}} \ (P_{\text{grav}}$ - the mechanical pressure on the magnetostrictive cylinder cross section) that corresponds to deviation of the transverse metric tensor component $\delta g_{ij} / \sqrt{\Delta f_{\text{SQUID}}}$. It may be noted that the signal in the famous LIGO experiment was on the level of $10^{-21}$. Additional advantage of the considered system is its compactness and cheapness. It permits to construct a phase array of n such detectors. Creation of phase array may enable both to determine the direction to the source of gravitational waves and to enhance the sensitivity $\sim V_{\text{SQUID}}$ times.

Keywords: SQUID-magnetostrictor system, gravitational wave, LIGO, phased array

Introduction

Registration of gravitational waves—the sensational discovery of the 2015 by Abbott BP et al. was not awarded the Nobel Prize. The main objective reason for this seems to be the low level of confirmation of the event. The second factor causing the “disbelief” may be some estimates which noted extremely low probability to find in the Universe a pair of black holes of appropriate scale. LIGO group proved the existence of the outburst by the mutually consistent record of gravitational noise on two giant gravitational antennas. Besides there were some “foggy messages” about time correlation with Gamma bursts. As the result the localization of this hypothetical pair of black holes on the sky is characterized by the wide arc with the angular area 600 square degrees, or otherwise it is not localized. No wonder that only on two detectors the fact of registration has been declared. LIGO collaboration has staked on the geometry of the metric tensor $\delta g_{ij}$ to be registered by the superconducting quantum interference device (SQUID). It is obvious that the system SQUID-magnetostrictor (Figure 1) should be much more sensitive than the Weber’s “electrometric amplifier” – piezoelectric. The main reason is that the amplifier is a sensitive device, and SQUID is a quantum one. In whole high sensitivity of the system SQUID-magnetostrictor is the consequence of unifying on one hand the high tensometric effectiveness of magnetostrictor sensor, acting on principles of the inverse magnetostrictive effect, based in its turn on the collective quantum solid-state effects, and on the other hand the high (of the quantum scale level) sensitivity of SQUID to the magnetic response of the sensor. The basic scale of the measured magnetic field induction is the quantum of magnetic field $\Phi_0 = \pi h / e \approx 2.07 \times 10^{-15} Wb$. The change of magnetic field flux through the SQUID ring by one quantum $\Phi_0$ corresponds to the one period change of the interference of the Cooper condensate wave in the SQUID working ring.

A plane-polarized gravitational wave is shown schematically in the foreground, and in the background is the registering SQUID/magnetostrictor system consisting of a magnetostrictic cylinder connected with the DC SQUID by flux transducer loops. Arrows show the regions of the trial body contraction and expansion caused by the action of the gravitational wave. The wavelength-to-antenna (cylinder) size ratio is deliberately distorted ($\lambda$ is “actually” about five orders of magnitude greater than $L$), and for obviousness, the effect of change in the trial body geometrical sizes in the gravitational wave field is also enlarged by 20 orders of magnitude.

We suggest using a magnetostrictor transducer or even making the test body from the magnetostrictor material. In order to achieve the maximum sensitivity, the magnetic response of the test body originated due to the inverse magnetostrictive effect should be registered by the superconducting quantum interference device (SQUID). It is obvious that the system SQUID-magnetostrictor (Figure 1) should be much more sensitive than the Weber’s “electrometric amplifier” – piezoelectric. The main reason is that the amplifier is a sensitive device, and SQUID is a quantum one. In whole high sensitivity of the system SQUID-magnetostrictor is the consequence of unifying on one hand the high tensometric effectiveness of magnetostrictor sensor, acting on principles of the inverse magnetostrictive effect, based in its turn on the collective quantum solid-state effects, and on the other hand the high (of the quantum scale level) sensitivity of SQUID to the magnetic response of the sensor. The basic scale of the measured magnetic field induction is the quantum of magnetic field $\Phi_0 = \pi h / e \approx 2.07 \times 10^{-15} Wb$. The change of magnetic field flux through the SQUID ring by one quantum $\Phi_0$ corresponds to the one period change of the interference of the Cooper condensate wave in the SQUID working ring.

The compact detector system SQUID/magnetostrictor

In the classical Weber work low frequency oscillations of the test body, aluminum cylinder of one meter length, arisen under the action of “tidal” forces of the gravitational wave were transformed into the low frequency electric signal by the piezoelectric transducer.
Gravitational observatory on the base of phase array of compact high-sensitive detectors SQUID/magnetostrictor

Figure 1 Qualitative correlation between the geometry of gravitational wave propagation and the magnetostrictor antenna, the resultant change in the magnetic flux of which is registered by SQUID.

Where one (RF-SQUID) or two (DC_SQUID) Josephson junctions are included. It is just the smallness of the flux quantum value corresponding to the change of the Cooper condensate phase on $2\pi$, provides the high sensitivity of SQUID, which registers flux change on the small part of $\phi_0$. The estimation of the dilatometric sensitivity of the Weber system electrometric amplifier - piezoelectric gives the value $\langle\delta\Phi / \ell\rangle_{\text{magn}} \approx 10^{-17} / \sqrt{\text{Hz}}$. The corresponding limit value for the system SQUID-magnetostrictor is $\langle\delta\Phi / \ell\rangle_{\text{SQUID-magn}} \approx 10^{-24} / \sqrt{\text{Hz}}$.

Let us see now how we can get this so high value of sensitivity for the system SQUID-magnetostrictor. High but not the record value of magnetostrictive sensitivity $\Lambda^{(-1)}$ for instance for the alloy 54%Fe-46%P is $\Lambda^{(-1)} \approx 7 \times 10^{-5} \text{T/Pa}$. The magnetostrictive sensitivity is defined as the differential ratio of the magnetic inductance change to the change of the elastic stress causing this enhance in concrete magnetostrictive material, $\Lambda^{(-1)}$ is calculated from the $\Lambda$ on the base of the reciprocity theorem.

In the limit case the registration of magnetic response $\Delta\Phi$ by the SQUID correspond to the condition $\Delta\Phi = \delta\Phi$, i.e. equality of the magnetic response to the resolving power of the SQUID or otherwise to the magnetic noise reduced to the SQUID input. The change of the magnetic inductance flux in the cross-section area $S_m$ of the magnetostrictor cylinder is $\delta\Phi = \Delta\Phi = S_m\Delta B = S_m\Lambda^{(-1)}\Delta P$.

Hence the limit sensitivity of the system SQUID-magnetostrictor to the tension or factually pressure $\Delta P$, compressing or elongating the cylinder, is expressed through the possibility of the quantum interferometer as $\delta P = \delta\Phi / (S_m\Lambda^{(-1)})$.

The possibility of the system to the tension could be transformed into the possibility of the elongation $\delta\ell / \ell = \Delta P / E = \delta\Phi / (ES_m\Lambda^{(-1)})$ through the Hook law $\Delta P = E\Delta\ell / \ell$, here $E$ - Yung module, for solid matter it is about 100 GPa.

If we use the resolution of a good modern but not the record SQUID $\langle\delta\Phi / \ell\rangle_{\text{SQUID}} \approx 10^{-6} \Phi_0 / \sqrt{\text{Hz}} = 2.07 \times 10^{-21} \text{Wb} / \sqrt{\text{Hz}}$, transformation coefficient of magnetic signal by the superconducting flux transformer $\kappa \approx 0.001$ and take into account mechanical quality factor of longitudinal deformation oscillations of cylinder $Q \approx 1000$ as the factor arising the amplitude of $\delta\ell / \ell$ we shall get

$$\langle\delta\ell / \ell\rangle_{\text{SQUID}} \approx \frac{\kappa Q}{ES_m\Lambda^{(-1)}} \frac{\delta\Phi}{\sqrt{\text{Hz}}} \approx 10^{-24} / \sqrt{\text{Hz}}.$$

In this estimation we used the cross-section area of the cylinder $S_m \approx 3 \times 10^{-3} \text{m}^2$ and magnetostrictor sensitivity $\Lambda^{(-1)} \approx 7 \times 10^{-6} \text{T} / \pi \text{Pa}$. The phase array of gravdetectors SQUID-magnetostrictor

This rather optimistic estimation does not seem absolutely unreal in the context of supposed using of the system SQUID-magnetostrictor as a part of the many element phase array, each node of which will be such detector. In this case different noises not taken into account in previous calculations will be suppressed into $\sqrt{\text{Hz}}$ times. Here $n$ is the number of detectors united in the phase array. We may note that in passive hydrolocative systems a phase array of hydrophones permit not only to determine the direction on to the source of sound, produced by the marine screw, but also due to the large number hydrophones to detect the acoustic signal with the amplitude of $1 \text{mkPa} / \sqrt{\text{Hz}}$ in conditions of one ball sea storm, which corresponds to the noise on the level $10 \text{mkPa} / \sqrt{\text{Hz}}$. The sensitivity of phase array consisting of $n=100$ gravdetectors to the variation of the metric tensor is formally evaluated on the level $\langle\delta\ell / \ell\rangle_{\text{array}} \approx 10^{-25} / \sqrt{\text{Hz}}$. It may be even more important on practice that using of such a phase array of gravitational detectors would permit to weaken “not considered noises” by an order. (By the way, realistic financial estimates of the cost of such phase array of gravitational detectors of the type SQUID-magnetostrictor give the value of about ten million dollars).

Let us consider the action of such an array specifically. The period of the nodes in the array should be of the order of gravitational wave length $\min[\text{L}_p - \text{L}_g] = \delta\lambda$. In the classical Weber experiment’ the supposed wave length $\lambda$ was 300 km, i.e. the resonance frequency of the test body-aluminum cylinder of one meter length, was of the order of 1 kHz. The system SQUID-magnetostrictor can operate both in resonant and nonresonant regimes, but in the last case the sensitivity will decrease for about 3 orders $10^{-24} / \sqrt{\text{Hz}} \rightarrow 10^{-21} / \sqrt{\text{Hz}}$, since the mechanical Q-factor decreases from 1000 down to 1. This decline of sensitivity is partially compensated, at list on the order, by the array factor $\sqrt{n}$.

For illustration let us consider the simplified particular case. The square flat phase array of $n$ nodes is on the Earth surface. The compact gravitational waves detector SQUID-magnetostrictor is placed in the each node. The incidence plane of gravitational wave is parallel to one side of the phase array (parallel to the lines of phase array) and perpendicular to another side (perpendicular to the rows of phase array). Let us consider two nodes lying in the $k$-th line and separated by the distance $|L_k - L_{k-1}|$. Then time difference between the moments of detecting of the signal with the same phase on these two notes is defined by the relation

\[ t_n - t_p = \frac{t_p - t_n}{\pi} \sin \phi, \]

Where \( \phi \) - the angle between the normal to the phase array plane and the direction to the source of the gravitational waves. We suppose that the distance to the source is many orders more than the phase array dimensions. In this case the ratio

\[ \frac{c(t_n - t_p)}{t_p - t_n} = \sin \phi. \]

For any \( 1 < i < \sqrt{n} \) and \( 1 < j < \sqrt{n} \), is to be invariable and random deviations caused by the noise of the system will be suppressed through the averaging for all possible combinations \( i \) and \( j \). The whole number of these combinations is

\[ \binom{\sqrt{n}}{2} = \frac{1}{2} \sqrt{n(n-1)} \frac{n^2}{2}. \]

And consequently, the mistake of determining of the angle \( \phi \) of the direction to the source is suppressed by \( \sqrt{n}/2 \) times. Furthermore if we have the square array with equal distances between all nearest neighboring nodes the magnitude of index \( k \) does not matter, second indexes of nodes (k) may be arbitrary and different, i.e. we may consider nodes \( ik \) and \( jl \). We must write the distance \( (L_i - L_j) \) between \( i \)-th and \( j \)-th rows in the denominator of the formula

\[ \frac{c(t_n - t_p)}{L_i - L_j} = \sin \phi. \]

It will be correct for all \( k \) and \( j \). It means that number of all possible combinations is

\[ \binom{n}{2} = \frac{n!}{2(n-2)!} = \frac{n(n-1)}{2}. \]

And hence the mistake is suppressed in \( (2\sqrt{n}) \) times. We have considered the simplified case but it may be easily generalized on the case of arbitrary orientation of the phase array with respect to the direction of the gravitational wave source.

**Conclusion**

In conclusion we want to emphasize that the realization of the possibilities of the phase array of gravitational detectors: statistical suppression of the noise of alone detector, more precise determination of the angular coordinates of the source, registration of the gravitational waves of more complicated structure, corresponding to the paired turning sources, -is capable only with compact and relatively cheap detectors such as proposed system SQUID-magnetostrictor.

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**Conflicts of interest**

Authors declare there is no conflict of interest.

**References**
