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On mathematical analysis for bianchi type–I string cosmological model in modified theory of relativity

Review Article

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Abstract

The present paper deals with a spatially homogenous Bianchi Type- I massive string cosmological model with bulk viscosity and decaying vacuum energy density. The Einstein's field equations are solved by applying a special law of variation of Hubble parameter that yields a constant deceleration parameter. Assuming the constant deceleration parameter negative and suitable form of decaying vacuum energy density, an accelerating model of the universe is derived in the presence of string scenario. The strings eventually disappear from the universe for sufficiently large-times. The physical and geometric properties of the models are also discussed.

Keywords: cosmic strings Bianchi type–I model, bulk viscosity, accelerated expansion

Introduction

The study of cosmic strings has been a subject of much interest for cosmologists. It is believed that these strings give rise to density perturbations leading to the formation of galaxies.[1] String theory is a hypothetical framework in which the point-like particles are replaced by one-dimensional objects called strings. On distance scales larger than the string scale, a string in believed to work just like an ordinary particle. In the early universe (strings dominated era), the strings produce fluctuations in the density of particles. We may speculate that as strings vanish and particles become important, then the fluctuations grow in such a way that finally we shall end up with galaxies. The presence of string in the early universe can be explained using grand unified theories (GUT) as already discussed.[2–4] These strings have stress energy and are classified as geometric and massive strings.

Letelier PS[5,6] studied a gauge on variant model of a cloud formed by geometric strings and used the model as a source of gravitational field. Stachel J[7] has developed the classical theory of geometric strings as a theory of simple surface forming time-like bivector field in an arbitrary background space-time. Letelier PS[5] has formulated the energy-momentum tensor for massive strings and explained that the massive strings are formed by geometric strings with particles attached its extension. Further, Letelier PS[8] used this idea for deriving cosmological models of Bianchi type- I and Kantowski-Sachs space-times in the presence of massive strings. Matraverse DR[9] has presented a class of exact solutions of Einstein's field equations with a two-parameter family of classical strings. Krori KD et al.[10] have obtained some exact solutions in string cosmology for homogenous spaces for Bianchi type II, VI, VIII and IX. Banerjee et al.[11] have studied Bianchi type-I string cosmological models with and without the source-free magnetic field. Tikekar R & Patel LK[12] have obtained some Bianchi type III cosmological solutions of massive strings in the presence of a magnetic field. Shri Ram & Singh JK[13] have presented some spatially homogenous type-I Bianchi massive string models with and without source free magnetic field. Chakraborty S[14] has discussed string cosmology in Bianchi type VI_o space-time. Bali R et al.[15,16] have studied spatially homogenous string cosmological models in different physical contexts. Wang has investigated a Bianchi type III massive string cosmological model with magnetic field. Pradhan A[17] has discussed anisotropic Bianchi type I magnetized string cosmological models with decaying vacuum energy density $\wedge$ (t). Saha B & Visinescu M[18] and Saha B et al.[19] have studied Bianchi Type I models with cosmic strings in the presence of magnetic flux.

In general relativity, cosmological models are usually constructed under the assumption that the matter content is an idealized prefect fluid. This assumption may be a good approximation to the actual matter content of the universe at present time. The evolution of isotropic cosmological models filled with perfect fluids has been extensively studied by many workers. It is certainly of considerable interest to study cosmologies with richer structure both geometrically and physically than the standard prefect fluid FRW models. It is of interest to take into account dissipative process such as viscosity and heat conduction in cosmological models. Misner CW[20] has studied the effect of viscosity on the evolution of cosmological models.

Several authors viz. Belinski VA & Khalatnikov IM,[21] Banerjee A & Santos NO,[22] Beesham A,[23] Bali et al.[24] Shri Ram et al.[25] have obtained exact solutions of Einstein's field equations by taking viscous effects in isotropic as well as anisotropic space-times. The recent observations of Riess AG et al.[26] and Perlmutter S[27] have led to the belief that a cosmological constant in a kind of repulsive pressure, dubbed as dark energy, and is most suitable candidate to explain the recent observations that the universe is not only expanding but also accelerating. The cosmological term $\wedge$ is also interpreted as the vacuum energy density. Cosmological models with time-dependent $\wedge$ term have been investigated so far by many authors. Recently, Bali R & Swati[28] have investigated a Bianchi type-I massive string cosmological model with bulk viscosity and vacuum energy density.

In this paper, we obtain a new spatially homogenous Bianchi type-I massive string cosmological models with bulk viscosity and vacuum energy density. The plan of the paper is as follows. In Section 2, we present the metric and field equations by assuming a special law of variation of Hubble parameter. We discuss the physical and kinematical properties of the model are Section 4. In Section 5 we outline some concluding remarks.

Metric and field equations

We consider the spatially homogenous Bianchi type-I line element given by

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \qquad (1)$$

Where the metric potentials A, B and C are functions of cosmic time t.

The energy-momentum tensor for a cloud of massive string dust with a bulk viscous fluid of string in given by Letelier PS[8] as

$$T_\mu^\upsilon = \rho\upsilon_\mu\upsilon^\nu - \lambda x_\mu x^\nu - \xi\theta\left(\delta_\mu^\upsilon + v_\mu v^\upsilon\right) \qquad (2)$$

Where υ_μ and x_μ satisfy conditions

$$\upsilon_\mu\upsilon^\mu = -x_\mu x^\mu = 1, \upsilon_\mu x^\mu = 0. \qquad (3)$$

Here ρ is the proper energy density for a could of strings with particles attached to them, λ is the string tension density, υ^μ is the four-velocity of the particles, x^μ is a unit space- like vector representing the direction of string, ξ is the coefficient of bulk viscosity and θ is the expansion scalar. If the particle density of the configuration is denoted by ρ_p, then we have

$$\rho = \rho_p + \lambda \qquad (4)$$

Einstein's field equations for a system of strings with vacuum energy density are given by

$$R_\mu^\upsilon - \frac{1}{2}R\delta_\mu^\upsilon = -T_\mu^\upsilon + \wedge\delta_\mu^\upsilon \qquad (5)$$

Where $R_{\mu\nu}$ is the Ricci tensor and R is the scalar curvature in the gravitational unit $8\pi G = 1$, c=1

We assume that the string's direction is along the x-axis, so that $x^\mu = \left(\frac{1}{A},0,0,0\right)$. In commoving coordinate system $v^\mu = \left(0,0,0,1\right)$, the field equations (5) together with (2) and (3) lead to the following system of equations.

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \lambda + \xi\theta + \wedge, \qquad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \xi\theta + \wedge, \qquad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \xi\theta + \wedge, \qquad (8)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = \rho + \wedge. \qquad (9)$$

Where an over dot denotes ordinary derivative with respect to t. For the model (1), the spatial volume (V) and average scale factor a are given by

$$V = a^3 = ABC. \qquad (10)$$

The expansion scalar (θ), shear scalar (σ), the anisotropy parameter (Am) and, Hubble parameter H are given by

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \qquad (11)$$

$$\sigma = \frac{1}{2}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B} + \frac{\dot{C}^2}{C}\right) - \frac{1}{6}\theta^2, \qquad (12)$$

$$Am = \frac{1}{3}\sum_{\mu=1}^{3}\left(\frac{H_\mu - H}{H}\right)^2, \qquad (13)$$

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right). \qquad (14)$$

Where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the direction of x, y and z-axes respectively. An important observational quantity in cosmology is the deceleration parameter (q) defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \qquad (15)$$

The sign of q indicates whether the model inflates or not. The positive value of q corresponds to the standard decelerating universe and the negative value indicates inflation.

Solutions of the field equations

We now obtain exact solutions of (6)-(9) which are four equations in seven unknowns A, B, C, ρ, λ, ξ and $\wedge$. Therefore to obtain a deterministic model, we need extra conditions depending upon the physical nature of the problem.

Subtracting (8) from (7), we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0. \qquad (16)$$

Equation (16), on integration, provides

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k}{a^3} \qquad (17)$$

Where k is an arbitrary constant. To treat (17) we consider the following relation

$$A^n = BC, \quad n \neq 0 \text{ (Constant)} \qquad (18)$$

As suggested by Goswami et al.[29] Future, we set

$$B = A^{n/2}D, \; C = A^{n/2}D^{-1} \qquad (19)$$

Where D(t) is an arbitrary function. Substituting (19) in (17) and integrating of the resulting equation we get

$$\frac{\dot{D}}{D} = \frac{K}{a^3}, \qquad (20)$$

Where $K = \frac{k}{2}$. From (10) and (18), we find that

$$V = a^3 = A^{n+1} \qquad (21)$$

We can determine the function D(t) from (20) if the average scale factor a is a known function of time. Hence to obtain an expression for the average scale factor a, we can apply the special law of variation of Hubble parameter proposed by Berman (1983) that yields constant value of the deceleration parameter. It may be noted that here most- of the well known models in general relativity and alternative theories including inflationary models are model with constant deceleration parameter. For accelerating models of the universe, we take the constant negative. Then (15) gives the solution

$$a=\left(ct+d\right)^{\frac{1}{1+q}} \tag{22}$$

Where $c\neq 0$ and d are integration constants. This equation implies that the condition of expansion is $1+q>0$.

Substituting (22) in (20) and integrating, we obtain

$$D=N\exp\left\{-\frac{K(1+q)}{c(2-q)}\left(ct+d\right)^{\frac{q-2}{1+q}}\right\} \tag{23}$$

Where N is integration constant. Without loss of generality we can take N=1. Therefore from (19), (21), (22) and (23), the solutions for the scale factor A, B and C are obtained as

$$A=\left(ct+d\right)^{\frac{3}{(n+1)(1+q)}} \tag{24}$$

$$B=\left(ct+d\right)^{\frac{3n}{2(n+1)(1+q)}}\exp\left\{-\frac{K(1+q)}{c(2-q)}(ct+d)^{\frac{q-2}{1+q}}\right\} \tag{25}$$

$$C=(ct+d)^{\frac{3n}{2(n+1)(1+q)}}\exp\left\{\frac{K(1+q)}{c(2-q)}(ct+d)^{\frac{q-2}{1+q}}\right\} \tag{26}$$

To get a deterministic model of universe, we assume that

$$\wedge=\frac{\alpha}{a^3} \tag{27}$$

Where α is a constant. Then, from (22) and (27), the vacuum energy density $\wedge$ has the value given by

$$\wedge=\frac{\alpha}{(ct+d)^{\frac{3}{1+q}}} \tag{28}$$

Physical feature of cosmological model

For the present model, we obtain

$$H_1=\frac{3c}{\left(1+q\right)\left(n+1\right)\left(ct+d\right)}, \tag{29}$$

$$H_2=\frac{3nc}{2\left(1+q\right)\left(n+1\right)\left(ct+d\right)}+\frac{K}{(ct+d)^{\frac{3}{1+q}}}, \tag{30}$$

$$H_3=\frac{3nc}{2\left(1+q\right)\left(n+1\right)\left(ct+d\right)}-\frac{K}{(ct+d)^{\frac{3}{1+q}}}, \tag{31}$$

$$H=\frac{c}{\left(1+q\right)\left(ct+d\right)}. \tag{32}$$

The expansion scalar, shear scalar and anisotropy parameter are obtained as

$$\theta=\frac{3c}{\left(1+q\right)\left(ct+d\right)}, \tag{33}$$

$$\sigma^2=\frac{9c^2\left(2n^2-2n+5\right)}{2\left(1+q\right)^2\left(n+1\right)^2\left(ct+d\right)^2}+\frac{2K^2}{(ct+d)^{6/1+q}} \tag{34}$$

$$\Delta=\frac{1}{2}\left(\frac{n-2}{n+1}\right)^2+\frac{2}{3}\frac{K^2c^2(1+q)^2}{(ct+d)^{4-2q/1+q}} \tag{35}$$

The energy density, string tensor density, particle density and bulk viscosity coefficient are given by

$$\rho=\frac{9n(n+4)c^2}{4(1+q)^2(n+1)^2(ct+d)^2}-\frac{K^2}{(ct+d)^{\frac{6}{1+q}}}-\frac{\alpha}{(ct+d)^{\frac{3}{1+q}}}, \tag{36}$$

$$\lambda=\frac{9c^2(n^2-n-2)}{2(1+q)^2(n+1)^2(ct+d)^2}+\frac{3c^2(2-n)}{2(1+q)(n+1)(ct+d)^2}-\frac{\alpha}{(ct+d)^{\frac{3}{1+q}}}, \tag{37}$$

$$\rho_p=\frac{9c^2(4+6n-n^2)}{4(1+q)^2(n+1)^2(ct+d)^2}+\frac{3c^2(n-2)c^2}{2(1+q)(n+1)(ct+d)^2}-\frac{K^2}{(ct+d)^{\frac{6}{1+q}}}, \tag{38}$$

$$\xi=\frac{3c(n^2+2n+4)}{4(1+q)(n+1)^2(ct+d)}-\frac{c(n+2)}{2(n+1)(ct+d)}+\frac{K^2(1+q)}{3c(ct+d)^{\frac{5-q}{1+q}}}-\frac{\alpha\left(1+q\right)}{(ct+d)^{\frac{2-q}{1+q}}} \tag{39}$$

We observe that the spatial volume is zero at $t=-b/a\equiv t_o$. As $t\to t_o$. The Hubble parameter, the scalar expansion and shear scalar assume infinitely large values whereas with the growth of cosmic time they decrease to null values as $t\to\infty$. At the instant $t=t_o$, p,λ,ρ_p are all infinite. Thus, the model starts evolving with a big-bang singularity at $t=t_o$. As t tends to infinity, the spatial becomes infinite and the physical and kinematical parameter all tend to zero. Therefore, the model essentially gives an empty space-time for large time. The anisotropy parameters Δ being infinite initially, tends to a constant as $t\to\infty$. This means that the anisotropy of the model is maintained throughout the passage of time. Since the deceleration parameter is negative, the present model represents an accelerating phase of the universe. The strings eventually disappear from the universe for sufficiently large time. The model is compatible with the results of recent observations on present-day universe from type Ia supernova.[26,27]

Conclusion

In this paper, a spatially homogenous and anisotropic Bianchi type I model representing massive strings with bulk viscosity and vacuum energy density has been studied. The exact solutions of the field equations have been obtained by applying a special law of variation of Hubble parameter which yields a constant value of the deceleration parameter. By assuming the negative constant deceleration parameter and a time-decaying form vacuum energy density, an accelerating

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model of the universe has been presented. The model starts evolving from a finite big-bang singularity. The anisotropy in the model is maintained throughout its evolution. The strings dominate in the early universe and eventually disappear from the universe for sufficiently large time. For sufficiently large time, the model would essentially give an empty space-time as ρ, λ and ρ_p all tend to zero as $t \to \infty$. The model presented here is physically meaningful as the associated parameters behave reasonably.

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Conflict of interest

Author declares there is no conflict of interest.

References

1. Zeldovich YB. Cosmological fluctuations produced near a singularity. *Monthly Notices of the Royal Astronomical Society.* 1980;192(4):663–667.
2. Kibble TWB. Topology of cosmic domains and strings. *Journal of Physics A: Mathematical and General.* 1976;9(8):1–13.
3. Everet AE. Cosmic strings in unified gauge theories. *Physical Review D.* 1981;24(4): 858–868.
4. Vilenkin A. Cosmic strings. *Physical Review D.* 1981;24(8).
5. Letelier PS. Clouds of strings in general relativity. *Physical Review D.* 1979;20(6):1294–1302.
6. Stachel J. Thickening the string. I. The string perfect dust. *Physical Review D.* 1980;21(8):2171–2181.
7. Stachel J. *Physical Review* D. 1980;(20):1294.
8. Letelier PS. String cosmologies. *Physical Review D.* 1983;28(10).
9. Matraverse DR. Solutions to Einstein's field equations with Kantowski–Sachs symmetry and string dust source. *General Relativity and Gravitation*. 1998;20(3):279–288.
10. Krori KD, Chaudhury T, MahantaCR, et al. Some exact solutions in string cosmology. *General Relativity and Gravitation*. 1990;22(2):123–130.
11. Banerjee A, Sanyal AK, Chakraborty SString cosmology in Bianchi I space–time. *Pramana*.1990;34(1):1–11.
12. Tikekar R, Patel LK. Some exact solutions of string cosmology in Bianchi III space–time. *General Relativity and Gravitation*. 1992;24(4):397–404.
13. Ram S, Singh JK. Some spatially homogeneous string cosmological models. *General Relativity and Gravitation*. 1995;27(11):1207–1213.
14. Chakrabotry S. *Indian Journal of Pure & Applied Physics*. 1991;29.
15. Bali R, Upadhyay RD. *Astrophysics and Space Science*. 2003;238:97.
16. Bali R, Banerjee R, Banerjee SK. Bianchi type VI_0 magnetized bulk viscous massive string cosmological model in General Relativity. *Astrophysics and Space Science*. 2008;317(1–2):21–26.
17. Pradhan A. Some Magnetized Bulk Viscous String Cosmological Models in Cylindrically Symmetric Inhomogeneous Universe with Variable Λ–Term. *Communications in Theoretical Physics*. 2011;55(2).
18. Saha B, Visinescu M. Bianchi Type–I Model with Cosmic String in the Presence of a Magnetic Field: Spinor Description. *International Journal of Theoretical Physics*. 2010;49(7):1411–1421.
19. Saha B, Rikhvitsky V, Visinescu M. Bianchi type–I string cosmological model in the presence of a magnetic flux: exact and qualitative solutions. *Central European Journal of Physics*. 2010;8:113.
20. Misner CW. The Isotropy of the Universe. *Astrophysical Journal*. 1968;151.
21. Belinski VA, Khalatnikov IM. Effect of Scalar and Vector Fields on the Nature of the Cosmological Singularity. *Soviet Journal of Experimental and Theoretical Physics*. 1973;36(4):591–810.
22. Banerjee A, Santos NO. Spatially homogeneous cosmological models. *General Relativity and Gravitation*. 1984;16(3): 217–224.
23. Beesham A. Cosmological models with a variable cosmological term and bulk viscous models. *Physical Review D*. 1993;48(8):3539.
24. Bali R, Singh JP. Bulk Viscous Bianchi Type I Cosmological Models with Time–Dependent Cosmological Term Λ. International *Journal of Theoretical Physics*. 2008;47(12):3288–3297.
25. Palcu A. The Electric Charge Assignment In $Su(4)_L$ □ $\mathbf{U(1)}_Y$ **Gauge Models.** Modern Physics Letters A. 2009;24(16):1247–1255.
26. Riess AG, Filippenko AV, Challis P, et al. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *The Astronomical Journal*. 1998;116(3):1009–1038.
27. Perlmutter S, Aldering G, Valle MD, et al. Discovery of a supernova explosion at half the age of the Universe. *Nature*. 1998;391:51–54.
28. Bali R, Bola SC. Bianchi Type I Massive String Cosmological Model with Bulk Viscosity and Vacuum Energy Density. *International Journal of Applied Mathematics and Computer Science*. 2016;3(6):211–214.
29. Goswami GK. Dewangan RN, Yadav AK. Anisotropic universe with magnetized dark energy. *Astrophysics and Space Science*. 2016;361:119.

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