Statistical Model of Nuclide Shell Structure

Abstract

This thesis, after a systematic and in-depth analysis of known nuclides, proposes a new model of nuclides’ shell structure and offers a table of the shell structures of 935 nuclides. With this theoretical approach, the thesis studies the shell combination with a bias towards the statistical analysis of nuclide structures. This thesis distinguishes between the basic models of nuclides and gives 7 criteria for nuclide binding, the maximal nucleonic number of each shell (ΔA), combination of proton and neutron (p/n) and graphs of the nuclide growth. Based on magnetic moment, it also conducts a quantitative analysis of p/n on the shell. The nuclide structure has the characteristic of a shell and on every shell the combination of proton and neutron features clear regularity. Among the 263 elements from $^1$H to $^{263}$Sg the serial number of the most outside shell in structure are 7, and nuclides $^{262}$Hf and $^{263}$Sg are respectively even A and odd A 7 shells. It is not a coincidence but a reflection of the shell structure. The thesis uses the result of a statistical analysis to confirm the existence of “the magic Number” and reveals the fact that the magic number is a reflection of p/n on nuclide shell, particularly on the outer shells. The statistical analysis reveals that the nuclide stability and its way of decay are dependent on the nucleonic combination on the most outside shell and the matching between full-filled and semi-filled p/n, thus unveiling the general law governing the stability and decay of nuclides.

Keywords: Nuclide shell structure; p/n (mass rate of proton and neutron); Criteria of nuclide binding; Graphs of nuclide growth; Table of nuclide shell structure

Introduction

In 1940s M.G. Mayer discovered that the number of protons and neutrons is 2, 8, 20, 28, 50, 82 and 126 and so on. These kinds of nuclides are stable in a special way. This characteristic is called “the magic number” law. The existence of “the magic number” indicates that the nuclide is characteristic of a shell structure. Afterwards, M.G. Mayer and J.H. D. Jensen proposed, with the nuclide independent motion as its theoretic basis, the shell structure of nuclides and as a result explained the “magic number” law [1].

The Mayer’s shell structure model solved the magic numbers of 2, 8 and 20 first by using potential energy function of nuclear central force field in the model of harmonic oscillator potential well and square potential well. Then, with the analysis of splitting of energy levels, other magic numbers are obtained.

Mayer’s shell structure is good in many ways. For an example, it successfully explains the characters of double magic-number nuclides and their near-by ones in both theory and experiment. But there are many examples showing great differences between prediction and experiment such as electric quadruple moment of baryon odd-A nuclear. To solve the problem, A.Bohr B.R Mottelson and L.J. Rainwater proposed the model of collective motion. However neither of the models gave specific from of the nuclide shell structure.

We hold that circumstances inside and outside nuclear are entirely different. Inside it is similar to a free space while outside it has a powerful nuclear force. So there is no electronic orbiting motion and no steady-state distribution inside nuclear. Therefore, the model of nucleon shell structure is most likely an approximate description of the nuclide shell structure. The model we offer in this paper is different from the thought of Mayer, A.Bohr et al and it is based on classification of basic models of nuclides and on statistical analysis of nuclides.

Statistical Characteristics of Nuclide Shell Structure

Shell structure of a nuclear is a necessary result of direct proportion between its volume and its nucleon number. The volume V of a nuclear is

$$V = \frac{4}{3} \pi r_0^3 A$$

(1)

In which radius $r_0$ ($r_0 = 1.21 \times 10^{-15}m$) is a constant obtained from experiment [2]. It is known that each nucleon has similar mass and identical volume. Suppose the nucleon inside the nuclear takes up an average space of a sphere with a radius of $r_n$, the nuclide shell structure could be composed with diameter $r_n$. Suppose the average space between shells is a sphere with diameter $r_n$, and the distance between two nearby shells is $r_n$, too, the geometric space $\Delta N_i$ of Nu.“i” shell is as follows, for the volume is directly proportional to nucleon number:

$$\Delta N_i = \left(\frac{4}{3} \pi (r_n)^3 - \frac{4}{3} \pi \left((i-1)r_n\right)^3\right) \frac{4}{3} \pi r_0^3 = i^3 - (i - 1)^3$$

(2)

In the formula (2), if “i” is 1, 2, 3, 4, 5, 6 or 7, $\Delta N_i$ must respectively be 1, 7, 19, 37, 61, 91 or 127, indicating geometric
space of shell layers in terms of nuclide numbers. To make a distinction, nuclide with "k" shells is called k-shell nuclide. For instance $^{16}_{8}$O, could be called 3-shell nuclide and its second shell has 4 nucleons.

There is nuclear force between nucleons, so they cannot be indefinitely close to each other. Except for $i=1$, no shell can be covered with the maximal number of nucleons as given in formula (2). Nucleons do not fully occupy the geometric space of the shell either. The nuclide shell structure is shown in Figure 1. Suppose the actual maximal number of nucleons contained on I shell structure is $\Delta A_i$, $\Delta A_i \leq \Delta N_i$, $\Delta A_i / \Delta N_i$ represents the ratio of nucleons' occupation of the shell space and reflects the fullness of nucleons. The proton-neutron ratio of each shell, called $p/n$ for short indicates the nucleonic combination on each shell. Interdigitational distribution of nucleons on shells.

\[ \Delta A_i / \Delta N_i < \Delta A_2 / \Delta N_2 < \Delta A_3 / \Delta N_3 < \Delta A_4 / \Delta N_4 < \Delta A_5 / \Delta N_5 < \Delta A_6 / \Delta N_6 < \Delta A_7 / \Delta N_7 < \alpha \]  

(3)

$\alpha$ being an actual number smaller than 1.

The $\Delta A_i$ may be deduced from the relationship shown in Formula (3) and the stability of corresponding nuclide. Taking even A nucleus as an example, we know that the helium (He) nucleus has stable nuclides of sphere symmetry and is often used as bullet to attack other nuclei. From this we infer that $^4$He is the even A full-filled nuclide of the second shell level and $\Delta A_2=4$, $\Delta A_2 / \Delta N_2 = 4/7 = 0.571$. For even A nuclides of the 3rd shell level, Because

\[ \Delta \alpha / \Delta N = \Delta \alpha / 19 > 0.571 \] (4)

So \( \Delta \alpha > 10.894 \). Noticing the characteristic of even integer of even A, \( \Delta \alpha \) can only be chosen from among 12, 14, 16 and 18. Since 18/19 → 1, as a matter of fact \( \Delta \alpha \) can only be chosen from 12, 14 and 16. Again, because the nucleus number of full-filled nuclides of even A of 2\textsuperscript{nd} shell level is 4, the nucleus number of full-filled nuclides of even A of 3\textsuperscript{rd} shell level can only be taken from 16, 18 and 20. Seeing that the nuclides have nucleus numbers are 18 and 20 lack high abundance stability, the nucleus number of even A full-filled nuclides of the 3\textsuperscript{rd} shell level can be none other than 16 and the corresponding nuclide is \( ^{16}\text{O}_{16} \).

\[ \Delta \alpha = 12, \Delta \alpha / \Delta N = 12 / 19 = 0.6316 \]

For full-filled nuclides of even A of the 4\textsuperscript{th} shell level, because

\[ \Delta \alpha / \Delta N = \Delta \alpha / 37 > 0.6316 \] (5)

\[ \Delta \alpha > 23.369 , \text{ so } \Delta \alpha \text{ can only be chosen from among 24, 26, 28 and 30. The corresponding full-filled nucleus numbers of even A are respectively 40, 42, 44 and 46. We notice that none of the nucleus numbers 42, 44 and 46 have nuclides of high abundance stability while A=40 has two nuclides of high abundance of stability: }^{40}\text{Ar}_{22} \text{ and }^{40}\text{Ca}_{20} \text{ and their graduations are 99.60 and 96.94. The full-filled nuclides have good stability, and from this we can judge that the full-filled nucleus number of even A of the 4}\textsuperscript{th} \text{ shell level is 40. } \Delta \alpha = 24, \Delta \alpha / \Delta N = 24 / 37 = 0.6486 . \]

For full-filled nuclides of even A of the 5\textsuperscript{th} shell,

\[ \Delta \alpha / \Delta N = \Delta \alpha / 61 > 0.6486 \] (6)

\[ \Delta \alpha > 39.56 , \text{ so } \Delta \alpha \text{ can only be chosen from among 40, 42, 44, 46, 48 and 50. Because even A nucleus } A > 40, \text{ the full-filled nucleus numbers of even A of the 5}\textsuperscript{th} \text{ shell level are respectively 80, 82, 84, 86, 88 and 90. From the analysis of the natural abundance of stable nuclides, we can infer that the full-filled nucleus number of even A of the 5}\textsuperscript{th} \text{ shell level is 88, the corresponding nuclide is }^{88}\text{Sr}_{50} \text{ and the abundance is 82.60. } \Delta \alpha = 48, \Delta \alpha / \Delta N = 48 / 61 = 0.7869 . \]

For full-filled nuclides of even A of the 6\textsuperscript{th} shell level,

\[ \Delta \alpha / \Delta N = \Delta \alpha / 91 > 0.7869 \] (7)

\[ \Delta \alpha > 71.608 , \text{ so } \Delta \alpha \text{ can only be selected from among 72, 74, 76, 78 and 80 and the corresponding full-filled nucleus numbers of even A are respectively 160, 162, 164, 166 and 168. From the ratio between } \Delta \alpha \text{ and } \Delta N , \text{ we can see that the } \Delta \alpha / \Delta N \text{ values of the 3}\textsuperscript{rd} \text{ and } 4\textsuperscript{th} \text{ shell levels are close to each other and the } \Delta \alpha / \Delta N \text{ value of the } 5\textsuperscript{th} \text{ shell level is clearly enlarged. Because } \Delta \alpha / \Delta N \text{, when } \beta < 1, \text{ the } \Delta \alpha / \Delta N \text{ and } \Delta \alpha / \Delta N \text{ cannot possibly maintain the increase rate of } \Delta \alpha / \Delta N . \text{ From the analysis of the abundance of stable nuclides, it can be determined that the full-filled nucleus number of even A of the } 6\textsuperscript{th} \text{ shell level is 160 and the corresponding nuclides are respectively }^{160}\text{Dy}_{72} \text{ and }^{160}\text{Yb}_{72} . \]

\[ \Delta \alpha = 72, \Delta \alpha / \Delta N = 72 / 91 = 0.7912 . \]

Experiments reveal that the maximal nucleus number of even A nucleus is 262, which conforms to the characteristics of even A full-filled nucleus number of the 7\textsuperscript{th} shell level. Since 262 is the biggest nucleus number of even A, it must be the number of full-filled nuclides. If \( \Delta \alpha = 262, \Delta \alpha / \Delta N = 102, \Delta \alpha / \Delta N = 102 / 127 = 0.8031 \), which fully agrees to the relationship shown in Formula 3. From this we can come to the following judgment: 262 is the full-filled nucleus number of even A of the 7\textsuperscript{th} shell level and the corresponding nuclides are respectively \( ^{262}\text{Hf}_{157} \) and \( ^{262}\text{Rh}_{157} \).

After the full-filled nucleus number of even A is determined, that of odd A is at the same time determined. Because the even A nuclei are hollow nuclei and the odd A nuclei are neutron-star nuclides, the addition of one nucleus to even A full-filled nuclei does not alter the nucleus number at various shell levels. From this we know that the full-filled nucleus numbers of odd A are respectively \( A = 1, A = 5, A = 17, A = 41, A = 89, A = 161 \) and \( A = 263 \).

The corresponding nuclides are respectively \( ^1\text{H}_{1}, ^{17}\text{O}_{8}, ^{41}\text{K}_{22}, ^{89}\text{Y}_{50}, ^{161}\text{Dy}_{66} \) and \( ^{263}\text{Sr}_{157} \). Here, the nucleus numbers 41, 89 and 161 correspond to the full-filled 4\textsuperscript{th} shell level and the corresponding nucleus numbers 106, 155 and 233. This is an important enlightenment for us to better understand the fundamental categorization of nuclides. Especially, the heaviest nuclide \( ^{263}\text{Sr}_{157} \) of the laboratory exactly fills up the position of odd A full-filled nucleus of the 7\textsuperscript{th} shell level. It provides a convincing evidence for the fundamental classification method of nuclides.

**Magnetic Moments and Combinations of Nuclei**

The nucleus number \( \Delta \alpha \) of the shell level offers a general description of the nuclei of the level. To conduct an in-depth analysis of the shell-level structure, we have to probe into the combinations of the shell level nuclei, so as to find the specific forms combination between protons and neutrons. The proton-neutron combination ratio \( p/n \) at the shell level is determined by the pairing characteristics of nuclei and the quantitative relation of nuclear magnetic moments.

A nucleus pairing is the basic condition for the formation of nuclides. The fact that nuclei have magnetic moments means that, apart from nuclear force, there also exists electromagnetic force. Experiments show that the force between nuclei is related to the included angle of the nucleus’s spin angular momentum [3].

Let’s take even A nuclei as an example. He is the nuclide of full-filled 2\textsuperscript{nd} shell level. The nucleus of the first shell level is vacant. The 2\textsuperscript{nd} level has 4 nuclei and the proton-neutron is \( p / n = 2 / 2 \). 08 is the nuclides of the full-filled 3\textsuperscript{rd} shell level. The proton-neutron ratios of the 2\textsuperscript{nd} and 3\textsuperscript{rd} levels are \( p / n = 2 / 2 \) and \( p / n = 3 / 6 = 1 / 2 \). Ca20 refers to the nuclides of the full-filled 4\textsuperscript{th} shell level and the proton-neutron ratios of the 2\textsuperscript{nd}, 3\textsuperscript{rd} and 4\textsuperscript{th} levels are \( p / n = 2 / 2 \).
The statistical model of nuclide shell structure is characterized by one-to-one pairing between protons and neutrons. This is evident in the stable combinations of proton-neutron ratios of the 2nd, 3rd, and 4th shell levels: $p_2/n_2=2/2$, $p_3/n_3=6/6$, and $p_4/n_4=12/12$. This type of combination is indicative of a balance in the number of protons and neutrons, with a ratio of 1:1. The stability of these combinations is further supported by the high abundance and stability of nuclides with these ratios.

With more shell levels and increased neutron counts, the nuclei of higher levels do not maintain one-to-one pairing between protons and neutrons. However, the proton-neutron ratio ($p/n$) can be quantitatively analyzed to determine the ratio of nucleus magnetic moments. The magnetic force of nucleons is shown in Figure 3.

**Figure 3:** Magnetic Force of Nucleons.

Magnetic moments of the same kind of nucleons attract each other when in the same direction and repel each other when in opposite directions. Those of different kinds attract each other when in the same direction and repel each other when in opposite directions.

Statistical analysis reveals that $p/n=5/7$, $p/n=10/14$, and $p/n=20/28$ are respectively a stable combination of the full-filled $p/n$ of the 3rd, 4th, and 5th shell. It is no coincidence that the ratio of nucleons in these shells is 1/1.4. It is a manifestation of the magnetic moment of protons and neutrons and also a manifestation of the strength level of the eddy field caused by proton or neutron spin. A stable shell is in a balance state of electromagnetic force and is full-filled with neutrons. For any shell, if the quantity of protons is $p_i$, the quantity of neutrons $n_i \approx 1.46 p_i$. Thus, we have the following:

$$ p_i + n_i = p_i + 1.46 p_i = \Delta i. $$

In the formula $\Delta i$, the quantity of nucleons on the i-th shell is used. If $p_i$ is figured out, the nucleonic combination on the i-th shell can be known.

For the 3rd shell

$$ p_3 + 1.46 p_3 = 12, \ p_3 = 4.88, $$

After rounding it off to an integer

$$ P_3 = 5, \ p_3/n_3 = 5/7. $$

For the 4th shell

Experiment tests show that the proton magnetic moment $\mu_p = 2.792847386 (63) \mu_N$, the neutron magnetic moment $\mu_n = -1.91304275 (45) \mu_N$, and their relative rate is

$$ \frac{\mu_p}{\mu_n} = 1.46 $$

(8)

This relative rate represents the strength level of the eddy field caused by proton or neutron spin. Nucleons on stable shells are in a state of balance of electromagnetic force. For example, the eddy fields of protons and neutrons are in mutual balance. Except such nuclides as $^{16}O$ and $^{40}Ca$, the total balance of the proton and neutron eddy fields on all shells should be maintained. Formula (8) indicates that, when the $p/n$ of all shells approaches $1/1.46$, the electromagnetic force is in balance on the whole. The nucleon number is a natural one and even-even nucleons tend to be stable. With the above characteristics in mind, we can give a semi-quantitative explanation about the combinations of $p/n$ on all shells.
\[ p_4 + 1.46\rho_4 = 24, \quad p_4 = 9.76, \]

After rounding it off to an integer
\[ p_4 = 10, \quad n_4 = 14, \quad p_4/n_4 = 10/14. \]

For the 5\textsuperscript{th} shell
\[ p_5 + 1.46\rho_5 = 48, \quad p_5 = 19.51, \]

After rounding it off to an integer
\[ p_5 = 20, \quad n_5 = 28, \quad p_5/n_5 = 20/28. \]

For the 6\textsuperscript{th} shell
\[ p_6 + 1.46\rho_6 = 72, \quad p_6 = 29.27, \]

After rounding it off to an integer
\[ p_6 = 28, \quad n_6 = 44, \quad p_6/n_6 = 28/44. \]

For the 7\textsuperscript{th} shell
\[ p_7 + 1.46\rho_7 = 102, \quad p_7 = 41.46, \]

After rounding it off to an integer
\[ p_7 = 42, \quad n_7 = 60, \quad p_7/n_7 = 42/60. \]

The calculated results are in agreement with the stable combination of protons and neutrons on full-filled shells and also with the growing graph of nuclides. If these results reflect the overall balance of nucleonic electromagnetic force on full-filled shells, the \( p/n \)'s \( p/n_4 = 2/2, p/n_5 = 6/6 \) and \( p/n = 12/12 \) reflect the single balance of the nucleonic electromagnetic force. It is a manifestation of the one-to-one pairing between protons and neutrons. And the other \( p/n \)'s \( p/n_6 = 18/30, p/n_7 = 26/46 \) and \( p/n = 44/58 \) are stable combination of dynamic balance of nucleonic electromagnetic force.

Although there is a confirmed \( \Delta A \) (maximal number of nucleons) and a stable combination of protons and neutrons on each shell, the nucleons do not remain unchanged, for they keep exchanging nucleons against the background of energy exchange with the outside world. In an instant, a quasi-full-filled shell state is formed. In general, if the combination of nucleons on the 3\textsuperscript{rd} shell in a nuclide is a full-filled one \( (p/n_3 = 6/6) \), the combination of nucleons on the 4\textsuperscript{th} shell is mostly a full-neutron one \( (p/n_4 = 10/14) \). After a pair of nucleons are exchanged, we have \( p_4/n_4 = 5/7, p_5/n_5 = 11/13 \), of which \( p_4/n_4 = 11/13 \) is a stable combination of inside quasi-filled shell (Figure 4 (a)).

If the nucleonic combination on the 4\textsuperscript{th} shell is a full-proton one \( (p/n_4 = 12/12) \), that on the 5\textsuperscript{th} shell is a full-neutron combination \( (p/n_5 = 18/30) \). The exchange of two pairs of nucleons results in the combination \( p_5/n_5 = 10/14, p_4/n_4 = 20/28 \), which is shown in Figure 4(b). The newly-formed combination may be restored to the original state after exchange of two pairs of nucleons. Similarly, such exchange may take place between the 5\textsuperscript{th} and 6\textsuperscript{th} shells and between the 6\textsuperscript{th} and 7\textsuperscript{th} shells. To sum up, nucleons on shells fluctuate and exchange between individual balance \( (e.g. \ p/n = 6/6, p/n = 12/12) \) and overall balance \( (e.g. \ p/n = 5/7, p/n = 10/14, p/n = 20/28) \), which is controlled by the relative rate between the magnetic moment of protons and that of neutrons.

**Magic Numbers and Nuclide Stability**

In the preparation of the table, no particular attention is attached to the condition of magic numbers which nevertheless do exist as a natural character of the shell structures. Let’s take \(^4\text{He} \) as an example. Its \( p/n = 2/2 \) and it is a 2-shell nucleide with full-filled structure. \(^{16}\text{O} \) is a 3-shell nucleide with full-filled shell structure and its \( \Sigma p/n = 8/6 \). \(^{40}\text{Ca} \) its \( \Sigma p/n = 20/20 \), is a 4-shell nucleide with full-filled structure. \(^{88}\text{Sr} \) its \( \Sigma p/n = 38/50 \), is a 5-shell nucleide with full-filled shell structure. Nuclides with full-filled structures are stable in character.

Another feature of stable nuclides is that the \( p/n \) of most outside shell equals. Nuclides with \( N = 20 \) have 5 kinds of stable nuclides of which 4 have their \( p/n \) equal one on the most outside shells. They are \(^{37}\text{Cl} \), \(^{38}\text{Ar} \), \(^{39}\text{K} \) and \(^{40}\text{Ca} \). Their \( p/n \)s of most outside shells are respectively \( 10/10 \), \( 11/11 \) and \( 12/12 \). Of the 5 kinds of stable nuclides with \( N = 28 \), 4 have the characteristic of \( p/n \) equaling one on the most outside shell. They are \(^{50}\text{Ni} \), \(^{51}\text{V} \), \(^{52}\text{Cr} \) and \(^{54}\text{Fe} \). Their \( p/n \)s of the most outside shells are respectively \( 5/5, 5/6, 6/6 \) and \( 7/7 \).

There are 6 kinds of stable nuclides with \( N = 50 \); \(^{56}\text{Kr} \).
It is known from Table 1 that, of all the p/n’s of full-filled shells on the 5th shell have two combinations of $p/n = 20/28$ and $p/n = 18/30$. The $p/n$’s of the most outside shells of $^{86}$Kr and $^{87}$Rb are 18/28 and 18/28, close to the $p/n$ of the 5th shell. For $^{38}$Sr, $^{39}$Y, and $^{89}$Y, its $p/n$’s are both 20/28, identical with the combinations of the 5th full-filled shell. $^{90}$Zr and $^{92}$Mo have 6-shell nuclei with their $p/n$’s on the 6th shell being 1/1 and 2/2.

Of the heavy nuclei with $N = 82$, seven are stable ones: $^{136}$Xe, $^{138}$Ba, $^{139}$La, $^{140}$Pr, $^{142}$Nd, $^{144}$Sm. They are all 6-shell nuclei and their $p/n$’s of the most outside shells are respectively 18/30, 30/20, 20/20, 20/20, 22/30, 22/30, 24/30 and 24/32.

Nuclides have shell structures and stable nuclides have stable $p/n$ on the most outside shell. But with the increase of nucleon number $A$, the filling level of neutrons grows higher and the $p/n$ of the most outside shell is smaller than one. For the 7 stable nuclides with $N = 82$, the $p/n$ value of the most outside shells changes around 1/1.5. So it is known that this characteristic is relevant to the magnetic moment of nucleons.

With $N = 126$ are 7-shell structured and there are two stable nuclides: $^{208}$Pb and $^{209}$Bi. Their $p/n$’s of the most outside shells are 18/30 and 20/28, identical with the two stable combinations of the 5th shell when it is full-filled. This shows that a nuclide may become stable when its combination of protons and neutrons of the most outside shells in consistent with the stable nucleonic combination of an inside shell.

Heavy nuclides with $Z > 84$ are unstable except $^{232}$Th, $^{235}$U, and $^{238}$U. They are 7-shell structured and their $p/n$’s of the most outside shells are respectively 26/46, 24/46 and 28/50. Except even-even nucleonic combination of the most outside shells, being identical with or close to the $p/n$’s of the 6th shell is also a prerequisite for the nuclide stability.

To sum up, it could be presumed that the stability of a nuclide is decided by $p/n$ combination on the most outside shells and on the $p/n$ filling level of each shell. The “magic number” is the reflection of this feature. Except the case that the numbers of protons or neutrons are 2, 8 or 20, other magic numbers reflect nucleonic number of unfilled shells. So magic numbers reflect the combinations of protons and neutrons of stable full-filled shells.

Decay modes of unstable nuclides are dependent on the nucleonic combinations of outside shells. The Table of Nuclide Shell Structure (See the appendix) indicates that a nuclide decays in the $(\alpha)$ way when its $p-n$ of the most outside shell is 2, 4 or 6, and it decays in the $(\beta^{-})$ way when its outside shell $n-p$ is 2, 4 or 6. This characteristic remains true after nearly 1000 unstates nuclides are tested.

Unstable nuclides decaying in the $(\alpha)$ way are characterized by the nucleonic numbers on outside shells being even numbers of 2, 4, 6, etc. Judging from the condition of forming a nuclide, we know from Figure 2(a) that pairing of protons in the abnormal (reverse) direction caused by magnetic moment is a kind of pairing style. The magnetic moment of a proton is $1.46$ times more powerful than that of a pair of a neutron and the electromagnetic force of a pair of protons is $1.46$ times more powerful than that of a pair of neutrons. We know that the pairing of protons and neutrons is an important pre-requisite for a nuclide to form. Therefore, a pair of protons is unstable which can distribute on the most outside shell for a short time. By absorbing an electron, a proton turns into a neutron, thus forming a stable nucleonic pair. So the nuclide becomes stable and this is the cause of $(\alpha)$ way of decay.

The unstable nuclides which decay in the $(\beta^{-})$ way are characterized by the even number of nucleons on the outside shells, 2, 4, 6, etc. When protons and neutrons on outside shells fail to strike individual or overall balance, superfluous neutrons pair in reverse direction caused by magnetic moment, as is shown in Figure 2(b). But the electromagnetic force between pairs of neutrons is $1.46$ greater than that between proton-neutron pairs. It is less powerful than the combination ability of proton-neutron pairs, so the neutron pair is also unstable and can only be distributed on outside shells. By the force of proton-neutron pairs in the neighboring field, one of the neutrons becomes a proton after discharging an electron. A stable nucleonic pair is formed and the nuclide is made stable. And this is the cause of $β^{-}$ way of decay.

It is surprising that No. 42 element Mo and No. 44 element Ru each have 7 stable isotopes while No. 43 element Tc between them has no stable nuclides at all. The Table of Nuclide Shell Structure tells us that element Tc could not form structure with suitable $p/n$ among the cells and its $p/n$ of the most outside shell is not one.

The $(\alpha)$ way of decay of heavy nuclides is a reflection of the evolution of the $p/n$ combination of most outside shells from unstable to stable. For instance, the outside shell $p/n$ of stable nuclide $^{209}$Bi is 20/28. As for unstable nuclide $^{213}$At, its outside shell $p/n$ is 22/30 and the product after its $(\alpha)$ way of decay is $^{205}$Bi, tending to be stable. Let’s cite another example, the stable nuclide $^{238}$U has its outside shell $p/n$ at 28/50. The unstable nuclide $^{242}$Pu has it at 30/52, tending to be stable after its decay in the $(\alpha)$ way. Thus, the conclusion is drawn that the decay mode depends on nucleonic combination of the most outside shells. Unstable nuclides which decay in the $(\beta^{-})$ way result from the imbalance of $p/n$’s between shells.

The heavy nuclide confirmed by experiment is $^{237}$Sg. It is a neutron-filled nuclide with a 7-shell filled-structure. It decays by free fission. If there exists a heavier nuclide with a super-large $N$ number, it must be 8-shell structured. It is presumed from Table 1 about the specific value $(\Delta A, \Delta N)$ of the shell space nucleons take up that the maximal number of nucleons $\Delta A$ which can be accommodated by the 8th full-filled shell should be 136 ($[(8^2 - 7^2) \times 0.81]$). If the number of protons on the 8th shell is...
equal to that of neutrons on the 7th shell, it is a pre-requisite for the stable combination of p/n’s of the 8th shell. The p/n’s of full-filled 8th shell are 60/76 and 58/78. From this we may calculate that the nucleonic number A of an even-A nuclide with 8 full-filled shells is 398.

Table 1: Table of Nuclide Shell Structure.

<table>
<thead>
<tr>
<th>Number of Shell</th>
<th>$\Delta N_i$</th>
<th>$\Delta A_i$</th>
<th>$\frac{\Delta A_i}{\Delta N_i}$</th>
<th>Structure of Full-filled Nucleon $^a$</th>
<th>$\Delta A_i$ of Even A Kind Nuclear $^b$</th>
<th>$\Delta A_i$ of Odd A Kind Nuclear $^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I (p/n)</td>
<td>II (p/n)</td>
<td>A</td>
<td>I (p/n)</td>
<td>II (p/n)</td>
<td>A</td>
</tr>
<tr>
<td>7</td>
<td>127</td>
<td>102</td>
<td>0.8031</td>
<td>44/58</td>
<td>42/60</td>
<td>262</td>
</tr>
<tr>
<td>6</td>
<td>91</td>
<td>72</td>
<td>0.7912</td>
<td>28/44</td>
<td>26/46</td>
<td>160</td>
</tr>
<tr>
<td>5</td>
<td>61</td>
<td>48</td>
<td>0.7869</td>
<td>20/28</td>
<td>18/30</td>
<td>88</td>
</tr>
<tr>
<td>4</td>
<td>37</td>
<td>24</td>
<td>0.6486</td>
<td>12/12</td>
<td>10/14</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>19</td>
<td>12</td>
<td>0.6316</td>
<td>6/6</td>
<td>5/7</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>4</td>
<td>0.5714</td>
<td>2/2</td>
<td>2/2</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

1. $\Delta N_i = i^3 - (i-1)^3$ is the geometric space of the $i^{th}$ shell indicated by the nucleonic number.
2. $\Delta A_i$ is the maximal number of nucleons contained in the $i^{th}$ shell. The determination of $\Delta A_i$ and $p/n$ is the basis for compilation of Table of Nuclide Shell Structure.
3. The combinations of "I" type belong to the category of full-filled protons while those of "II" type are of the category of full-filled neutrons.
4. The even-A nuclides are of the hollow type and are indicated with "\( \bigcirc \)" the odd-A nuclides are of the neutron-filled type and mad are indicated with "\( \bigcirc \)".

M.G. Mayer predicted the existence of Z=114 supper-heavy nuclide. At the end of last century, scientists of Joint Institute for Nuclear Research announced that they had successfully produced Z=114 nuclide, its atomic weight being 289 and its half of decay being 30 seconds which is much longer than other nearby nuclides [5]. Our theory on nuclide shell structure tells us that, if the nuclide whit Z=114 and A=287 tends to be stable, its structure should be as follows:

<table>
<thead>
<tr>
<th>$A$</th>
<th>2/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/6</td>
<td>10/14</td>
</tr>
<tr>
<td>18/30</td>
<td>26/46</td>
</tr>
<tr>
<td>42/60</td>
<td>10/14</td>
</tr>
<tr>
<td>$\Sigma$ : 114/173</td>
<td></td>
</tr>
</tbody>
</table>

### Binding Energy of a Nuclear and Characteristics of Nuclear Force

The mass average of a nuclear is lighter than the mass sum of free nucleons of the nuclear. The difference between the two is called mass loss. Take $\Delta m(Z, A)$ for an example,

$$\Delta m(Z, A) = Z m_p + (A-Z) m_n - m(Z, A)$$

(10)

In the formula, $m(Z, A)$ is the mass of the nuclide. All nuclears suffer mass loss, i.e. $\Delta m(Z, A) > 0$.

When the nuclear mass is represented by $M(Z, A)$,

$$\Delta m(Z, A) = \Delta M(Z, A) = Z (\text{H}) + (A-Z)m_n - M(Z, A)$$

(11)

In the formula (\text{H}) stands for the mass of atom hydrogen. According to the relationship between mass and energy in relativity theory, the binding energy of an atom is

$$B(Z, A) = \Delta m(Z, A)c^2$$

(12)

The nucleonic radium calculated by the mass formula is $R = 1.21 (F)$, but it is $R = 0.8 (F)$ according to the test conducted by R. Hofstadter in his experiment on electronic scattering. This proves that in the nuclide is a rim with a thickness of $t = 0.4 (F)$ [6].

The binding energy of a nuclide increases as the nucleonic number grows larger. The binding energy difference between different nuclides is great, but no regularity is discovered. Theoretically, the average binding energy of each nuclide is used to represent the level of tightness of the binding energy. The specific binding energy is

$$\epsilon(Z, A) = B(Z, A) / A$$

(13)

It represents the average work done on each nucleon when the nuclear with mass number A and electric charge number Z is fragmented into free nucleons. Graph of specific binding energy obtained from experiments is shown in Figure 5 [7].

Both theory and experiments prove that the binding energy $\epsilon$ value of an even-A nuclide with full-filled shells is relatively high at its peak value. This rule can be confirmed by working out the binding energy of the last nucleon. The significance of the last nucleon’s binding energy refers to the energy released when a free nucleon and other nucleons of the nuclear combine into a nuclide. In other words, it is the energy needed to separate a nucleon from the nuclear.

The binding energy of the last proton is

\[ S_p(Z, A) = B(Z, A) - (Z - 1, A - 1). \]  

(14)

The binding energy of the last neutron is

\[ S_n(Z, A) = B(Z, A) - B(Z, A - 1). \]  

(15)

From the value surplus in \( \Delta(Z, A) \) Table of Nuclide Shell Structure, the definition of \( \Delta(Z, A) \) and Formula

\[ \Delta(Z, A) = [M(Z, A) - A]c^2, \]  

(16)

Can be used to work out the binding energy of a nuclide and that of the last nucleon. \( ^{16}_8 \text{O} \) is a 3-shelled full-filled nuclide of "category. From the definition of \( S_p, S_n \) we can work out the following:

\[ S_p(16_8 \text{O}) = 12.12\text{MeV}, S_n(16_8 \text{O}) = 15.66\text{MeV} \]

\[ S_p(17_8 \text{F}) = 0.61\text{MeV}, S_n(17_8 \text{O}) = 4.15\text{MeV} \]

Table 2: Binding Energy of Isotopes on Full-filled Shells.

<table>
<thead>
<tr>
<th>Nuclides</th>
<th>B(ZA)(MeV)</th>
<th>( S_p(\text{MeV}) )</th>
<th>Nuclides</th>
<th>B(ZA)(MeV)</th>
<th>( S_n(\text{MeV}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{14}_{8} \text{O} )</td>
<td>98.73</td>
<td></td>
<td>( ^{16}_{8} \text{Y} )</td>
<td>742.87</td>
<td></td>
</tr>
<tr>
<td>( ^{15}_{8} \text{O} )</td>
<td>111.96</td>
<td>13.23</td>
<td>( ^{17}_{8} \text{Y} )</td>
<td>754.72</td>
<td>11.85**</td>
</tr>
<tr>
<td>( ^{16}_{8} \text{O} )</td>
<td>127.62</td>
<td>15.66*</td>
<td>( ^{88}_{38} \text{Sr} )</td>
<td>764.07</td>
<td>9.35</td>
</tr>
</tbody>
</table>

*The nuclide \( S_n \) with full-filled shells in high in binding energy.

**Such nuclides have protons with high filling level and are strong in electromagnetic force and high in binding energy. With the increase of the filling level of protons, the binding energy decreases.
<table>
<thead>
<tr>
<th>Nuclide</th>
<th>Z</th>
<th>N</th>
<th>Energy (MeV)</th>
<th>$\Delta A$</th>
<th>$\Delta N$</th>
<th>$\frac{\Delta A}{\Delta N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{17}_{8}$O</td>
<td>8</td>
<td>9</td>
<td>131.77</td>
<td>4.15</td>
<td>$^{9}_{39}$Y</td>
<td>775.54</td>
</tr>
<tr>
<td>$^{18}_{8}$O</td>
<td>8</td>
<td>10</td>
<td>139.81</td>
<td>8.04</td>
<td>$^{90}_{39}$Y</td>
<td>782.40</td>
</tr>
<tr>
<td>$^{19}_{8}$O</td>
<td>8</td>
<td>11</td>
<td>143.77</td>
<td>3.96</td>
<td>$^{91}_{39}$Y</td>
<td>790.34</td>
</tr>
<tr>
<td>$^{20}_{8}$O</td>
<td>8</td>
<td>12</td>
<td>154.37</td>
<td>7.60</td>
<td>$^{92}_{39}$Y</td>
<td>796.88</td>
</tr>
<tr>
<td>$^{38}_{20}$Ca</td>
<td>20</td>
<td>18</td>
<td>313.13</td>
<td></td>
<td>$^{157}_{66}$Dy</td>
<td>1285.00</td>
</tr>
<tr>
<td>$^{39}_{20}$Ca</td>
<td>20</td>
<td>19</td>
<td>362.42</td>
<td>13.29</td>
<td>$^{158}_{66}$Dy</td>
<td>1294.06</td>
</tr>
<tr>
<td>$^{40}_{20}$Ca</td>
<td>20</td>
<td>20</td>
<td>342.06</td>
<td>15.64*</td>
<td>$^{159}_{66}$Dy</td>
<td>1300.89</td>
</tr>
<tr>
<td>$^{41}_{20}$Ca</td>
<td>20</td>
<td>21</td>
<td>350.32</td>
<td>8.26</td>
<td>$^{160}_{66}$Dy</td>
<td>1309.47</td>
</tr>
<tr>
<td>$^{42}_{20}$Ca</td>
<td>20</td>
<td>22</td>
<td>361.90</td>
<td>11.58*</td>
<td>$^{161}_{66}$Dy</td>
<td>1315.92</td>
</tr>
<tr>
<td>$^{43}_{20}$Ca</td>
<td>20</td>
<td>23</td>
<td>369.83</td>
<td>7.93</td>
<td>$^{162}_{66}$Dy</td>
<td>1324.12</td>
</tr>
<tr>
<td>$^{44}_{20}$Ca</td>
<td>20</td>
<td>24</td>
<td>380.96</td>
<td>11.13*</td>
<td>$^{163}_{66}$Dy</td>
<td>1330.39</td>
</tr>
<tr>
<td>$^{83}_{38}$Sr</td>
<td>38</td>
<td>45</td>
<td>716.86</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{84}_{38}$Sr</td>
<td>38</td>
<td>46</td>
<td>728.91</td>
<td>12.05**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{85}_{38}$Sr</td>
<td>38</td>
<td>47</td>
<td>737.44</td>
<td>8.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{86}_{38}$Sr</td>
<td>38</td>
<td>48</td>
<td>748.92</td>
<td>11.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{87}_{38}$Sr</td>
<td>38</td>
<td>49</td>
<td>757.44</td>
<td>8.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{88}_{38}$Sr</td>
<td>38</td>
<td>50</td>
<td>768.47</td>
<td>11.03*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{89}_{38}$Sr</td>
<td>38</td>
<td>51</td>
<td>774.83</td>
<td>6.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{90}_{38}$Sr</td>
<td>38</td>
<td>52</td>
<td>782.63</td>
<td>7.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{91}_{38}$Sr</td>
<td>38</td>
<td>53</td>
<td>788.44</td>
<td>5.81</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Criteria of Nuclide Shell Structure and the Graph of Its Growth**

Table 1 shows that the ratio $\Delta A / \Delta N$ is between 0.571 and 0.803 and increases as the number of shells becomes larger, which is consistent with the fact that the distance between nearby nucleons decreases as the radius of curvature becomes larger.

The full-filled shell nuclide refers to the nuclide, each of whose shells has been filled with $\Delta A$. Only after the inside shells are fully filled, will the outside ones begin to full fill. Therefore, the unfull-filled shells only refer to those outside ones whose proton is smaller than $\Delta A$. 

The statistical analysis shows that, except for $^{40}$Ar in the stable nuclides of the $2^{\text{nd}}$, $3^{\text{rd}}$ and $4^{\text{th}}$ shells, the $p/n$ is 1 and the protons and neutrons are very likely to pair with each other. If the shells are naturally stable, the $p/n$ of most outside shells is 1.

When the 5$^{\text{th}}$, 6$^{\text{th}}$ and 7$^{\text{th}}$ shells are full-filled, the $p/n$'s of the most of their outside shells are 1, showing a big regularity. So we can make out the shell structure table of all the nuclides with the principle of the table, their pairing characteristic and the combining criteria shown in the nuclide structure. The major criteria of nuclide combination are the following 7.

1) The proton cannot occupy the first shell of a nuclide except for element H.

2) Every shell of a nuclide is filled with nucleons of even number except the first shell which is either unfilled or filled with a neutron. The nuclide with an unfilled first shell is called hollow nuclide symbolized by "$\circ$". The nuclide with a filled first shell is called neutron-filled nuclide symbolized by "$\circ$". They are two basic kinds of nuclides. The even $A$ nuclides are "$\circ$" kind and the odd $A$ nuclides belong to "$\circ$" kind.

3) For nucleons show the characteristic of pairing with each, nucleonic number of the most outside shells is even with the exception of element H. The $p/n$ of the most outside shells of stable nuclides is most likely to be 1.

4) For any nuclides with $k$ shells, there are only 2 kinds of combinations of $p/n$ in full-filled shells except for the $2^{\text{nd}}$ shell, as is shown in the following:

- $2^{\text{nd}}$ shell: $p/n = 2/2$, and if $k > 2$, then $p_2 = 2, n_2 = 2$;
- $3^{\text{rd}}$ shell: $p/n = 6/6, p/n = 5/7$; and if $k > 3$, then $p_3 \leq 6, n_3 \leq 7$;
- $4^{\text{th}}$ shell: $p/n = 12/12, p/n = 10/14$; and if $k > 4$, then $p_4 \leq 12, n_4 \leq 14$;
- $5^{\text{th}}$ shell: $p/n = 22/28, p/n = 18/30$; and if $k > 5$, then $p_5 \leq 20, n_5 \leq 30$;
- $6^{\text{th}}$ shell: $p/n = 28/44, p/n = 26/46$; and if $k > 6$, then $p_6 \leq 28, n_6 \leq 46$;
- $7^{\text{th}}$ shell: $p/n = 44/58, p/n = 42/60$; and if $k > 7$, then $p_7 \leq 44, n_7 \leq 60$.

5) If the $p/n$ of the most outside shell is not 1, generally $|p-n| = 2$. For the nuclide of $|p-n| \neq 2$, the $p/n$ is an even number.

6) The mode of nuclide decay depends on the nucleonic combination of its most outside shell.

7) The stability of a nuclide depends on the $p/n$ relationship between shells. Generally, the filling level of the $p/n$ of each shell of a stable nuclide is invariably 1, but its shells are full-filled or unfilled-filled alternatively in the II kind of nuclides.

To determine values of $\Delta A$ and the $p/n$ combinations is the principal basis for the preparation of the nuclide shell structure table. The regularity shown in $\Delta A$ and $p/n$ is embedded in the graph of nuclide growth. The developing route of full-filled nuclides is shown in Figure 6(a) & (b). Figure 6(a) is the route of the development of even $A$ full-filled nuclides and Figure 6(b) is that of odd $A$ full-filled nuclides. The 1$^{\text{st}}$, 2$^{\text{nd}}$, 3$^{\text{rd}}$ and 4$^{\text{th}}$ shells are the same as the shell structure proposed by Mayer, but the 5$^{\text{th}}$, 6$^{\text{th}}$ and 7$^{\text{th}}$ shells are obviously different in the number of nucleons [8].

Figure 6(a): Developing Route of Even A nuclides.

Even $A$ nuclide belongs to "$\circ$" category and its first shell is vacant. With the second shell full-filled, its stable nuclide is $^{2}$He; with the third shell full-filled, its stable nuclide is $^{4}$O; with the fourth shell full-filled, its stable nuclides are $^{40}$Ar$^+_2$ and $^{40}$Ca$^+_2$; with the fifth shell full-filled, its stable nuclide is $^{38}$Sr$^+_2$; with the sixth shell full-filled, its stable nuclides are $^{160}$Dy$^+_2$ and $^{160}$Ho$^+_2$; with the seventh shell full-filled, its stable nuclides are $^{202}$Hg$^+_2$ and $^{202}$Tl$^+_2$. Bh$^+_2$ are exactly the nuclides of the seventh shell. This figure derives from Table 1.

Any nuclide is first of all categorized according to the nature of nucleon $A$, i.e. whether it is odd or even in number, and then it is filled with nucleons one shell after another from inside to outside.
The $p/n$ of each shell is determined by the afore-mentioned 7 criteria. For an example, $^{35}_{17}$Cl, is an odd $A$ nuclide belonging to "O" kind. So its first shell is filled with one neutron and its $p/n$'s on the 2nd, 3rd, 4th and the most outside shells are respectively 2/2, 6/6, 9/9 and 1.1. It is therefore identified as a stable nuclide.

Figure 6(b): Developing Route of Odd $A$ nuclides.

Odd $A$ nuclide belongs to "O" category. Except for Element H, the first shell is invariably filled with neutrons. With the second shell full-filled, its nuclide $^{5}_{2}$He, is unstable; with the third shell full-filled, it is the isotope $^{17}_{8}$O, of the lowly full-filled O; with the fourth shell full-filled, the nuclides include stable nuclide $^{39}_{19}$K and unstable nuclide $^{41}_{20}$Ca; with the fifth shell full-filled, the nuclide $^{89}_{39}$Y, is stable; with the sixth shell full-filled, the nuclide $^{166}_{66}$Dy, is stable. $^{260}_{106}$Sr, is exactly the odd $A$ nuclide of the full-filled seventh shell. The figure derives from Table 1.

Nuclide shell structures may either be directly indicated or shown in a table. For instance, the shell structures of $^{17}_{8}$O and $^{238}_{92}$U are illustrated as follows:

$$
\begin{array}{c|c|c|c|c|c|c}
1 & 2/2 & 6/6 & \Sigma & 8/9
\end{array}
$$

$$
\begin{array}{c|c|c|c|c|c|c}
8 & 92 \text{Sr} & 1 & 2/2 & 6/6 & 10/14 & 20/28 & \Sigma & 38/56
\end{array}
$$

$$
\begin{array}{c|c|c|c|c|c|c}
238 & 92 \text{U} & 1 & 2/2 & 6/6 & 10/14 & 20/28 & 26/46 & 28/50 & \Sigma & 92/146
\end{array}
$$

Shell structures of stable nuclides are illustrated in Table II which is prepared in accordance with the afore-mentioned criteria. All tile stable nuclides are included and special nuclides are marked with an asterisk "*". The clear regularity shown in the shell structures of stable nuclides is the basis for the preparation of this table [9].

The Table of Nuclide Shell Structure is completed on the basis of The Table of Shell Structures of Stable Nuclide, giving consideration to the stability and decay modes of nuclides and even to the above-mentioned 7 criteria. Consideration should be given to matching between full-level and unfull level of $p/n$'s between shells and to decay modes of unstable nuclides in the combination of nucleons on the most outside shells. The Table of Nuclide Shell Structures prepared in this way can very well explain and predict the stability of nuclides and the decay patterns of unstable nuclides. Please refer to the Appendix for the shell structures, with special nuclides marked with asterisks. Tables of Shell Structure of Stable Nuclides are included in Appendix One. Tables of Shell Structure of Nuclides are included in Appendix Two.

Conclusion

We've arrived at the following conclusions after statistics and analysis of nuclides. The known highest position of nuclides is a structure of 7 shell levels and the structure is composed with the nucleus ratios $r0$ as its unit. Nuclides are categorized into two: odd and even nuclides. Except for hydrogen nucleides, even $A$ are hollow and odd $A$ are neutron-star type. The statistic model based on the fundamental categorization of nuclides and the tables of shell structure of nuclides prepared on the basis of the model reveal the general law governing the stability and decay of nuclides. This law is both the effect and a proof of the fundamental categorization method of nuclides.

Acknowledgement

None.

Conflict of Interest

Author declares there is no conflict of interest.

References


6. Hofstadter R (1960) Conducted the experiment on electronic scattering of high energy and discovered that the average radius of charged particles is $0.8$ (F) and the rim thickness is $t \approx 0.4$ (F). History and Status Quo of the Atomic Theory, Beijing University press, Beijing, China, pp. 220.

