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Potts Model With q=3 States on Directed Erdös-Rènyi Random Graphs

Abstract

We study the behavior critical of the Potts model with 3 states on Erdös-Rènyí random graphs using Monte Carlo simulations. Our results show that this present a first-ordem phase transition. This result is different of the Potts model with 3 states on a square lattice that present a second-oredem phase transistion.

Keywords: Potts; Networks; Spins

Research Article

Volume 1 Issue 6 - 2017

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Received: November 29, 2017 | **Published:** December 29, 2017

Introduction

An Erdös–Rényi (ER) random graph is a set of N vertices (sites) connected by B links (bonds) [1,2]. The probability p that a given pair of sites is connected by a bond $p = 2B / N (N-1)$ The connectivity of a site is defined as the total number of bonds connected to it, that is $k_i = \sum_j l_{ij}$, where l_{ij} if there is a link between the sites i and j and $l_{ij}=1$ otherwise. Random graphs are completely characterized by the mean number of bonds per site, or the average connectivity $z = p (N-1)$. In the limit $N \to \infty$, the distribution of connectivities is given by the Poisson distribution.

It is known from the literature that the Ising and Potts models in two-dimension (d=2) present phase transitionat finite temperature T, for any number of states q. However in d=2 there are a second-order phase transition and a first-order transition for $q \leq 4$ and $q \geq 5$, respectivelly [3].

Silva et al. [4] have studied through Monte Carlo simulations a two-dimensional Potts models with q=3 and q=4 states on a directed small-world network. From this study they found both, a first-order and second order phase transition for q=3 depending on the rewiring probability p. Otherwise, for q=4 the system shows only a first-order phase transition for any value of a rewiring probability p.

In this paper we return to this subject considering the Potts model Erdös-Rènyí random graphs with q=3 states. On this system we perform a set of Monte Carlo simulations using the spin-flip heat bath algorithm to update the spins.

Model and Simulations: Potts Model on ER Graphs

The time evolution of the system is given by a single spin-flip like dynamics [5] with a probability P_i described by

$$P_i = 1/\left[1+\exp\left(2E_i/K_{BT}\right)\right] \tag{1}$$

Where T is the temperature, K_B is the Boltzmann constant, and E_i is the energy of the configuration obtained from the Hamiltonian

$$H = -J\sum_{i=1}^{N} E_i, \tag{2}$$

With

$$E_i = \sum_{j=1}^{k} \delta_{s_i s_j} \tag{3}$$

Where the sum is carried out over the k neighbours l of site i (including the nearest-neighbor and the long-range connections determined by the probability p). In the above equation J is the exchange coupling.

The simulations have been performed on different (ER) random graph comprising a number N=1000, 2000, 4000, 8000, 1600, 32000 and 64000 of sites. For each system size quenched averages over the connectivity disorder are approximated by averaging over independent realizations. For eachsimulation we have started with a uniform configuration of spins. We ran $4x10^5$ Monte Carlo steps (MCS) per spin with $2x10^5$ configurations discarded for thermalization using the “perfect” random-number generator [3]. We do not see any significant change by increasing the number of replicas (R) (for example R=50) and MCS. So, we keep these values constant once they seem to give reasonable results for all simulations.

Results and Discussions

Here, we have employed the heat bath algorithm [3] and for every MCS, the energy per spin, e=E/N, and the magnetization per spin, m=M/N with M= (q. Max [n_i]-N)/(q-1), were evaluated. Where $n_i \leq N$ denotes the number of spins with “orientation” *i=1,...,q*.

From the energy measurements we can compute the average energy, specific heat, and also the fourth-order Binder cumulant of the energy, given respectively by

$$u(T) = \left[\langle e\rangle\right]_{av}/N, \tag{4}$$

$$C(T) = \frac{N}{T^2}\left(\left[\langle e^2\rangle\right]_{av} - \left[\langle e\rangle^2\right]_{av}\right), \tag{5}$$

$$B_e(T)=1-\frac{\langle e^4\rangle_{av}}{3\left[\langle e\rangle\right]_{av}^2}. \tag{6}$$

In the above equations $<\ldots>$ stands for thermodynamic averages and $[\ldots]_{av}$ for averages over different realizations. Similarly, we can derive from the magnetization measurements the average magnetization, the susceptibility, and the fourth-order magnetic cumulant,

$$m(T)=\left[\langle|m|\rangle\right]_{av}, \tag{7}$$

$$\chi(\mathrm{T})=\frac{\mathrm{N}}{\mathrm{T}}\left(\left[\langle m^2\rangle\right]_{av}-\left[\langle m\rangle^2\right]_{av}\right), \tag{8}$$

$$U_4(T)=1-\frac{\langle m^4\rangle_{av}}{3\left[\langle m\rangle\right]_{av}^2}, \tag{9}$$

A more quantitative analysis can be carried out through the FSS of the specific heat C_{max} , the susceptibility maxima $\chi_{\max}$ and the minima of the Binder parameter $B_{i,min}$. If the hypothesis of a first-order phase transition is correct, we should then expect, for large system sizes, an asymptotic FSS behavior of the form [6,7],

$$C_{max}=a_C+b_C N+\ldots \tag{10}$$

$$\chi_{\max}=a_\chi+b_\chi N+\ldots \tag{11}$$

$$B_{i,min}=a_{B_i}+b_{B_i}/N+\ldots \tag{12}$$

The $B_e(T)$ (equation (6)) also known as the Binder parameter, gives a qualitative as well as a quantitative description of the order of the transition [8]. It is known that this param-eter takes a minimum value $B_{i,m}$ at the effective transition temperature Tc(N). One can show that for asecond-order transition $\lim_{N\to\infty}\left(2/3-B_{i,m}\right)=0$, even at Tc, while at a first-order transition the same limit measures the latent heat $|e_+-e_-|$, i.e., $\lim_{N\to\infty}\left(2/3-B_{i,m}\right)\neq 0.$

Figure 1 shows the probability density function (PDF) of the order parameter. From this PDF, one can see that the phase transition is of is discontinuous or first-order for q=3 and N=8000 sites.

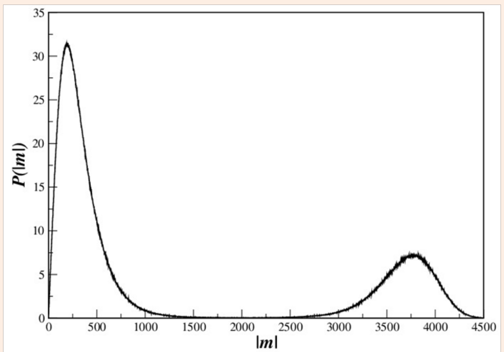

Figure 1: PDF of $|m|$ for q=3 and N=8000 sites. The double peak in the magnetization distribution indicates that the transition is of the first order.

In the Figure 2, we plot the magnetization and energy versus temperature. Both magnetization and energy show a discontinuity near the critical point indicating that the system presents a first-order phase transition.

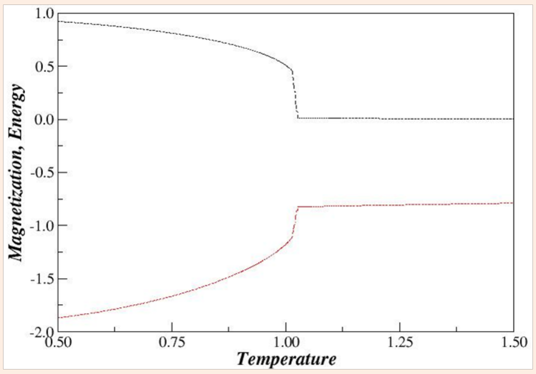

Figure 2: Magnetization and energy versus temperature for N=64000 sites.

As depicted in Figure 3, our results for the scaling of the specific heat and susceptibility are consistent with equations (10, 11). In Figure 4 we show the scaling of the Binder parameter minima, and again the first order phase transition is verified. Next, we calculate the energetic fourth- order Binder parameter and estimate the critical temperatures. For the largest N, we obtain $T_c=1.0168(3)$.

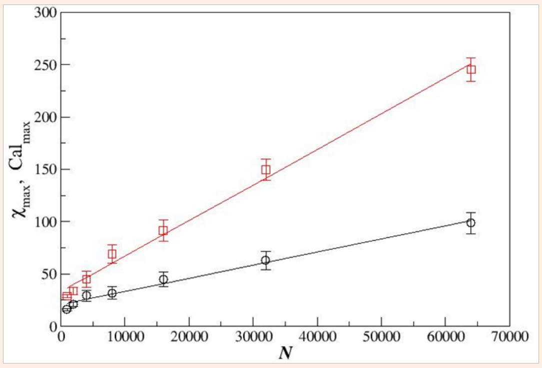

Figure 3: Plot of the susceptibility and heat at its maximum value versus N=1000, 2000, 4000, 8000, 1600, 32000 and 64000 sites.

The Figure 4 display $\left(2/3-B_{i,m}\right)$ versus $1/N$, where see that $\lim_{N\to\infty}\left(2/3-B_{i,m}\right)\neq 0$. Here, $\lim_{N\to\infty}\left(2/3-B_{i,m}\right)\approx 0.005$. Confirming that there is a first-order phase transition.

Conclusion

In the present work, we have shown that, by considering the three-states ferromagnetic Potts model on

ER random graphs there are a phase trasition. Different from the Potts model with q=3 on square lattice that presents a second-order phase transition, here, we show that this same model on ER random graphs presents a first-order phase transition. Therefore, our results agree with the Harris-Luck criterion for ER random graphs.

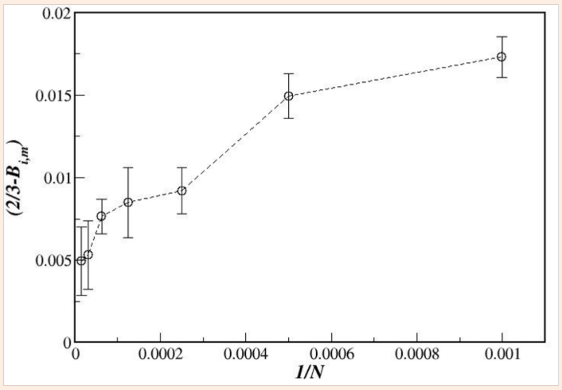

Figure 4: Plot of $(2/3 - B_{i,m})$ versus $1/N$ with N=1000, 2000, 4000, 8000, 1600, 32000 and 64000 sites.

Acknowledgement

The author would like to thank the Brazilian agencies CNPq and Capes.

Conflict of Interest

The Author states that there is no conflict of interest.

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