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Cosmological parametrizations and their scalar field descendants

Abstract

In literature, there exist numerous cosmological solutions based upon some specific scheme of parametrization of cosmological parameters. Our present work is an attempt to reconstruct the field potentials in case of (non)phantom fields for different models resulting from parametrization of $H(t)$, $H(t)$ and $a(t)$ in the framework of Friedmann Robertson Walker (FRW) geometry. In addition we carry out similar procedure to reconstruct the field potentials for tachyonic field for the same models. In this note, we reconstructed the field potentials for some known models e.g. constant deceleration parameter model, linearly varying deceleration parameter model and a model based on a specific parametrization of Hubble parameter. The procedure adopted here shows that in principle, the scalar field potentials for quintessence, phantom and tachyonic fields can be reconstructed for any scheme of parametrization of cosmological parameters $a(t)$, $q(t)$, $w(t)$, $w(t)$, $p(t)$ or $p(t)$.

Keywords: scalar field potential; parametrization; dark energy

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Introduction

The revolution in observational cosmology during the past two decades has provided sufficient evidence for late time acceleration of the Universe.[1–8] This phenomenon can be explained in several ways such as by incorporation of an extra term in the right hand side of Einstein's field equations or by modifying the left hand side of the field equations. In general relativity the concept of dark energy seems to be more relevant to the observed accelerated expansion of the Universe. In this framework, dark energy constitutes nearly 69% of the total energy budget of the Universe along with other components - dark matter (27%) and the baryonic matter (4%) (Plank 2015 results). However, important questions concerning the nature of dark energy, its interaction with other material components in the Universe, yet remain to be answered. A large number of candidates for dark energy including cosmological constant have been proposed in the recent years.[9–16] Phenomenologically quintessence field[17–20] with standard kinetic term and minimally coupled to gravity can be considered as a very good candidate for dark energy. In slow roll approximation (potential dominated scalar field i.e. $H(t)$), it can also act as a cosmological constant. The scalar field with the wrong sign in the kinetic term, dubbed phantom[21–27] is also allowed observationally. There are other scalar field models relevant to dark energy namely, quintom,[28–31] k-essense,[32,33] tachyon[34–39] light mass Galileons,[40–45] chameleon[46–48] etc. There is plethora of field potentials that can describe the smooth transition from deceleration to acceleration. In this context, various canonical as well as non-canonical scalar field potentials (e.g. exponential potential, flat potential, linear potential, quadratic potential etc.) for different fields have been proposed that can lead to different theoretical and observational consequences.

On the other hand, the inclusion of one more component (dark energy) into the evolution equations in the form of scalar field adds an extra degree of freedom. And for a unique solution, one requires a constrain equation. This can be achieved, in particular, by parameterizing the deceleration parameter $H(t)$, Hubble parameter $H(t)$, the equation of state parameter $a(t)$ or the scale factor $a(t)$ (for a recent review on various parametrization one can see[49]). An interesting article[50] can be found in literature wherein tachyonic potential is reconstructed on the FRW brane. There are other reconstructions of scalar field potentials describing the late-time acceleration of the Universe e.g. reconstructions of scalar field potential to unify early-time and late-time Universe based on phantom cosmology,[51,52] reconstruction of scalar field potential in light of supernovae data,[53] reconstruction of phantom scalar potentials in two-field cosmological models,[54] holographic reconstruction of scalar field dark energy models[55] and many more. In this paper, following the recommendation of,[50,51] we reconstruct the scalar field potentials for models obtained by various parametrization of $q(t)$, $a(t)$ or φ in case of quintessence, phantom and tachyonic fields.

Scalar field potentials for quintessence and phantom field

We consider an action describing a general scalar field ϕ as

$$S=\int d^4x\sqrt{-g}\left\{\frac{M_p^2}{2}R-\frac{1}{2}\omega\partial_\mu\phi\partial^\mu\phi-V(\phi)+L_{Matter}\right\}, \tag{1}$$

Where $\omega=+1$ or $V(\phi)$ for quintessence and phantom field respectively and $V(\phi)$ is the potential function for the scalar field. In the flat FRW background the energy density ρ_ϕ and pressure p_ϕ of the scalar field can be written as

$$\rho_\phi=\frac{1}{2}\omega\dot{\phi}^2+V(\phi), \tag{2}$$

$$p_\phi=\frac{1}{2}\omega\dot{\phi}^2-V(\phi). \tag{3}$$

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From equations (2) and (3) we may obtain

$$V(\phi)=\frac{1}{2}\left(\rho_\phi-p_\phi\right) \quad (4)$$

And

$$\omega\phi(t)=\int\left(\rho_\phi+p_\phi\right)^{\frac{1}{2}}dt+\phi_i, \text{where } \phi_i \text{ is a constant of integration.} \quad (5)$$

The effective energy density and pressure can be written as

$$\rho_{eff}=\rho_\phi+\Sigma\rho_i \text{ and } p_{eff}=p_\phi+\Sigma p_i, \quad (6)$$

Where p_i and p_i are the energy densities and pressures of all relativistic and non-relativistic components of the Universe. Using the perfect fluid equation of state $p_i=w_i\rho_i$ ($0\leqslant w_i\leqslant 1$) for the matter fields and substituting (6) in (4) and (5), we may obtain the expressions

$$V(\phi)=\frac{1}{2}\left[\left(1-w_{eff}\right)\rho_{eff}-\Sigma\left(1-w_i\right)\rho_i\right] \quad (7)$$

And

$$\omega\phi(t)=\int\left[\left(1+w_{eff}\right)\rho_{eff}-\Sigma\left(1+w_i\right)\rho_i\right]^{\frac{1}{2}}dt+\phi_i. \quad (8)$$

Where $w_{eff}=\frac{p_{eff}}{\rho_{eff}}$ is the effective equation of state parameter. For flat ($k=0$) case, Friedmann equations reduce to

$$\rho_{eff}=3M_p^2H^2, \quad (9)$$

$$p_{eff}=-M_p^2\left(3H^2+2\dot{H}\right). \quad (10)$$

Observations suggest that the dominant constituents in the Universe are dark energy and cold dark matter. So, considering a two fluid Universe (dark energy and cold dark matter), equations (7) and (8) reduce to

$$V(\phi)=\frac{1}{2}\left[\left(1-w_{eff}\right)\rho_{eff}-\rho_m\right] \quad (11)$$

And

$$\omega\phi(t)=\int\left[\left(1+w_{eff}\right)\rho_{eff}-\rho_m\right]^{\frac{1}{2}}dt+\phi_i. \quad (12)$$

Furthermore, if we assume the minimal interaction between matter and the scalar field then from the conservation equation, we have $\dot{\rho}_m+3H\left(1+w_m\right)\rho_m=0,$ and

$$\dot{\rho}_m+3H\left(1+w_m\right)\rho_m=0, \quad (13)$$

Which yields $\rho_m=\rho_0a^{-3}$, where ρ_0 is a constant of integration and is generally attributed to present value of matter energy density. Here and afterwards a suffix ' 0 ' for any variable refers to present value of the concerned quantity. Hence, the potential for the scalar field can be written as

$$V(\phi)=\frac{1}{2}\left[\left(1-w_{eff}\right)\rho_{eff}-\rho_0a^{-3}\right], \quad (14)$$

Together with the expression of the scalar function

$$\omega\phi(t)=\int\left[\left(1+w_{eff}\right)\rho_{eff}-\rho_0a^{-3}\right]^{\frac{1}{2}}dt+\phi_i. \quad (15)$$

From the two Friedmann equations (7) and (8), it is easy to derive

$$w_{eff}=-1-\frac{2}{3}\frac{\dot{H}}{H^2}. \quad (16)$$

Which can also be represented as

$$w_{eff}=-\frac{1}{3}+\frac{2}{3}q=-\frac{1}{3}-\frac{2}{3}\frac{a\ddot{a}}{\dot{a}^2}. \quad (17)$$

We can observe that, for any parametrization of the parameters $q(t)$, $H(t)$ or $a(t)$, all the quantities w_{eff}, w_{eff}, a can easily be obtained using equations (9) and (16) (or (17)). Hence, we can obtain scalar function $\phi(t)$ using equation (15) and eliminating $\phi(t)$ from $\phi(t)$ and using in (14), we can obtain the potential function $q(t)$ for any model resulting from the parametrization of $q(t)$, $a(t)$ or $a(t)$. It is to be noted that for quintessence field ($\omega=+1$), from equation (15) we can have $\phi(t)=\phi_i+\int\left[\left(1+w_{eff}\right)\rho_{eff}-\rho_0a^{-3}\right]^{\frac{1}{2}}dt$ while for phantom field ($\omega=-1$), we can write the scalar function $\phi(t)=\phi_i-\int\left[\left(1+w_{eff}\right)\rho_{eff}-\rho_0a^{-3}\right]^{\frac{1}{2}}dt$.

Potential in $q(t)$ parametrized model

Equations (14) and (15) can be written as a single unknown variable $q(t)$ as

$$V(\phi)=\frac{(2-q)M_p^2}{\left\{q_0+\int(1+q)dt\right\}^2}-\frac{\rho_0}{2a_0^3}\exp\left\{-3\int\frac{dt}{q_0+\int(1+q)dt}\right\}, \quad (18)$$

Where q_0 and a_0 are integrating constants. The scalar function $\phi(t)$ is given by

$$\omega\phi(t)=\phi_i+\int\left[\frac{2(1+q)M_p^2}{\left\{q_0+\int(1+q)dt\right\}^2}-\frac{\rho_0}{a_0^3}\exp\left\{-3\int\frac{dt}{q_0+\int(1+q)dt}\right\}\right]^{\frac{1}{2}}dt. \quad (19)$$

The potential for the Berman's parametrization[56] of constant deceleration parameter $q(t)=m-1$, is then obtained as

$$V(\phi)=\frac{1}{2}\left[\frac{(3-\beta)M_p^2}{\left(q_0+\beta t\right)^2}-\frac{\rho_0}{a_0^3\left(q_0+\beta t\right)^{\frac{3}{\beta}}}\right] \quad (20)$$

Together with

$$\omega\phi(t)=\phi_i+\int\left[\frac{2mM_p^2}{\left(q_0+mt\right)^2}-\frac{\rho_0}{a_0^3\left(q_0+mt\right)^{\frac{3}{m}}}\right]^{\frac{1}{2}}dt. \quad (21)$$

At late times, when the dark energy overtakes the matter energy i.e. $\rho_{eff}=\rho_\phi$, we have $\omega\phi(t)-\phi_i=\sqrt{\frac{2}{m}}M_p\ln\left(q_0+mt\right)$ and the potential is found to an exponential potential in the form

$$V(\phi)=(3-2m)M_p^2\exp\left\{-\frac{\sqrt{2m}}{M_p}(\omega\phi-\phi_i)\right\}. \quad (22)$$

Similarly, the potential for Linearly Varying Deceleration Parameter model (LVDP)[57] $q(t)=-2\alpha t+\beta-1$ (at late times) is given as

$$V(t)=\frac{(3-\beta+2\alpha t)M_p^2}{\left(q_0+\beta t-\alpha t^2\right)^2}, \quad (23)$$

Where t is to be eliminated from

$$\frac{\left(4q_0\alpha+\beta^2\right)^{\frac{1}{4}}}{2\sqrt{2}M_p}(\omega\phi-\phi_i)=\tan^{-1}\frac{\sqrt{\beta-2\alpha t}}{\left(4q_0\alpha+\beta^2\right)^{\frac{1}{4}}}-\tanh^{-1}\frac{\sqrt{\beta-2\alpha t}}{\left(4q_0\alpha+\beta^2\right)^{\frac{1}{4}}}. \quad (24)$$

Potential in $a(t)$ parametrized model

Equations (14) and (15) can be written as a single unknown variable $a(t)$ as

$$V(\phi)=M_p^2\left(2+\frac{a\ddot{a}}{\dot{a}^2}\right)\frac{\dot{a}^2}{a^2}-\frac{\rho_0}{2a^3} \quad (25)$$

Together with the scalar function

$$\omega\phi(t)=\phi_i+\int\left[2M_p^2\left(1-\frac{a\ddot{a}}{\dot{a}^2}\right)\frac{\dot{a}^2}{a^2}-\frac{\rho_0}{a^3}\right]^{\frac{1}{2}}dt. \quad (26)$$

The potential for the power law cosmology[58] $a(t)=\beta t^n$, is given by

$$V(\phi)=M_p^2\frac{n(3n-1)}{t^2}-\frac{\rho_0}{2\beta^3t^{3n}} \quad (27)$$

Together with

$$\omega\phi(t)=\phi_i+\int\left[2M_p^2\frac{n}{t^2}-\frac{\rho_0}{\beta^3t^{3n}}\right]^{\frac{1}{2}}dt. \quad (28)$$

At late times, when the dark energy overtakes the matter energy i.e. $\rho_{eff}=\rho_\phi$, we have $\omega\phi(t)-\phi_i=\sqrt{2n}M_p\ln t$ and the potential is found to be again an exponential potential in the form

$$V(\phi)=n(3n-1)M_p^2\exp\left\{-\sqrt{\frac{2}{n}}\frac{1}{M_p}(\omega\phi-\phi_i)\right\}. \quad (29)$$

Potential in $H(t)$ parametrized model

Equations (14) and (15) can be written as a single unknown variable $H(t)$ as

$$V(\phi)=M_p^2\left[3H^2+\dot{H}\right]-\frac{\rho_0}{2a_0^3}\exp\{-3\int H(t)dt\} \quad (30)$$

Together with the expression of scalar function

$$\omega\phi(t)=\phi_i+\int\left[-2M_p^2\dot{H}-\frac{\rho_0}{a_0^3}\exp\{-3\int H(t)dt\}\right]^{\frac{1}{2}}dt. \quad (31)$$

The potential for the parametrized Hubble function of the form $H(t)=\frac{\beta t^m}{\left(t^n+\alpha\right)^p}$,[49] is found to be

$$V(\phi)=M_p^2\left[\frac{3\beta^2t^{2m}}{\left(t^n+\alpha\right)^{2p}}+\beta\left(\frac{mt^{m-1}}{\left(t^n+\alpha\right)^p}-\frac{npt^{m+n-1}}{\left(t^n+\alpha\right)^{p+1}}\right)\right]-\frac{\rho_0}{2a_0^3}\exp\left\{-3\beta\int\frac{t^m}{\left(t^n+\alpha\right)^p}dt\right\} \quad (32)$$

Together with

$$\omega\phi(t)=\phi_i+\int\left[-2M_p^2\beta\left(\frac{mt^{m-1}}{\left(t^n+\alpha\right)^p}-\frac{npt^{m+n-1}}{\left(t^n+\alpha\right)^{p+1}}\right)-\frac{\rho_0}{a_0^3}\exp\left\{-3\beta\int\frac{t^m}{\left(t^n+\alpha\right)^p}dt\right\}\right]^{\frac{1}{2}}dt. \quad (33)$$

For a specific model with $m=0$, $n=1$, $p=\frac{1}{2}$ (Model-VI of[49]), we have $\rho_{eff}=\rho_\phi$. At late times, when the dark energy overtakes the matter energy i.e. $\rho_{eff}=\rho_\phi$, we have $\omega\phi(t)-\phi_i=4M_p\sqrt{\beta}(t+\alpha)^{\frac{1}{4}}$ and the potential is obtained as

$$V(\phi)=\frac{256M_p^3\beta^{\frac{3}{2}}}{(\omega\phi-\phi_i)}\left[3\beta-\frac{8\left(M_p\sqrt{\beta}\right)^{\frac{1}{2}}}{\sqrt{\omega\phi-\phi_i}}\right]. \quad (34)$$

Potential for tachyonic field

We consider an action describing a general tachyon field ϕ as

$$S=-\int d^4xV(\phi)\sqrt{-\det\left(g_{\mu\nu}+\partial_\mu\phi\partial^\mu\phi\right)}, \quad (35)$$

Where $V(\phi)$ is the potential function for the tachyon field. In the flat FRW background the energy density ρ_ϕ and pressure p_ϕ of the tachyon field can be written as

$$\rho_\phi=\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \quad (36)$$

$$p_\phi=-V(\phi)\sqrt{1-\dot{\phi}^2}. \quad (37)$$

Here also, we consider two fluid (tachyons and matter) models. If we assume the minimal interaction between matter field and tachyon field and making use of the Friedmann equations (9) and (10) along with the perfect fluid equation of state, we obtain the tachyonic potential as

$$V(\phi)=\sqrt{-w_{eff}\rho_{eff}\left(\rho_{eff}-\rho_0a^{-3}\right)} \quad (38)$$

And

$$\phi(t)-\phi_i=\int\sqrt{\frac{\left(1+w_{eff}\right)\rho_{eff}-\rho_0a^{-3}}{\left(\rho_{eff}-\rho_0a^{-3}\right)}}dt,\phi_i \text{isanintegratingconstant.} \quad (39)$$

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As in the case of quintessence and phantom fields, we can obtain the tachyon potential $V(\phi)$ and the tachyon field $\phi(t)$ using the relation (38) and (39) for any parametrization of any cosmological parameter $q(t)$, $q(t)$, $H(t)$ where the quantities ρ_{eff}, $_{a(t)=\beta t^n}$, $_{a(t)=\beta t}$ can easily be obtained using equations (9) and (16) (or (17)).

Tachyonic potential for power law cosmology[58] $a(t)=\beta t^n$, is obtained as

$$V(\phi)=M_p\sqrt{\frac{(3n^2-2n)}{t^2}\left(\frac{3n^2M_p^2}{t^2}-\frac{\rho_0}{\beta^3t^{3n}}\right)} \tag{40}$$

Together with

$$\phi(t)-\phi_i=\int\left[\left(\frac{2nM_p^2}{t^2}-\frac{\rho_0}{\beta^3t^{3n}}\right)/\left(\frac{3n^2M_p^2}{t^2}-\frac{\rho_0}{\beta^3t^{3n}}\right)\right]^{\frac{1}{2}}dt. \tag{41}$$

At late times, when $\rho_{eff}=\rho_\phi$, we have $\phi-\phi_i=\sqrt{\frac{2}{3n}}t$ and the potential

$$V(\phi)=2M_p^2\sqrt{n^2-\frac{2}{3}n}\frac{1}{(\phi-\phi_i)^2}. \tag{42}$$

Tachyonic potential for Berman's model of constant deceleration parameter[56] $q(t)=m-1$, is given by

$$V(\phi)=\frac{\sqrt{3-2m}M_p}{(q_0+mt)}\sqrt{\frac{3M_p^2}{(q_0+mt)^2}-\frac{\rho_0}{a_0^3(q_0+mt)^{\frac{3}{m}}}} \tag{43}$$

Together with

$$\phi(t)-\phi_i=\int\left[\left(\frac{2mM_p^2}{(q_0+mt)^2}-\frac{\rho_0}{a_0^3(q_0+mt)^{3/m}}\right)/\left(\frac{3M_p^2}{(q_0+mt)^2}-\frac{\rho_0}{a_0^3(q_0+mt)^{3/m}}\right)\right]^{\frac{1}{2}}dt. \tag{44}$$

At late times, when $\rho_{eff}=\rho_\phi$, we have $\phi-\phi_i=\sqrt{\frac{2m}{3}}t$ and the potential is given as

$$V(\phi)=\frac{\sqrt{3(3-2m)}M_p^2}{\left\{q_0+\sqrt{\frac{3m}{2}}(\phi-\phi_i)\right\}^2}. \tag{45}$$

Similarly, the potential for LVDP model[57] $q(t)=-2\alpha t+\beta-1$, is given by

$$V(t)=\sqrt{3}M_p^2\frac{\sqrt{3+2(2\alpha t-\beta)}}{\left(q_0+\beta t-\alpha t^2\right)^2}, \tag{46}$$

Where $_{H(}$ is to be eliminated from $\phi(t)=\phi_i-\frac{\sqrt{6}}{9\alpha}(\beta-2\alpha t)^{3/2}$.

Tachyonic potential for the $H(t)$ parametrized model[49] $H(t)=\frac{\beta}{\sqrt{t+\alpha}}$ (Model-VI in[49]) is obtained as

$$V(\phi)=M_p\sqrt{\beta}\sqrt{\frac{(3\beta\sqrt{t+\alpha}-1)}{(t+\alpha)^{\frac{3}{2}}}\left(\frac{3\beta^2M_p^2}{t+\alpha}-\frac{\rho_0}{a_0^3\exp(6\beta\sqrt{t+\alpha})}\right)} \tag{47}$$

Together with

$$\phi(t)-\phi_i=\int\left[\left(\frac{\beta M_p^2}{(t+\alpha)^{3/2}}-\frac{\rho_0}{a_0^3\exp(6\beta\sqrt{t+\alpha})}\right)/\left(\frac{3\beta^2M_p^2}{t+\alpha}-\frac{\rho_0}{a_0^3\exp(6\beta\sqrt{t+\alpha})}\right)\right]^{\frac{1}{2}}dt. \tag{48}$$

At late times, when $\rho_{eff}=\rho_\phi$, we have $\phi-\phi_i=\frac{4}{3\sqrt{3\beta}}(t+\alpha)^{3/4}$ and the potential is given as

$$V(\phi)=\frac{8}{9}\frac{1}{2^{1/3}}\frac{1}{\beta^{1/6}}M_p^2\left[\frac{9\beta^{4/3}}{(\phi-\phi_i)^{8/3}}-\frac{2^{2/3}}{(\phi-\phi_i)^{10/3}}\right]^{\frac{1}{2}}. \tag{49}$$

Following the same procedure, scalar field potentials can be constructed either explicitly or implicitly for any cosmological parametrization.

Conclusion

In this paper, we considered models based upon a specific scheme of parametrization. We have constructed the scalar field potentials in $H(t)$, $H(t)$ and $H(t)$ parametrized models for quintessence, phantom and tachyonic fields in the FRW framework. In case of constant deceleration parameter or power law cosmology, the scalar field potential reduces to exponential form as expected. In case of tachyon field, the potential corresponding to scaling solution is provided by inverse power law, $V(\phi)\sim\phi^{-2}$ as noted earlier. For a specific model (model-VI in[49]) resulting from a parametrization of H, the potential $V(\phi)\sim[V_1(\phi)+V_1(\phi)]$ where $V_1(\phi)\sim\phi^{-1}$ and $V_1(\phi)\sim\phi^{-3/2}$ for (non) phantom case and $V(\phi)\sim[V_3(\phi)+V_4(\phi)]$ where $V_3(\phi)\sim\phi^{-8/3}$ and $V_1(\phi)\sim\phi^{-10/3}$ in case of tachyon. Similarly, we can also constructed the scalar field potentials for all other $H(t)$ parametrized models obtained in [49]. The potentials for the linearly varying deceleration parameter model have also been obtained for both (non) phantom and tachyonic fields as implicit functions of ϕ and $_{a(t}$. In principle, for any scheme of parametrization of $a(t)$, $q(t)$, $w(t)$, $w(t)$, $\rho(t)$, the scalar field potentials for quintessence, phantom and tachyonic fields can be constructed.

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Conflicts of interest

Author declares that there is no conflict of interest.

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