1. Appendix: A Matrix Elements of Dispersion Equation

For model1, the matrix elements in Equation (26) are listed as following:

$m_{11} = -\alpha_1 I_1(\alpha_1 r)$; $m_{12} = -\mathrm{i}k\beta_1 I_1(\beta_1 r)$;

$m_{13} = \alpha_{21}K_1(\alpha_{21}r)(1-\beta) + \beta\eta_1\alpha_{21}K_1(\alpha_{21}r)$; $m_{14} = \alpha_{22}K_1(\alpha_{22}r)(1-\beta) + \beta\eta_2\alpha_{22}K_1(\alpha_{22}r)$;

$m_{15} = \mathrm{i}k\beta_{21}K_1(\beta_{21}r)(1-\beta) + \beta\eta_3\mathrm{i}k\beta_{21}K_1(\beta_{21}r)$;

$m_{21} = \mathrm{i}kI_0(\alpha_1 r)$; $m_{22} = \beta_1{}^2 I_0(\beta_1 r)$; $m_{23} = -\mathrm{i}kK_0(\alpha_{21}r)$;

$m_{24} = -\mathrm{i}kK_0(\alpha_{22}r)$; $m_{25} = -\beta_{21}{}^2 K_0(\beta_{21}r)$;

$$m_{31} = (-2\mu_1\alpha_1 - \alpha_1\lambda_1)[\alpha_1 I_0(\alpha_1 r) - \frac{1}{r}I_1(\alpha_1 r)] - \lambda_1 k^2 I_0(\alpha_1 r) - \frac{\alpha_1}{r}\lambda_1 I_1(\alpha_1 r) ;$$

$$m_{32} = -2\mathrm{i}k\mu_1\beta_1[\beta_1 I_0(\beta_1 r) - \frac{1}{r}I_2(\beta_1 r)] ;$$

$$m_{33} = (\alpha_{21}{}^2 + k^2)K_0(\alpha_{21}r)(A + \eta_1 Q + Q + \eta_1 R) + 2N\alpha_{21}[\alpha_{21}K_0(\alpha_{21}r) - \frac{1}{r}K_1(\alpha_{21}r)]$$

$$m_{34} = (\alpha_{22}{}^2 + k^2)K_0(\alpha_{22}r)(A + \eta_2 Q + Q + \eta_2 R) + 2N\alpha_{22}[\alpha_{22}K_0(\alpha_{22}r) - \frac{1}{r}K_1(\alpha_{22}r)]$$

$$m_{35} = 2\mathrm{i}kN\beta_{21}[\beta_{21}K_0(\beta_{21}r) - \frac{1}{r}K_1(\beta_{21}r)] ;$$

$m_{41} = -2\mathrm{i}ku_1\alpha_1 I_1(\alpha_1 r)$;

$m_{42} = \mu_1(k^2\beta_1 - \beta_1{}^3)I_1(\beta_1 r)$;

$m_{43} = 2\mathrm{i}kN\alpha_{21}K_1(\alpha_{21}r)$; $m_{44} = 2\mathrm{i}kN\alpha_{22}K_1(\alpha_{22}r)$;

$m_{45} = -N(k^2\beta_{21} - \beta_{21}{}^3)K_1(\beta_{21}r)$;

$m_{51}=0$; $m_{52} = 0$; $m_{53} = -\alpha_{21}(1-\eta_1)K_1(\alpha_{21}r)$;

$m_{54} = -\alpha_{22}(1-\eta_2)K_1(\alpha_{22}r)$; $m_{55} = -\mathrm{i}k\beta_{21}(1-\eta_3)K_1(\beta_{21}r)$;

$b_1 = \alpha_1 K_1(\alpha_1 r)$; $b_2 = \mathrm{i}kK_0(\alpha_1 r)$;

$$b_3 = (2\mu_1\alpha_1{}^2 + \lambda_1\alpha_1{}^2 + \lambda_1 k^2)K_0(\alpha_1 r) - 2\mu_1\frac{\alpha_1}{r}K_1(\alpha_1 r) ;$$

$b_4 = 2\mathrm{i}k\mu_1\alpha_1 K_1(\alpha_1 r)$;

Where r is the radius of the rod, λ_1, μ_1 are lame constants of the internal elastic media. For model2, The matrix elements in Equation (35) are listed as following:

$m_{11} = -\alpha_{11}I_1(\alpha_{11}r)(1-\beta) - \beta\eta_1\alpha_{11}I_1(\alpha_{11}r)$; $m_{12} = -\alpha_{12}I_1(\alpha_{12}r)(1-\beta) - \beta\eta_2\alpha_{12}I_1(\alpha_{12}r)$

$m_{13} = -\mathrm{i}k\beta_{11}I_1(\beta_{11}r)(1-\beta) - \mathrm{i}k\beta\eta_3\beta_{11}I_1(\beta_{11}r)$; $m_{14} = \alpha_2 K_1(\alpha_2 r)$; $m_{15} = \mathrm{i}k\beta_2 K_1(\beta_2 r)$;

$m_{21} = \mathrm{i}kI_0(\alpha_{11}r)$; $m_{22} = \mathrm{i}kI_0(\alpha_{12}r)$; $m_{23} = \beta_{11}{}^2 I_0(\beta_{11}r)$;

$m_{24} = -\mathrm{i}kK_0(\alpha_2 r)$; $m_{25} = -\beta_2{}^2 K_0(\beta_2 r)$

$$m_{31} = [-\alpha_{11}{}^2 I_0(\alpha_{11}r) - k^2 I_0(\alpha_{11}r)](A + \eta_1 Q + Q + \eta_1 R) - 2N\alpha_{11}{}^2 I_0(\alpha_{11}r) + \frac{2N\alpha_{11}}{r}I_1(\alpha_{11}r) ;$$

$$m_{32} = [-\alpha_{12}{}^2 I_0(\alpha_{12}r) - k^2 I_0(\alpha_{12}r)](A + \eta_2 Q + Q + \eta_2 R) - 2N\alpha_{12}{}^2 I_0(\alpha_{12}r) + \frac{2N\alpha_{12}}{r}I_1(\alpha_{12}r) ;$$

$$m_{33} = -2\mathrm{i}kN\beta_{11}[\beta_{11}I_0(\beta_{11}r) - \frac{1}{r}I_1(\beta_{11}r)] ;$$

$$m_{34} = (\lambda_2\alpha_2^2 + \lambda_2 k^2 + 2\mu_2\alpha_2^2)K_0(\alpha_2 r) - \frac{2\mu_2\alpha_2}{r}K_1(\alpha_2 r) ;$$

$$m_{35} = 2\mathrm{i}k\mu_2\beta_2[\beta_2 K_0(\beta_2 r) - \frac{1}{r}K_1(\beta_2 r)] ;$$

$m_{41} = -2\mathrm{i}kN\alpha_{11}I_1(\alpha_{11}r)$; $m_{42} = -2\mathrm{i}kN\alpha_{12}I_1(\alpha_{12}r)$; $m_{43} = (\beta_{11}k^2 - \beta_{11}^3)NI_1(\beta_{11}r)$; $m_{44}=2\mathrm{i}k\mu_2\alpha_2 K_1(\alpha_2 r)$;

$m_{45} = -(\beta_2 k^2 - \beta_2^3)\mu_2 K_1(\beta_2 r)$;

$m_{51} = -\alpha_{11}(1-\eta_1)I_1(\alpha_{11}r)$; $m_{52} = -\alpha_{12}(1-\eta_2)I_1(\alpha_{12}r)$; $m_{53} = -\mathrm{i}k\beta_{11}(1-\eta_3)I_1(\beta_{11}r)$; $m_{54} = 0$; $m_{55} = 0$;

$c_1 = [\alpha_{11}(1-\beta) + \alpha_{11}\beta\eta_1]K_1(\alpha_{11}r) - [\alpha_{12}(1-\beta) + \alpha_{12}\beta\eta_2]K_1(\alpha_{12}r)$;

$c_2 = -\mathrm{i}kK_0(\alpha_{11}r) - \mathrm{i}kK_0(\alpha_{12}r)$;

$$c_3 = [\alpha_{11}{}^2 K_0(\alpha_{11}r) + k^2 K_0(\alpha_{11}r)](A + \eta_1 Q + Q + \eta_1 R) + 2N\alpha_{11}{}^2 K_0(\alpha_{11}r) - \frac{2N\alpha_{11}}{r} K_1(\alpha_{11}r)$$

$$+[\alpha_{12}{}^2 K_0(\alpha_{12}r) + k^2 K_0(\alpha_{12}r)](A + \eta_2 Q + Q + \eta_2 R) + 2N\alpha_{12}{}^2 K_0(\alpha_{12}r) - \frac{2N\alpha_{12}}{r} K_1(\alpha_{12}r)$$

$c_4 = 2\mathrm{i}kN[\alpha_{11}K_1(\alpha_{11}r) + \alpha_{12}K_1(\alpha_{12}r)]$; $c_5 = (1-\eta_1)\alpha_{11}K_1(\alpha_{11}r) + (1-\eta_2)\alpha_{12}K_1(\alpha_{12}r)$

Where r is the radius of the rod, λ_2, $\mu 2$ are lame constants of the external elastic media.