Open Access Journal of Mathematical and Theoretical Physics

Research Article

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Electromagnetic-radiation effect on alpha decay

Abstract

The effect of the electromagnetic radiation on the spontaneous charge emission from heavy atomic nuclei is estimated in a model which may be relevant for proton emission and alpha-particle decay in laser fields. Arguments are given that the electronic cloud in heavy atoms screens appreciably the electric field acting on the nucleus and the nucleus "sees" rather low fields. In these conditions, it is shown that the electromagnetic radiation brings second-order corrections in the electric field to the disintegration rate, with a slight anisotropy. These corrections give a small enhancement of the disintegration rate. The case of a static electric field is also discussed.

PACS: 23.60.+e; 23.50.+z; 03.65.xp; 03.65.Sq; 03.50.de

Keywords: alpha decay, electromagnetic radiation, proton emission, laser radiation

Volume 1 Issue 4 - 2018

M Apostol

Department of Theoretical Physics, Romania

Correspondence: M Apostol, Department of Theoretical Physics, Institute of Atomic Physics, Magurele-Bucharest MG-6, POBox MG-35, Romania, Email apoma@theory.nipne.ro

Received: July 12, 2018 | **Published:** August 13, 2018

Introduction

In the context of an active topical research in laser-related physics,[1–5] the problem of charge emission from bound-states under the action of the electromagnetic radiation is receiving an increasing interest. Some investigations focus especially on the effect the optical-laser radiation may have on the spontaneous alpha-particle decay of the atomic nuclei,[6–11] or nuclear proton emission,[12,13] but the area may be extended to atom ionization or molecular or atomic clusters fragmentation.[14–17] The aim of the present paper is to estimate the effect of the adiabatically-applied electromagnetic radiation upon the rate of spontaneous nuclear alpha decay and proton emission. Specifically, the paper is motivated by the interest in computing the rate of tunneling through a Coulomb potential barrier in the presence of electric fields. It is claimed that the rate of alpha decay is practically not affected by electric fields,[8] or it is greatly enhanced by strong electric fields.[18] On the other side, the atomic electron cloud may screen appreciably the external electric fields, such that the atomic nucleus may experience, in fact, rather low electric fields. It is this point, related to low electric fields, which may raise technical difficulties in estimating the small effect of these external fields upon the alpha decay.

We adopt a nuclear model with Z protons and A−Z neutrons, where A is the mass number of the nucleus, moving in the nuclear mean field. The experiments proceed usually by placing a collection of heavy atoms in the focal region of a laser beam, and focusing radiation pulses upon that collection of atoms. We consider an optical-laser radiation with a typical frequency ω of the order $10^{15}s^{-1}$ (corresponding to a period $T \simeq 10^{-15}s$ and a wavelength $\lambda \simeq 0.8\mu m$). We assume that the motion of the charges under the action of the electromagnetic radiation remains non-relativistic, *i.e.* $qA_0/mc^2 << 1$, where q is the particle charge, m is the particle mass and A_0 is the amplitude of the vector potential (c denotes the speed of light in vacuum). For protons in atomic nuclei $(q=4.8\times 10^{-10} esu, m \simeq 2\times 10^{-24} g, c = 3\times 10^{10} cm/s)$ this condition yields a very high electric field $E_0 = 3\times 10^{13} V/cm$ (10^{11} electrostatic units), which corresponds to a maximum intensity of the laser beam in the focal region of the order $I = cE_0^2/8\pi = 10^{24} w/cm^2$. Typically, the duration of the laser pulse is of the order of tens of radiation period (or longer), such that we may consider the action of the electromagnetic radiation much longer than the period of the radiation. The repetition rate of the laser pulses is usually much longer than the pulse duration. For simplification we consider linearly-polarized radiation plane waves; the calculations can be extended to a general polarization. The laser-beam shape or multi-mode operation has little relevance upon the results presented here.

The electric fields are appreciably screened by the electronic cloud of the heavy atoms. The screening effects on the thermonuclear reactions, alpha decay and lifetimes have been considered previously.[10,11,19,20] A convenient means of treating the electron cloud in heavy atoms is the linearized Thomas-Fermi model.[21] According to this model, the radial electron distribution is concentrated at distance $R = a_H/Z^{1/3}$ (screening distance), where $a_H = \hbar^2/mq^2$ is the Bohr radius and Z is the atomic number ($Z \gg 1$); q and m denote the electron charge and mass, respectively. The atomic binding energy depends on R, and the atom exhibits an eigenmode related to the change in R (a breathing-type mode), with an eigenfrequency $\omega_0 \simeq Z|q|/\sqrt{ma_H^3} \simeq 4.5Z\times 10^{16}s^{-1} (\hbar\omega_0 \simeq 28Z(eV))$. We recognize in ω_0 the plasma frequency $\simeq 4\pi\overline{n}q^2/m$ of a mean electron density $\overline{n} \simeq Z/R^3 = Z^2/a_H^3$. It corresponds to the atomic giant-dipole oscillations discussed in Ref.[21] In the presence of an external electric field E oriented along the z-direction the electrons are displaced by u(with fixed nucleus), a displacement which produces an energy change $\sim z^2u^2/R^2$. By integrating over z, we get a factor $1/\sqrt{3}$ in the eigenfrequency ω_0, as expected. It follows that the displacement u obeys the equation of motion $\ddot{u} + \Omega^2 u = qE/m$, where $\Omega = \omega_0/\sqrt{3}$; the internal field is $E_i = -4\pi\overline{n}qu$ (polarization $P=\overline{n}qu$ and the dipole moment $p=Zqu$). For $E=E_0\sin\omega t$ the solution of this equation is $u = u_0\sin\omega t, u_0 = -(qE_0/m)/(\omega^2-\Omega^2)$, and the internal field is $E_i = \Omega^2 E/(\omega^2-\Omega^2)$; the total electric field acting upon the atomic nucleus is

$$F = E + E_i = \frac{\omega^2}{\omega^2-\Omega^2}E_0\sin\omega t; \qquad (1)$$

since $\omega << \Omega$, we may use the approximation $F \simeq -(\omega^2/\Omega^2)E \simeq -10^{-3}/Z^2$ (where $\omega=10^{15}s^{-1}$); we can see that the total field acting upon the nucleus is appreciably reduced by the electron screening. For $Z=50$ this reduction factor is $\simeq 4\times 10^{-7}$; the maximum field 3×10^{13}V/cm is reduced to 10^7V/cm ($\simeq 10^4$ electrostatic units). It follows that we may limit ourselves here to low fields acting upon the atomic nuclei. The cases of strong fields have been analyzed in Refs.[8,9,18,22,23] At the same time, an induced electric field generated by the dipolar eigenmodes occurs inside the atom, which oscillates with the higher eigenfrequency Ω.

If the field is low, the bound-state charge oscillates, emits higher-order harmonics of electromagnetic radiation and tunneling may appear; in this latter case, the charge accommodates itself in the field, in a long time, which amounts to an adiabatically-introduced interaction; this regime allows the usual, standard application of the tunneling approach. As we shall see below, the threshold field which separates the two regimes (low-field regime from high-field regime) can be obtained from $|q|E_0 / m\omega^2 a = 1$, where a is a distance of the order of the bound-state dimension ($a = 10^{-13}cm$) (for protons, the threshold field is $E_0 \simeq 10^5 V / cm$ (10^2 electrostatic units)).

Originally, the charge emission from bound states, like atom ionization, has been treated by using adiabatic hypothesis, either by time-dependent perturbation theory, or by imaginary-time tunneling, or other equivalent approaches.[24–32] Quasi-classical tunneling through the potential barrier generated by the field has been applied in classical works to static fields and the hydrogen atom (in parabollic coordinates).[33–35] For alpha-particle decay or proton emission the situation is different. First, in spontaneous decay, the alpha particle (and, in general, the ejected charge) is preformed and, second, the tunneling through the Coulomb potential barrier must be included.[36–40] We analyze below the spontaneous charge emission, affected by the presence of an adiabatically-introduced electromagnetic radiation, in the presence of a Coulomb barrier; the problem may exhibit relevance for studies of alpha-particle decay or proton emission.

The standard model of spontaneous alpha decay is based on Bohr's concept of compound nuclei.[41] In an alpha-unstable nucleus the pre-formed alpha particle acquires a kinetic energy and penetrates (tunnels through) the Coulomb potential barrier. Consequently, the alpha-unstable nucleus is in fact in a "metastable state". In this simple model, the spontaneous alpha-particle decay and proton emission proceed by tunneling through the Coulomb potential barrier, as a result of many "attempts" the charge makes to penetrate the barrier. The (high) frequency of this process is of the order $1/t_a$, where t_a corresponds, approximately, to the energy level spacing; in atomic nuclei this spacing, for the relevant energy levels, is of the order $\Delta\varepsilon = 200keV$, which gives[41] $t_a \simeq 3\times 10^{-21} s$; also, the broadening of the charge energy levels introduces an energy uncertainty (we leave aside the so-called tunneling through the internal potential barrier and the pre-formation factor of the alpha particle). The order of magnitude of the energy of the charge is a few MeV, which ensures a quasi-classical tunneling. The effect of the electromagnetic radiation upon the initial preparation of the charge for tunneling may be neglected. Similarly, we consider a sufficiently low electromagnetic radiation, such that we may neglect its effects upon the mean-field potential. We limit ourselves to the effect of the electromagnetic interaction on the tunneling rate.

Let us consider a charge $q>0$ with mass m in the potential barrier $V(r)$ in the presence of an electromagnetic radiation with the vector potential $\mathbf{A} = \mathbf{A_0}\cos(\omega t - \mathbf{kr})$, where $\mathbf{A_0}$ is the amplitude of the vector potential, ω is the radiation frequency and k is the radiation wavevector ($\omega = ck$); the electromagnetic field is transverse, *i.e.* $\mathbf{kA} = 0$. Since the phase velocity of the non-relativistic charge is much smaller than the speed of light c in vacuum, we may neglect the spatial phase $\mathbf{kr}$ in comparison with the temporal phase ωt; consequently, the vector potential may be approximated by $\mathbf{A} \simeq \mathbf{A_0}\cos\omega t$. This approximation amounts to neglecting the effects of the magnetic field. It is assumed that this potential is introduced adiabatically. The charge is immersed in the radiation field, such that we may start with the standard non-relativistic hamiltonian

$$H = \frac{1}{2m}(\mathbf{p} - \frac{q}{c}\mathbf{A})^2 + V(\mathbf{r}), \qquad (2)$$

where the momentum $\mathbf{p}$ includes the electromagnetic momentum qA/c beside the mechanical momentum $m\mathbf{v}$, where $\mathbf{v}$ is the velocity of the particle. We consider the Schrodinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi; \qquad (3)$$

since the interaction is time-dependent we need the time evolution of the wavefunction. Consequently, in equation(3) we perform the well-known Kramers-Henneberger transform[42–45] (with a vanishing electromagnetic interaction for $t \to -\infty$)

$$\psi = e^{iS}\varphi,$$

$$S = \frac{q}{\hbar mc\omega}\mathbf{A_0 p}\sin\omega t - \frac{q^2 A_0^2}{8\hbar mc^2\omega}(2\omega t + \sin 2\omega t); \qquad (4)$$

the Schrodinger equation becomes

$$i\hbar\frac{\partial\varphi}{\partial t} = \frac{1}{2m}p^2\varphi + \tilde{V}(\mathbf{r})\varphi,$$

$$\tilde{V}(\mathbf{r}) = e^{-is}V(\mathbf{r})e^{is} = V(\mathbf{r} - q\mathbf{A_0}\sin\omega t / mc\omega); \qquad (5)$$

it is convenient to introduce the electric field $\mathbf{E} = \mathbf{E_0}\sin\omega t, \mathbf{E_0} = \omega\mathbf{A_0}/c$; we get

$$S = \frac{q}{\hbar m\omega^2}\mathbf{E_0 p}\sin\omega t - \frac{q^2 A_0^2}{8\hbar mc^2\omega}(2\omega t + \sin 2\omega t) \qquad (6)$$

and $$\tilde{V}(\mathbf{r}) = V(\mathbf{r} - q\mathbf{E}/m\omega^2). \qquad (7)$$

We can see that for high-intensity fields the potential (including the mean- field potential) is rapidly vanishing along the field direction. Here we assume that the field intensity is low; specifically we assume $qE_0/m\omega^2 << a$, where a is the dimension of the region the charge moves in (the atomic nucleus); for protons this inequality means $E_0 << 3\times 10^4 V/cm$ (10^2 electrostatic units), as stated above. The preformed alpha particle (or emitted proton) may tunnel through the potential barrier given by equation (7); the "attempt" frequency to penetrate the barrier and the energy uncertainty are practically not affected by the low-intensity field.

We adopt a model of nuclear decay by assuming a Coulomb potential barrier $V(r) \simeq Zq^2 / r$ (with the center-of-mass of the original nucleus placed at the origin); in the absence of the field the tunneling proceeds from $r_1 = a$ to $r_2 = Zq^2/\varepsilon_r$, where ε_r is the radial energy of the charge; it is convenient to introduce the parameter $\xi = qE_0/m\omega^2 a << 1$, which includes the effect of the field. In the presence of the field these limits become

$$\tilde{r}_1 = |\mathbf{a} - q\mathbf{E}/m\omega^2| \qquad (8)$$

and $\tilde{r}_2 = r_2$, where $\mathbf{a} = a\mathbf{r}/r$. We expand $\tilde{r}_1$ in powers of ξ and get

$$\tilde{r}_1 = a(1 - \xi \sin\omega t.\cos\theta + \frac{1}{2}\xi^2 \sin^2 \omega t.\sin^2\theta) + ..., \quad (9)$$

where θ is the angle the radius vector r makes with the electric field $\mathbf{E_0}$.

To continue, we assume that the free charge attempting to penetrate the potential barrier has momentum $\mathbf{p}_n$ and kinetic energy $\varepsilon_n = p_n^2/2m$, where n is a generic notation for its state; we may leave aside the orbital motion and denote by $\mathbf{p}_{rn}$ the radial momentum and by ε_{rn} the radial energy. Let $\mathbf{p}_r$ and $\varepsilon = p^2/2m$ be the highest radial momentum and, respectively, the highest radial energy; they correspond to the total momentum p and, respectively, total energy $\varepsilon = p^2/2m$ (in general, a degeneration may exist). This charge may tunnel through the potential barrier $V(r)$ from $\tilde{r}_1$ to $\tilde{r}_2$. The relevant factors in the wave function ψ given by equation (4) are

$$e^{\frac{iqE(t)}{\hbar m\omega^2}\cos\theta.(p_2 - p_1) + \frac{i}{\hbar}\int_{\tilde{r}_1}^{\tilde{r}_2} dr.p_r(r)} \quad (10)$$

Where $p_r(r) = \sqrt{2m[\varepsilon - V(r)]}$, $p_{1,2} = p_r(\tilde{r}_1,2) = \sqrt{2m[\varepsilon - V(\tilde{r}_{1,2})]}$; it is easy to see that $p_2 = 0$. It follows that the tunneling probability (transmission coefficient) is given by $w = e^{-\gamma}$, where

$$\gamma = -A\xi \sin\omega t.\cos\theta + B,$$

$$A = \frac{2a|p_1|}{\hbar}, \xi = \frac{qE_0}{m\omega^2 a}, B = \frac{2}{\hbar}\int_{\tilde{r}_1}^{\tilde{r}_2} dr|p_r(r)| \quad (11)$$

and $|p_1| = \sqrt{2m[V(\tilde{r}_1) - \varepsilon]}$, $|p_r(r)| = \sqrt{2m[V(r) - \varepsilon]}$ (the condition $V(\tilde{r}_1) > \varepsilon$ ensures the existence of the bound state). We expand the coefficient A in powers of ξ and take the average with respect to time; we get

$$\gamma = -\frac{Zq^2}{2\hbar}\sqrt{\frac{2m}{Zq^2/a - \varepsilon}}\xi^2\cos^2\theta + B...; \quad (12)$$

the same procedure applied to the coefficient B leads to

$$B = \gamma_0 - \frac{a\xi^2}{2\hbar}\sqrt{2m(Zq^2/a - \varepsilon)} + \frac{a\xi^2}{2\hbar}\sqrt{\frac{2m}{Zq^2/a - \varepsilon}}(3Zq^2/2a - \varepsilon)\cos^2\theta, \quad (13)$$

where γ_0 corresponds to the absence of the radiation; finally, we get

$$\gamma = \gamma_0 - \frac{a\xi^2}{2\hbar}\sqrt{2m(Zq^2/a - \varepsilon)}\left[1 - \frac{Zq^2/2a - \varepsilon}{Zq^2/a - \varepsilon}\cos^2\theta\right] \quad (14)$$

We can see that the effect of the radiation is to increase the rate of charge emission by a factor proportional to the square of the electric field (ξ^2) and to introduce a slight anisotropy. It is worth noting that the radiation field contributes not only to the tunneling factor, as expressed by the coefficient B, but it is present also in the coefficient A, via the time-dependence of the wave function provided by the Kramers-Henneberger transform.

We can define a total disintegration probability

$$w_{tot} \simeq \left\{1 + \frac{a\xi^2}{2\hbar}\sqrt{2m(Zq^2/a - \varepsilon)}\left[1 - \frac{Zq^2/2a - \varepsilon}{3(Zq^2/a - \varepsilon)}\right]\right\} w_{tot}^0 \quad (15)$$

by integrating over angle θ, where $w_{tot}^0 = e^{-\gamma_0}$ The disintegration rate per unit time is $(1/\tau)w_{tot}$, where τ is related to the time t_a estimated above and the time introduced by the energy uncertainty.[41]

The exponent γ_0, corresponding to the absence of the radiation, is

$$\gamma_0 = \frac{Zq^2}{\hbar}\sqrt{2m/\varepsilon}\left(\arccos\sqrt{\varepsilon a/Zq^2} - \sqrt{\varepsilon a/Zq^2}\sqrt{1 - \varepsilon a/Zq^2}\right); \quad (16)$$

since $Zq^2 / a >> \varepsilon$ (for protons $q^2/a = 2.5$M eV), we may use the approximate formulae

$$\gamma_0 \simeq \frac{\pi Zq^2}{2\hbar}\sqrt{2m/\varepsilon} \quad (17)$$

and $$w_{tot} \simeq \left(1 + \frac{5a\xi^2}{12\hbar}\sqrt{2mZq^2/a}\right) w_{tot}^0 \quad (18)$$

As it is well know the interplay between the very large values of $1/\tau$ and the very small values of $e^{-\gamma_0}$, makes the disintegration rate to be very sensitive to the energy values, and to vary over a wide range.[41] The result can be cast in the form of the Geiger-Nuttall law, which, in the absence of the radiation, can be written as $\ln(w_{tot}^0/\tau) = -a_0 Z/\sqrt{\varepsilon} + b_0$, a_0 and b_0 being well-known constants;[41] the only effect of the radiation is to modify the constant b_0 into $b = b_0 + (5a\xi^2/12\hbar)\sqrt{2mZq^2/a}$. The correction to b_0 can $(5\xi^2/12)[(Zq^2/a)/(\hbar^2/2ma^2)]^{1/2}$ also be written as for $\xi << 1$. The maximum value of this correction is of the order of the unity; it follows that the decay rate is enhanced by the radiation by a factor of the order $\xi^2 << 1$.

After the emission of the charge, the mean-field potential suffers a reconfiguration (re-arrangement) process and the potential $V(r)$ is modified; this is the well-known process of "core shake-up" (or "core excitation"); a new bound state is formed and a new transformation process may begin for the modified potential $V(r)$. The tunneling probability w given above is a transmission coefficient (we can check that $w<1$); with probability $1-w$ the charge is reflected from the potential barrier; in these conditions the bound state is "shaken-up" and the charge resumes its motion, or its pre-formation process, until it tunnels, or is rescattered back to the core; the latter is the well-known recollision process.[8,46–50]

The case of a static field requires a special discussion. Within the present formalism a static electric field $\mathbf{E}$ can be obtained from a vector potential $\mathbf{A} = -c\mathbf{E}t$; the position vector in the mean-field potential is shifted to $\mathbf{r} \to \mathbf{r} + \boldsymbol{\zeta}$, where $\boldsymbol{\zeta} = q\mathbf{E}t^2/2m$; the special discussion is necessary because the parameter $\boldsymbol{\zeta}$ is unbounded in time. The distance a is covered in time $t_0 = \sqrt{2ma/qE}$; for proton, a is of the order a = 10^{-13}cm and the threshold field is $E = E_0 = 3\times10^4$ V/cm (10^2 electrostatic units) given above; we get $t_0 \simeq 10^{-15}s$. This is a very long duration, in comparison with the relevant nuclear times, in particular the attempt time τ ($t_a \simeq 10^{-21}s$ estimated above). In general, the condition of adiabatic interaction reads $t_0 << \hbar/\Delta\varepsilon$, where $\Delta\varepsilon$ is the mean separation of the energy levels; it implies $qEa << (\Delta\varepsilon)^2/(\hbar^2/ma^2)$, which allows for high static fields. In these conditions the protons accommodate themselves to the electric field, which is absorbed into slightly modified energy levels; this change, which can be estimated by perturbation theory, is irrelevant for our discussion, since the field strength is small. However, it has an important consequence in that the electric field, once taken in the energy levels, is not available anymore for the Kramers-Henneberger transform given by equation (4); therefore, the present time-dependent formalism cannot be applied. Instead of using the hamiltonian given by equation (2), we start with the (equivalent) dipole hamiltonian which includes the interaction term $-q\mathbf{Er}$. Consequently, the potential barrier $V(r) \simeq Zq^2/r$ is changed into

$$V(\mathrm{r}) = \frac{Zq^2}{r} - q\mathbf{Er} = \frac{Zq^2}{r}\left(1 - \frac{Er^2}{Zq}\cos\theta\right). \qquad (19)$$

We compute the tunneling rate by using this potential barrier. In view of the small value of the correction parameter proportional to E in equation (19), we may expand the momentum $p_r = \sqrt{2m\left[\varepsilon - V(r)\right]}$ in powers of this parameter and replace the powers of r^2 by their mean values over the tunneling range from $r_1 = a$ to $r_2 = Zq^2/\varepsilon$; since $r_2 \gg a$, we get the small parameter $\alpha = Er_2^2/Zq = Eqr_2/\varepsilon << 1$ in equation (19). For $Z = 100$ and $\varepsilon = 1MeV$ this parameter is $\alpha = 10^{-4}E$, which is much smaller than unity for any usual static field. Integrating over angles and assuming $\alpha\gamma_0 << 1$, where γ_0 is given by equation (17), we get finally

$$w_{tot} \simeq \left(1 + \frac{\alpha^2\gamma_0^2}{108}\right) w_{tot}^0 . \qquad (20)$$

We can see that the correction brought by a static electric field to the decay rate is extremely small, as expected.

Finally, it is worth discussing the case of intermediate fields, i.e. field strengths which satisfy the inequality $qE_0/m\omega^2 > a(\xi > 1)$ (in our case, fields from 3×10^4 V/cm to 10^7 V/cm).[51] In this case the adiabatic hypothesis cannot be used anymore, and the initial conditions for introducing the interaction are important. The corresponding Kramers-Henneberger transform diminishes appreciably the potential barrier and the charge is set free in a short time, which is the reciprocal of the decay rate; this rate may exhibit oscillations as a function of the field strength.

Conclusion

We may say that in low-intensity electromagnetic radiation the bound-states charges accommodate themselves in the field, which amounts to an adiabatically-introduced interaction, as it is well known. In these conditions, besides oscillating and emitting higher harmonics, the charge may tunnel out from the bound state. This is the standard ionization process, which was widely investigated for atom ionization. In spontaneous alpha decay or proton emission the situation is different, because of the preformation stage and the tunneling through the Coulomb potential barrier. We have analyzed above the disintegration rate for the charge emission from atomic nuclei in the case of the adiabatic introduction of electromagnetic interaction, with application to nuclear alpha-particle decay and proton emission. Under these circumstances, it has been shown in this paper that the tunneling rate (through Coulomb potential) is slightly enhanced by the presence of the radiation, by corrections whose leading contributions are of second-order in the electric field, with a slight anisotropy. Similar results are presented in this paper for static fields.

Acknowledgements

The author is indebted to S. Misicu and the members of the Laboratory of Theoretical Physics at Magurele-Bucharest for many fruitful discussions. This work has been supported by the Scientific Research Agency of the Romanian Government through Grants 04-ELI / 2016 (Pro- gram 5/5.1/ELI-RO), PN 16 42 01 01 / 2016 and PN (ELI) 16 42 01 05 /2016.

Conflict of interest

The author declares that there is no conflict of interest.

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