Advance to customized eye modeling and precision refractive surgery

Abstract

The conventional laser surgery wouldn’t treat the inhomogeneous or irregular refractive distributions in the bulk of human eyes, while the change of the asphericity at the cornea anterior post-surgery usually leads to severe imaging distortion and vision acuity reduction in dark or twilight environment. In order to understand this problem, we established a customized eye-model for the representative myopia at -2D or -5D, simulated the retina imaging resolution through implementing modern optometry technique, and analyzed the results for three major cornea ablation schemes: i) Spherical, ii) Optimized-Q and iii) Wave-front guided cornea. Our research results showed that the wave-front guided scheme proposed in this report is better than the other two, and able to optimally compensate various low or high-orders of optical aberrations within a customized eye to achieve the best retina resolution or even the super vision ultimately.

Keywords: medical optics, human eye model, corneal astigmatism, ocular residual astigmatism, refractive surgery, high order optical aberrations

Introduction

Human Eye is a complex and delicately evolved optical imaging system, casting the colorful world onto the Retina, where the optical signals are converted into the electric impulses and then delivered through optic nerves to Brain to reconstruct the image of objects. The optical system is the front-end of the eye, transporting optical beams within the acceptance angle of the Iris diaphragm and focusing at the retina to achieve the vision acuity. However the eye is an imperfect imaging system with compound structures possessing intrinsic optical aberrations, thus its ultimate spatial resolution is worse than the diffraction limit of an aberration-free aperture in equal-size. Restricting and minimizing the overall optical aberrations of the eye is the most essential determinant of the quality of image. Therefore ophthalmology and optometry have been well established to investigate and understand the optics of the eye, to calibrate and correct the optical aberrations in presence to improve vision, typically via taking spectacles or contact lens. Within the past few decades, modern laser techniques were successfully implemented in clinical practices to rectify the optical errors of human eyes, e.g. excimer laser or femtosecond laser for photorefractive keratectomy (PRK), laser-assisted in situ keratomileusis (LASIK), epioplis laser in situ keratomileusis (Epi-LASIK) and etc. However thorough or complete knowledge of eye’s physical properties and imaging mechanism is still lacking, accurate eye modeling are in great demand for exploring and unveiling these secrets, and then designing and developing the optimal laser eye surgery schemes to approach better imaging resolution.

Figure 1A presents a schematic eye model, where Cornea and Lens with convex anterior/posterior boundaries are the two major refractive components. Aqueous and Vitreous are transparent media located in front of or behind adjacent to the lens, iris has a variable optical aperture (Pupil) adapted to the ambient brightness to confine the incidence beams, and Retina works as a photo-detector to collect and convert photons into neuron signals. In emmetropic eye, the overall dioptr well matches with its axial length, so lights pass through various optical components and focus at the fovea zone (a small depression on the retina screen with the highest concentration of photo-sensors), achieving the best vision acuity; for ametropic eye (e.g. myopia or hyperopia), the diopter mis-matches with the axial length, the beams then defocus on the retina to blur the image. The conventional refractive surgery based on spherical correction to cornea anterior according to Munnerlyn equation (Figure 1B) (Figure 1C), only corrects the low-order optical aberrations of the eye dominated by “defocus”, hardly rectifying the high-order aberrations. (Here it is worthwhile to mention about that the scheme can also remove “astigmatism” distributed over cornea anterior, but not that in the bulk of the eye, e.g. “lenticular astigmatism”. For latter case, extra cylindrical correction should be applied against the polarization of the astigmatism in the eye’s transverse cross-section, similar as a compound frame glasses possess complementary cylindrical profile to provide proper refractive compensation in cylindrical equivalence- (CE). Remarkably, the spherical ablation would unavoidably modify the original asphericity (addressed in the next section) of the cornea anterior to introduce significant “spherical aberration” to the eye-ball. Generally it has little impact, but would become problematic when the optical zone (defined by the diameter of the pupil) crossing the visual axis is considerably larger. Many patients are disturbed by the post-surgery side-effects e.g. halo ghost dizziness etc. at dusk or night, mainly because the dimmer environment

© 2018 Huang et al. This is an open access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and build upon your work non-commercially.
enlarges the iris aperture, thus the higher-order aberrations of the eye-ball which are negligible for small pupil would become outstanding, causing substantial reduction of the visual acuity and severe image distortion.\textsuperscript{20} Therefore the \textit{Q-value optimized scheme} has been established, targeting to maintain the longitudinal asphericity of the cornea anterior boundary (comparable to pre-surgery), and there are reports on observing the reduction of post-surgery spherical aberration of the eye-ball for Q-scheme in clinic.\textsuperscript{\textbullet} The Q-scheme could constrain the spherical aberration pretty well, but couldn’t reduce the rest of high-order aberrations (Figure 1D) (Figure 1E). The post-surgery wave-fronts of the refractive beams for either spherical or optimized-Q schemes illustrated in the respective plots (Figure 1B or Figure 1D) are clearly deviated from the ideal one (i.e. the wave-front of Gaussian beam), corresponding to the uncompensated high-order aberrations. The most desired laser eye surgery should not only correct the diopter mis-match, but provide general correction for the overall optical aberrations (including defocus, astigmatism and high-order aberrations) of the eye. Thus the \textit{wave-front guided scheme} is proposed, where the cornea refractive surgery is designed to correct the specific optical aberration distributions of individual ametropic eyes precisely, aiming to gain the optimal retina vision acuity; and after the surgery the wave-front of the refractive beams would approach to the ideal case nicely\textsuperscript{22} (Figure 1F) (Figure 1G). However the wave-front guided scheme is currently at the preliminary stage, the calibration and ablation techniques are in developing and yet to be perfect, the real-time centering, positioning and motion-tracking of the eye-ball, fast scan and high-precision 3D cornea ablation technique and etc. are all extremely challenging, requiring further investigation and significant improvements.\textsuperscript{23} Nevertheless the motivation for exploring and achieving super vision for mankind is very strong, and developing the customized wave-front guided refractive surgery schemes is an intriguing research topic and a feasible route to be there, deserving sustained efforts.\textsuperscript{24,25}

Materials and methods

In order to develop a customized eye model and explore the optimal cornea refractive surgery schemes, we configured a representative eye-ball for majority groups according to the previous studies and statistics,\textsuperscript{26,27} and included arbitrary distribution functions to simulate the specifications of individual eyes. The various main components of the eye are parameterized and highlighted in Table 1, further details and interpretations are provided in the subscript notes underneath the table. The typical emmetropia and myopia are considered in the simulation, where the degree of ametropia is defined using the conventional unit of Refractive Error in Spherical Equivalence (SE);\textsuperscript{18} thus SE = 0D, -2D or -5D are implemented in the models.

The eye ball is described as a realistic non-spherical complex, which consists several major refractive media possessing conicoid boundaries with specific asphericities (in general the pupil axial symmetry is presumed, and refer to Figure 1A,

\begin{equation}
\rho j (\rho) = \frac{\rho^2}{R_j} + \frac{\rho^2}{Q_j} + \frac{\rho^2}{R_j} + C_j \quad [1]
\end{equation}

Where \( \rho \) and \( j \) are the longitudinal and transverse coordinates of the eye-ball respectively, \( j \) denotes the \( j \)-th boundary (e.g. anterior or posterior of cornea, anterior or posterior of lens, retina screen etc.), \( R_j \) and \( Q_j \) are the apex radius and asphericity (Q) of each boundary, \( C_j \) is a constant associated with the thickness of the current medium. More specifically, the anterior or posterior surface of cornea is depicted by prolate ellipsoid (-1<\( Q <0 \), the anterior or posterior Lens by hyperboloid (\(-Q<1\)), while the retina screen by a double-axis oblate ellipsoid (\( Q>0 \)), possessing different radii and asphericities in Meridian (horizontal) or Sagittal (vertical) coordinates.\textsuperscript{28} Especially the anterior radius of cornea (\( R_c \)) and vitreous chamber length (\( L_v \)) are both correlated to the refractive error of the eye (SE), which is illustrated in Figure 1A as well.\textsuperscript{27} (Refer to the specific contents in Table 1 and the corresponding subscript notes for more details).

Then we systematically explore the optical properties of the realistic eye model and investigate the refractive surgery schemes for achieving the optimal retina resolution. The conventional spherical correction, Q-value optimized scheme and wave-front guided scheme (involving the Zernike polynomials)\textsuperscript{19,20} are all investigated, analyzed, and compared. The principal optical beams are set to incident along the visual axis upon the paraxial approximation, with an oblique Kappa angle\textsuperscript{30} of 5° in horizontal and 2° in vertical with respect to the pupillary axis of the eye-ball, and sequentially penetrate the cornea, aqueous, iris, lens, vitreous, and eventually hits on the fovea zone.
located a bit off-center of the retina screen (refer to Figure 1A). The conventional optical ray-tracing program “Zemax” is implemented to analyze the refractive beams, the focal spot distributions and chromatic dispersions in the eye models; and self-made ray-tracing scripts and software are developed for this purpose. Eventually, the resulting spot diagrams at fovea zone are retrieved and plot, the Point Spread Function (PSF)\(^\text{11}\) for each case is exhibited. The normalized spatial Fourier Transformation is applied to PSF to obtain the Modulation Transfer Function (MTF)\(^\text{12}\) to reflect the spatial resolution on retina directly (where

\[ MTF(\lambda, f) = \frac{FT[PSF]}{FT[PSF]|_{\lambda=\lambda_0}=f_0}. \]

and \(\lambda_0, f_0\) denotes the spatial frequency in horizontal or vertical direction). Furthermore, the Wave-front Aberration Distribution Maps (using a monochromatic wavelength at 555nm) for each eye model are calculated and analyzed. In application of the aforementioned cornea ablation schemes (as illustrated in Figure 1; the details and formulas regarding to these schemes are further discussed in the supplementary material), the results of both before and after the surgery are plot and compared. More specifically, the typical defocusing refractive errors of -2D or -5D were concerned in the myopic model. And in order to understand how the optical system of eye would behave either at nominal day-light or at dim to dark environment, we conduct the simulations for both small and big pupil sizes in diameters of 3mm or 6mm respectively.

Table 1 Parameters of emmetropic (0D) and myopic eye models (-2D and -5D)

<table>
<thead>
<tr>
<th>Medium</th>
<th>Apex Radius (mm)</th>
<th>Asphericity (Q)</th>
<th>Refractive Index*</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cornea</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anterior</td>
<td>7.77 (7.726, 7.66)*</td>
<td>-0.15</td>
<td>1.376</td>
<td>0.55</td>
</tr>
<tr>
<td>Posterior</td>
<td>6.4</td>
<td>-0.28</td>
<td>-0.28</td>
<td></td>
</tr>
<tr>
<td>Aqueous</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iris</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Infinity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lens</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anterior</td>
<td>11</td>
<td>-5</td>
<td>(n_{L1}^{(1)})</td>
<td>1.44</td>
</tr>
<tr>
<td>Posterior</td>
<td>-6</td>
<td>-2</td>
<td>(n_{L1}^{(2)})</td>
<td>2.16*</td>
</tr>
<tr>
<td>Vitreous</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retina*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Meridian</td>
<td>-12.91 (-12.72, 12.44)</td>
<td>0.207 (0.218, 0.14)</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Sagittal</td>
<td>-12.72 (-12.73, 12.74)</td>
<td>0.25 (0.216, 0.165)</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

*Anterior radius of cornea \(R_0\) is correlated to the refractive error of the eye-ball in spherical equivalence \(SE\): \(R_0 [\text{mm}] = 2.77 \times 0.022 \times SE\) (The degrees of myopia i.e. the refractive errors of \(SE = -2D\) or \(-5D\) are used in the modelling.)

*Vitreous chamber length \(L_{G} [\text{mm}] = 16.28 - 0.299 \times SE\).

*The lens could be virtually divided by an intermediate transverse crossing-plane (refer to Figure 1(A) for the vertical dotted line within the lens) into two parts: an anterior compound of \textit{convex-hyperboloid-to-plana} with longitudinal thickness of 1.44mm, and a posterior \textit{plana-to-concave-hyperboloid} with thickness of 2.16mm; the optical refractive indexes of these two sections at the wavelength of 555nm (in green color) are (where the superscript ‘A’ or ‘P’ in the bracket denotes the ‘anterior’ or ‘posterior’ section of the lens):

\[ n^{(A)}_L = 1.371 + 0.0652774 \times \frac{\lambda}{2.0.026659} + 0.0020399 \times \frac{\rho^2}{r^5} \]

\[ n^{(P)}_L = 1.418 - 0.0100737 \times \frac{\rho^2}{r^5} - 0.0020399 \times \rho^2 \]

*Meridian (horizontal, by superscript ‘\(x\)’ or Sagittal (vertical, by ‘\(y\)’) radius of retina (by the subscript ‘\(R\)’) screen along with their specific asphericities are given as: \(R_0^{(x)} [\text{mm}] = -12.91 - 0.0944 \times SE\); \(R_0^{(y)} [\text{mm}] = -12.72 + 0.004 \times SE\); and \(Q_0^{(x)} = 0.27 \pm 0.026 \times SE\); \(Q_0^{(y)} = 0.25 \pm 0.017 \times SE\).

*In order to simulate a customized eye, a thin layer with non-uniform thickness across the optical zone representing the asymmetric distributions of optical aberrations in the eye is applied adjacent to the posterior surface of the lens (right behind the lens). Thus the longitudinal coordinate at the posterior boundary of the lens \(\rho_{LP}\) (LP: lens posterior) could be formulated in the transverse ones \(\rho\) or \(\rho_{LP}\) (as in unit of mm): \(\frac{\rho_{LP}(\rho, \theta)}{\rho_{LP} = 2.16} - \frac{\rho_{LP}}{\rho_{LP}^5/5.9} f(\rho, \theta).\)

Where the vertex radius \(R_{LP} = -5.89 \text{mm}\) and asphericity \(Q_{LP} = -2\) for lens posterior surface have been implemented, and \(f(\rho, \theta)\) (the attached thin layer) is an arbitrary function used to simulate the specs of a customized eye-model. Since this layer is located behind the lens (within the bulk of the eye-ball), so in principle it represents the intra-ocular wave-front aberration distributions (including both the astigmatism and high-order aberration terms, possessing transverse inhomogeneity and irregularity) other than those from cornea anterior (eye-front).

*Since the retina receptors are most sensitive to the green light, so the refractive indexes of various eye components in Table 1 mainly concern the wavelength at 555nm. The visible light spans the spectral range of 400-700nm, the chromatic effect and dispersion are also taken into account in the modelling according to \(n(\lambda) = n(A) = 0.0512 - 0.1455 \times \lambda + 0.0961 \times \lambda^2\), where \(n(A)\) represents the refractive index at any arbitrary wavelength \(\lambda\), and \(\delta\) is the wavelength of 555nm.

Citation: Huang Y, Li Z, Li B. Advance to customized eye modeling and precision refractive surgery. Open Acc J Math Theor Phy. 2018;1(3):90–100.
DOI: 10.15406/oajmtp.2018.01.00014
Results and discussion

Ray-tracing spot diagrams at retina

Table 2 shows the ray-tracing spot diagrams at fovea zone using a Fraunhofer (assuming the source point at infinity) chromatic (spanning visible spectral range including 400nm (blue), 555nm (green) and 700nm (red) waves) oblique incidence beam (as illustrated in Figure 1A) for the eye-models and surgery schemes introduced previously. The customized myopic models were equipped with the refractive errors (defocus) of -2D or -5D (SE), containing a relatively large astigmatism of ~0.5D (CE) polarized to the abscissa of the transverse cross-section in the eye ball with an angle of 21-23°, along with inhomogeneous high order aberration distributions (both simulated by the arbitrary function $f_\rho$ described in Table 1). The pre- or post-operative results upon executing the three typical cornea ablation schemes for 3mm or 6mm pupil diameter are presented for comparison. More specifically, for the myopia at -2D with 3mm pupil, the RMS radius of the spot diagram at the fovea before the surgery is 45.70μm, which leads to the vision blurring since the size of the conic retina cell at the fovea of a human eye is typically 2-4μm in diameter. After the spherical or Q-value optimized correction, the spot radius (RMS) decreases substantially down to 6.02μm or 5.35μm for their minor axis respectively, thus most of the defocus error of the eye-ball is corrected. And the column for “Q-scheme” display more compact features compared to that for “spherical scheme”, mainly due to the appreciable reduction of the spherical aberration provided by the Q-scheme. Parallely the wave-front guided correction (Zernike) results in the smallest focal radius of about 4.9μm (RMS), indicating that highest retina resolution is achieved among the three. More remarkably, the results of the spherical or Q-value scheme appear asymmetric features (the major axis polarized to the abscissa with a ~20° angle), implicating that the intra-ocular astigmatism is hardly reduced while substantial parts of the high-order aberrations are uncorrected either i.e. non-uniform refractive distributions or irregularities in the customized eye are still remained. Instead applying appropriate Zernike polynomials to obtain the ideal surface profiles for cornea anterior leads to symmetric and evenly distributed ray-tracing spot diagrams indicating that the wave-front aberrations in each customized eye were precisely compensated to rectify and enhance the vision optimally. Moreover in each spot diagram, the chromatic dispersions are clearly observable, where the red light generates a bigger spot distribution than the green or blue, just because the longer wavelength is associated with a larger diffractive Airy disk at the focal plane within an identical optical system.

Table 2 The ray-tracing spot diagram at the fovea zone on the retina for the myopic eye model of -2D and -5D before or after surgery, where the schematic eye model at an oblique incidence is illustrated in Figure 1, and the parameters for various eye components are given in Table 1. The spot diagram for the iris aperture diameters in 3 or 6mm after 3 types of corneal surgery schemes i) conventionally spherical correction, ii) optimized-Q correction, and iii) Zernike correction are compared.

<table>
<thead>
<tr>
<th>Refractive error in se</th>
<th>Pupil size (mm)</th>
<th>Before surgery</th>
<th>Spherical correction</th>
<th>Optimized-q correction</th>
<th>Zernike correction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>20μm</td>
<td>10μm</td>
<td>10μm</td>
<td>10μm</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>40μm</td>
<td>10μm</td>
<td>10μm</td>
<td>10μm</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>40μm</td>
<td>10μm</td>
<td>10μm</td>
<td>10μm</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>80μm</td>
<td>10μm</td>
<td>10μm</td>
<td>10μm</td>
</tr>
</tbody>
</table>

Citation: Huang Y, Li Z, Li B. Advance to customized eye modeling and precision refractive surgery. Open Acc J Math Theor Phy. 2018;1(3):90–100. DOI: 10.15406/oajmtp.2018.01.00014
**Modulation transfer function**

MTF in Table 3 are obtained through applying the spatial Fourier transformation to the ray-tracing spot diagrams in Table 2 and normalizing (refer to previous section), which could directly reflect the spatial resolution at the fovea zone on retina. The red solid curve in each diagram specifies the aberration free diffraction limit, corresponding to the highest spatial resolution for either 3mm or 6mm diameter pupil with cut-off frequency at 350l/mm or 700l/mm respectively, where the unit of “l/mm” means the number of lines per millimeter (or tiny periodical structures) which could be discriminated by the retina. Obviously the larger iris aperture (associated with a smaller Airy disk on retina) provides a higher diffraction-limit spatial resolution. The green dot-star-line represents the MTF before surgery, which drops sharply along with the spatial frequency, indicating the poor visual quality due to the defocus error of -2D or -5D. Upon implementing the three typical schemes to correct of the optical errors, the MTF values for each case are improved tremendously, revealing that the post-surgery vision is considerably enhanced. Comparing closely for the MTF plots in the left column and those in the right (Table 3), the larger iris aperture is associated with higher absolute MTF value when approaching the diffraction limit, however the bigger optical zone would spontaneously induce larger optical aberrations in the meantime, so the absolute MTF values associated with the bigger pupil are actually lower compared to the smaller pupil. Thus the post-surgery MTF functions on the right sides (6mm pupil) are decaying relatively faster than those on left (3mm), exhibiting a much sharper contrast respected to its own diffraction limit curve. Apparently among the three surgery schemes, the Zernike correction achieves the highest MTF values across the spatial frequency axis towards the cut-off, either for the low (-2D) or high (-5D) refractive errors, and no matter for the small (3mm) or large (6mm) iris sizes.

**Table 3** The Modulation Transfer Function (MTF) for the eye models presented in Table 1 & 2, at both the pre-surgery (myopia of -2D and -5D) and the post-surgery (after the three typical ablation schemes). The results are displayed in each corresponding plot and compared along with the respective diffraction limit of an aberration-free optical aperture of the same size (with a diameter of 3 or 6 mm).

**Wave-front aberration distribution map**

Table 4 presents the overall wave-front aberration distributions for identical customized eye models previously analyzed by the ray-tracing spot diagram (Table 2) and MTF technique (Table 3) in parallel. According to the pre-surgery aberration map (figures in the first column), the bowl-shaped aberration distributions indicate the presence of negative defocusing error (SE =-2D or -5D in the modelling), where the periphery domains are associated with higher aberration values (warmer colors) compared to the central zones (cold colors). Upon execution of the spherical (the second column) or optimized-Q (the third column) correction, the absolute values of the wave-front aberrations across the optical zone decrease significantly.
(to one order less), implicating the myopia is rectified and the majority of vision acuity is recovered. However the residual patterns turn out more asymmetrically distributed and an outstanding “saddle” feature appears perpendicular to the polarization of astigmatism, indicating the intra-ocular astigmatism along with the high-order aberrations are still remained. In contrast, the wave-front guided scheme would further reduce the optical aberration distributions leading to much more symmetric features (right-most column), exhibiting the excellent compensation of the astigmatism, higher-order aberrations and irregular refractive distributions in the bulk of an eye-ball, where the currently prevalent surgery schemes are ineffective.

Table 4 The calculated wave-front aberration distribution maps before or after the various surgery schemes for the identical eye models described in Table 1 to Table 3. The absolute magnitudes of the wave-front aberrations in each diagram are scaled within the color-bar in unit of the wavelength at 555nm.

<table>
<thead>
<tr>
<th>Refractive error (Se) (Mm)</th>
<th>Pupil size</th>
<th>Before surgery</th>
<th>Spherical correction</th>
<th>Optimized-Q correction</th>
<th>Zernike correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2D</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
<td><img src="image4" alt="Image" /></td>
</tr>
<tr>
<td>6</td>
<td>2D</td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
<td><img src="image7" alt="Image" /></td>
<td><img src="image8" alt="Image" /></td>
</tr>
<tr>
<td>3</td>
<td>5D</td>
<td><img src="image9" alt="Image" /></td>
<td><img src="image10" alt="Image" /></td>
<td><img src="image11" alt="Image" /></td>
<td><img src="image12" alt="Image" /></td>
</tr>
<tr>
<td>6</td>
<td>5D</td>
<td><img src="image13" alt="Image" /></td>
<td><img src="image14" alt="Image" /></td>
<td><img src="image15" alt="Image" /></td>
<td><img src="image16" alt="Image" /></td>
</tr>
</tbody>
</table>

The optical aberration analysis

Thus the results from the above techniques (spot diagram, MTF and wave-front map) show that the customized wave-front guided cornea ablation scheme could be a universal myopia treatment plan for achieving better retina imaging resolution than the conventional scheme, at either the day-light or dim environment, especially for those with the irregular optical aberrations distributed in the bulk of the eye. Table 5 utilizes the bar diagrams to reflect and compare the contributions from the low or high order optical aberrations before or after the aforementioned refractive surgery schemes. For the pre-surgery case, the low-order optical aberrations (defocus and astigmatism) are dominant in the overall aberrations, while the high-order aberrations (spherical aberration, coma and etc.) only occupy a few percent of the magnitude: e.g. for the 6mm pupil, 5.6 λ (low-order) vs. 0.34 λ (high-order) at -2D; and 14.2 λ (low-order) vs. 0.31 λ (high-order) at -5D. (Regarding to the pre-surgery discrete components of the wave-front aberrations, please refer to Figure S1 in the supplementary material, in terms of Zernike polynomials.) All three schemes could correct the low-order aberrations of myopia nicely to regain the vision, for both the low (-2D) and high (-5D) degrees of defocus error, either small (3mm) or big (6mm) iris aperture: again at 6mm pupil, the low-order aberrations decrease from 5.6 λ (pre-surgery) to <0.85 λ (post-surgery) for -2D, and from 14.2 λ (pre-) to <0.9 λ (post-) for -5D, which are clearly demonstrated on the left side of Table 5A & C. However if only the high-order aberrations are filtered out for analysis, as presented on the right hand-side (B, D), the results for various schemes are divergent. For the spherical correction, the higher order aberration values after surgery become relatively larger compared to the pre-surgery, especially the situation gets even worse for the large iris case (6mm), where the absolute RMS values of the uncorrected high-order aberrations are comparable with those of the residual low-order aberrations: 0.39 λ (high-order) vs. 0.85 λ (low-order) for -2D, and 0.34 λ (high-order) vs. 0.9 λ (low-order) for -5D; both are relatively higher than the pre-surgery values: 0.34 λ (-2D),
0.31 λ (-5D). So the high-order aberrations become non-negligible, essentially determining the ultimate vision acuity, which also provides the most-likely reason why patients who took conventional cornea surgery are frequently suffered from the bad-vision experiences in the evening or at dim background. The optimized-Q scheme through optimizing the post-surgery QM value for the cornea anterior, could further reduce the high-order aberrations beyond the spherical correction: 47% reduction for -2D, and 60% for -5D respectively, which are less than the pre-surgery values: 0.2 λ (after the surgery) vs. 0.34 λ (before the surgery) for -2D, 0.13 λ (after) vs. 0.31 λ (before) for -5D. So the Q-scheme effectively overcomes the troublesome spherical aberration enhancement induced by the spherical correction. Furthermore the wave-front guided correction would base on the exact optical aberrations distributed in the eye-ball, and compensate them precisely through modifying cornea anterior to the ideal profile in various orders of specific Zernike polynomials. According to Table S5B & Table S5D, the Zernike correction apparently achieves the minimal high-order aberrations among all three ablation schemes for both small (3mm) and big (6mm) iris, to deliver the best image in daylight or at dark. At 6mm case, the higher order aberrations decrease from 0.20 λ (Q-scheme) to 0.04 λ (Zernike) for -2D, and from 0.13 λ (Q-scheme) to 0.04 λ (Zernike) for -5D. Besides that, Table S5A, Table S5C show that Zernike correction also achieves the lowest post-surgery low-order aberrations for both -2D and -5D. (The dominant optical aberration terms after the various cornea surgery schemes are discussed in the supplementary materials as well, refer to Figure S2.)

![Figure S1](https://example.com/figureS1.png)

**Figure S1** The coefficients of various orders of the optical aberrations (in terms of Zernike polynomials) for both the -2D (filled green bar) and -5D (red slash bar) myopic eye models before the surgery (6mm pupil).

![Figure S2](https://example.com/figureS2.png)

**Figure S2** The coefficients of few significant optical aberrations for the -5D myopic model possessing (A) "eye-front astigmatism" (distributed at cornea anterior), or (B) "intra-ocular astigmatism" (lens related) before or after the laser ablation surgeries, within a 6mm optical zone. Different colored patterns were used to denote various corresponding cases.

**Citation:** Huang Y, Li Z, Li B. Advance to customized eye modeling and precision refractive surgery. *Open Acc J Math Theor Phy.* 2018;1(3):90–100.

DOI: 10.15406/oajmtp.2018.01.00014
Advance to customized eye modeling and precision refractive surgery

The proposed customized cornea surgery scheme

The accuracy in measuring the pre-surgery optical aberrations of the eye-ball is extremely important for the wave-front guided scheme. However according to the figures presented in the first column of Table 4, the low order aberrations are dominant at pre-surgery, overwhelming the high order terms and preventing them from being calibrated precisely. There were reports about this even referring to the most advanced calibration techniques currently available. Thus we proposed a pseudo three-step customized cornea surgery scheme illustrated in Figure 2A: i) Firstly the in-corrected diopter of the eye and the Q-value of cornea anterior are measured using the conventional techniques, then the optimized-Q surgery is carried out to remove the majority of the low-order aberrations while restricting the spherical aberration of the eye-ball as small as possible; ii) Once the eye recovers from the Q-surgery and becomes stable, the overall remaining wave-front aberrations are calibrated again by utilizing a monochromatic laser. Upon the low order aberrations being removed, the high orders turn out more prominent while the asymmetric or irregular refractive features of individual eyes are unfolded, so they could in principle be well corrected to gain the optimal vision acuity. The first three Zernike polynomials correlated to the wave-front longitudinal shift \( z_0 \) and wave-front tilts \( z_1 \) are set to zeros, since they have no influence to the image quality at all. And because the optimized-Q scheme has already been executed previously, so only a small \( z_2 \) term is needed to finely adjust the residual defocus a bit. Then relatively large quantities of \( z_3 \) and \( z_4 \) are applied to remove the intra-ocular astigmatisms which couldn’t be corrected by the Q-scheme. The comma terms \( z_{11} \) and the trefoil errors \( z_{33} \) mainly associated with the oblique incidence in the eye-model (Figure 1A) could be compensated pretty well too. The spherical aberration is always an essential factor for the vision quality, which has been well constrained previously within the Q-scheme and could be further modified according to \( z_2 \) term to achieve a better focal distribution and spatial resolution at fovea. Thus through the three-step cornea ablation scheme via specifying the projection values for the Zernike ablation schemes in details. (Refer to the supplemental material for an arbitrary ametropic eye (with inhomogeneous refractive distributions) could be further modified according to the optimization techniques currently available.)

In Figures 2B-C, the ablation depths for the -2D myopic model within a 6mm optical zone utilizing the optimized-Q or Zernike schemes are compared in the Meridional (horizontal) and Sagittal (vertical) crossing-plane (depicted in Figure 2A), where the optimized Q-scheme would ablate a layer of corneal stroma with maximal thickness of 23μm at the apex, while Zernike correction would remove a few microns more based on the Q-scheme, with maximal ablation depth of 28μm located at a bit off center. This additional ablation layer (associated with 10-20% more corneal stroma) is also plot for comparison in both the horizontal and vertical planes, exhibiting asymmetric features across the optical zone. For the -5D model (refer to Figure 2D and Figure 2E), the situation looks pretty similar and the maximal ablation depths for both schemes are close to the center of cornea anterior: 58μm (optimized-Q) and 65μm (Zernike). A set of concrete coefficients of Zernike polynomials listed in Figure 2F are associated with the extra fine ablation profiles for the cornea anterior, indicating the rest of optical aberrations for any arbitrary ametropic eye (with inhomogeneous refractive distributions) could be further modified according to the projection values for the Zernike ablation schemes in details. (Refer to the supplemental material for a customized eye could be optimally corrected (Table 4) (Table 5) for 3 or 6 mm pupil.

**Table 5** The low-order (A,C) and high-order (B,D) optical aberration distributions for myopic eye models -2D (A,B) and -5D (C,D), in the unit of the absolutely numeric RMS value of a monochromatic wavelength at 555nm. On the left side of the table (A,C), the yellow cylinder represents the overall low-order aberrations for a 3mm pupil and the green represents for a 6mm pupil; on the right (B,D), the blue or red cylinder represents the high-order aberrations for 3 or 6 mm pupil.

**Citation:** Huang Y, Li Z, Li B. Advance to customized eye modeling and precision refractive surgery. Open Acc J Math Theor Phy. 2018;1(3):90–100. DOI: 10.15406/oajmtp.2018.01.00014
Figure 2(A) presents a human eye model and coordinates for a laser ablation surgery, where the longitudinal crossing planes in horizontal (x) or vertical (y) define the Tangential or Sagittal plane of the eye-ball respectively, and a customized three-step refractive surgery scheme is illustrated. The ablation thickness within a 6-mm optical zone at the cornea anterior for the refractive error of -2D are compared for the optimized-Q (blue curve) and wave-front guided (red curve) schemes, in (B) Tangential or (C) Sagittal planes respectively. The results for -5D are given in (D) Tangential and (E) Sagittal planes as well. The ablation depth differences in-between these two schemes are sketched as the dashed lines in each plot (B-to-E) using the new scale in the right axis. The coefficients for various orders of Zernike polynomials used in the wave-front guided scheme to achieve the ideal post-surgery cornea anterior profiles are listed in (F).

**Conclusion**

In order to understand and resolve the frequently reported problems of significant reduction of vision acuity or imaging distortion during the evening, even after a successful conventional laser ablation surgery, we systematically investigated the principles and properties of the typical laser ablation schemes at the cornea anterior, mainly through exploring and unveiling their optical nature and mechanisms. Based on the frame works by David A Atchison and other pioneers,\textsuperscript{26,33,37} we established a customized myopic model at the representative refractive errors of -2D and -5D concerning both small and large iris, and employed advanced optical techniques in modern optometry: i) the refractive beam tracing in the eye and spot diagrams at fovea zone, ii) the modulation transfer function of the spot diagrams, and iii) the wave-front aberration distributions of the eye, to evaluate the post-surgery imaging quality and spatial resolution for the three cornea ablation schemes: A) spherical, B) optimized-Q, and C) wave-front guided. We find out that: spherical ablation scheme could well rectify the mis-matched diopter of myopia, but would simultaneously introduce substantial spherical aberration against vision improving further; Q-scheme could reduce the spherical aberration successfully, but not the other types of high-order aberrations; wave-front guided scheme through specifying Zernike polynomials in various orders could correct the low or high order aberrations in general to achieve the best focal spot and to approach the ultimate retina resolution, which exhibits remarkable advantages over the other two especially for bigger iris sizes.

The major contributions of the current research to the laser eye ablation theory and practice are summarized in below:

I. A customized eye-model based on the light ray-tracing through the eye-ball and coherent optics is established, concerning both the generosity and uniqueness.

II. The laser refractive surgery scheme for universal myopic model is proposed and demonstrated, to correct the optical aberrations of human eyes including both the low and high order aberrations, to deliver the optimal retina imaging resolution and achieve the ultimate super vision potentially.

III. According to our knowledge, this is the first time to build up such a realistic customized human eye model to thoroughly investigate the intra-ocular astigmatism and high order optical aberrations and to unveil the eye’s optical nature and imaging resolution. Our finding could potentially serve as a

---

**Citation:** Huang Y, Li Z, Li B. Advance to customized eye modeling and precision refractive surgery. Open Acc J Math Theor Phy. 2018;1(3):90–100. DOI: 10.15406/oajmtp.2018.01.00014
Advance to customized eye modeling and precision refractive surgery

valuable reference for ophthalmology and optometry, the design of future cornea refractive surgery, and development of the cutting-edge techniques or instrumentation in this field.

It is worthwhile to point out that the customized eye model developed in this manuscript is a universal type, and the primary parameters used in the simulations could be adjusted according to the experimental or clinical characterization of the optical errors in individual ametropic eyes. Thus we actually demonstrated that any customized myopia, even with large intra-ocular astigmatisms or irregular high-order aberrations could in principle be well treated, by in-combination of the optimized-Q and Zernike schemes sequentially to achieve the superb image quality. A rational proposal to gain human eye super vision is to fabricate the cornea anterior precisely through the wave-front guided scheme via implementing the specific spatial profiles well adapted to the genuine optical system of the eye. With more trials in clinical practices while more challenging techniques being realized, the day will come sooner or later.

Funding details

The work is supported by the National Science Foundation of China (grants # 11475249), and Youth 1000-Talent Program in China (grants # Y326021061).

Author contributions

BL conceived the ideas and wrote the manuscript, YQH developed the simulative model and Zemax scripts, YQH and ZL conducted the simulations and analysis. All the authors contribute to the model development/upgrade and finalizing the manuscript.

Acknowledgements

The authors acknowledge the staff and facility support from Chinese Academy of Sciences, Shanghai Institute of Applied Physics.

Conflict of interest

The authors declare no competing financial interests.

References


