Journal of Nanomedicine Research

A New Relativistic Bound States Solution for Modified Pseudoharmonic Potential in One-electron Atoms at Planck's and Nano-Scales

Abstract

In this paper, we present solutions of the modified Dirac equation (MDE) with modified pseudoharmonic (MPH) potential for spin-1/2 particles by means Bopp's shift method, in the framework of noncommutativity 3-dimensional real space (NC: 3D-RS) symmetries. The exact corrections for n^{th} excited states are found straightforwardly by means of the standard perturbation theory. It is found that the energy eigenvalues strongly depend on the potential parameters, two infinitesimal parameter (Θ and χ), which induced by position-position noncommutativity, in addition to the discreet atomic quantum numbers ($j=\tilde{l}(l)\pm 1/2, s=\pm 1/2, \tilde{l}(l)$ and $\tilde{m}$ (m)) and we have also shown that, the usual states in ordinary 3-.dimensions are canceled and has been replaced by new degenerated $2(2\tilde{l}+1)$ and $2(2l+1)$ sub-states under the pseudo spin symmetry and spin symmetry conditions, respectively in the new quantum group (NC: 3D-RS).

Keywords: Dirac equation; Pseudoharmonic potential; Noncommutative space; Star product; Bopp's shift method

Review Article

Volume 7 Issue 1 - 2018

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Received: November 24, 2017 | **Published:** January 12, 2018

Abbreviations: MPH: Modified Pseudo Harmonicpotential; NC: 3D-RSP: Noncommutativity Three Dimensional Real Spaces; CCRs: Canonical Commutations Relations; NNCCRs: New Noncommutative Canonical Commutations Relations; MDE: Modified Dirac equation, SP: Schrödinger Picture; HP: Heisenberg Picture

Introduction

The analytical solution of Dirac equation plays a vital role in relativistic quantum mechanics and solving the Dirac equation to obtain the bound-state energies for with different potential models using various methods for example Nikiforov-Uvarov method, The Laplace transform approach (LTA), factorization method and so on [1-6]. The quantum algebraic structure based to the ordinary CCRs in both of SP, HP and Dirac picture (the operators are depended on time), as ($c=\hbar=1$):

$$[x_i,p_j]=[x_i(t),p_j(t)]=i\delta_{ij} \text{ and } [x_i,x_j]=[x_i(t),x_j(t)]=[p_i,p_j]=[p_i(t),p_j(t)]=0 \quad (1)$$

Very recently, non-commutative geometry plays an important role in modern physics and has sustained great interest, for example [7-21] and our works in this context [22-47] in the case of relativistic and nonrelativistic quantum mechanics. The new quantum structure of NC space based on the following NC CCRs in both of SP, HP and Dirac picture respectively, as follows [7-25]:

$$[\hat{x}_i \overset{*}{,} \hat{p}_j]=[\hat{x}_i(t)\overset{*}{,}\hat{p}_j(t)]=i\delta_{ij}, [\hat{x}_i\overset{*}{,}\hat{x}_j]=[\hat{x}_i(t)\overset{*}{,}\hat{x}_j(t)]=i\theta_{ij} \text{ and } [\hat{p}_i\overset{*}{,}\hat{p}_j]=[\hat{p}_i(t)\overset{*}{,}\hat{p}_j(t)]=0 \quad (2)$$

The very small parameters $\theta^{\mu\nu}$ (compared to the energy) are element of antisymmetric real matrix and $(*)$ denote to the new star product, which generalized to two arbitrary functions ($f(x) \rightarrow \hat{f}(\hat{x})$ and $g(x)\rightarrow \hat{g}(\hat{x})$ t) $\hat{f}(\hat{x})\hat{g}(\hat{x})\equiv(f*g)(x)$ instead of the usual product $(fg)(x)$ [17-27]:

$$\hat{f}(\hat{x})\hat{g}(\hat{x})\equiv(f*g)(x)\equiv\exp(\frac{i}{2}\theta^{\mu\nu}\partial_\mu^x\partial_\nu^x(fg)(x,p)\equiv(fg-\frac{i}{2}\theta^{\mu\nu}\partial_\mu^x f\partial_\nu^x g\Big|_{(x^\mu=x^{\nu\nu})}+O(\theta^2) \quad (3)$$

The new term ($-\frac{i}{2}\theta^{\mu\nu}\partial_\mu^x f(x)\partial_\nu^x g(x)$) is induced by (space-space) noncommutativity properties. A Bopp's shift method can be used, instead of solving any quantum systems by using directly star product procedure [18-33]:

$$[\hat{x}_i,\hat{x}_j]=i\theta_{ij} \text{ and } [\hat{p}_i,\hat{p}_j]=0 \quad (4)$$

The new three-generalized coordinates $(\hat{x}=\hat{x}_1,\hat{y}=\hat{x}_2,\hat{z}=\hat{x}_3)$ are given by [22-34]:

$$\hat{x}=x-\frac{\theta_{12}}{2}p_y-\frac{\theta_{13}}{2}p_z,\quad \hat{y}=y-\frac{\theta_{21}}{2}p_x-\frac{\theta_{23}}{2}p_z \text{ and } \hat{z}=z-\frac{\theta_{31}}{2}p_x-\frac{\theta_{32}}{2}p_y \quad (5)$$

Where (x,y,z) and (p_x,p_y,p_z) are three-usual coordinates and momentum ,which allow us to getting the operator $\hat{r}^2$ on NC three dimensional spaces as follows [22-33]:

$$\hat{r}^2=r^2-\vec{\mathbf{L}}\vec{\Theta} \quad \text{with} \quad \mathbf{L}\Theta\equiv L_x\Theta_{12}+L_y\Theta_{23}+L_z\Theta_{13} \text{ and } \left(\Theta_{ij}=\frac{\theta_{ij}}{2}\right) \quad (6)$$

In recent years, the study of PH potential has attracted a lot of interest of different authors, it have the general features of the true interaction energy, inter atomic and dynamical properties in solid-state physics and play an important role in the history of molecular structures and interactions; this potential is considered as an intermediate between harmonic oscillator and Morse-type potentials which are more realistic anharmonic potentials, furthermore, the PH potential is extensively used to describe the bound state of the interaction systems, and has been applied for both classical and modern physics [5,6]. This work is aimed at obtaining an analytic expression for the eigenenergies of a MPH potential in (NC: 3D-RS) symmetries using Bopp's shift method to discover the new symmetries and a possibility to obtain another applications to this potential in different fields. This work based essentially on our previously works [22-43], it was studied in our works [34,35] in the case of nonrelativistic case. The organization scheme of the study is given as follows: In next section, we briefly review the DE with PH potential on based to [6]. Sec. 3 is devoted to studying the MDE for MPH potential by applying Bopp's shift method. In the fourth section and by applying standard perturbation theory we find the quantum corrections of spectrums of the n^{th} excited states in (NC-3D: RS) for relativistic spin-orbital interaction. In the next sub-section, we derive the magnetic spectrum for MPH potential. In the fifth section, we resume the global spectrum and corresponding NC Hamiltonian operator for MPH potential. Finally, the important results and the conclusions are discussed in last section.

Review of the Dirac Equation for PH Potential

The Dirac equation for a spherically symmetric potential in 3-dimensional reads for a single-nucleon with the mass of M and relativistic energy E moving in an attractive scalar potential $S(r)$ and a repulsive potential $V(r)$ in natural units [6]:

$$\left(\alpha p+\beta(M+S(r))\right)\Psi(r,\theta,\phi)=\left(E-V(r)\right)\Psi(r,\theta,\phi) \quad (7)$$

Here $\left(\alpha_i=\begin{pmatrix}0 & \sigma_i\\ \sigma_i & 0\end{pmatrix},\beta=\begin{pmatrix}I_{2\times2} & 0\\ 0 & I_{2\times2}\end{pmatrix}\right),\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix}$, $\sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix}$ and $\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ are the usual Dirac matrices while the PH potential $V(r)$ for the spin symmetric and the pseudo-spin spin-symmetry [6]:

$$V(r)=D_0\left(\frac{r}{r_0}-\frac{r_0}{r}\right)^2\equiv ar^2+\frac{b}{r^2}+c \quad (8)$$

Where, D_0 and r_0 are two constants related to the dissociation energy of a molecule and an equilibrium distance, respectively while $a=D_0r_0^{-2}$, $b=D_0r_0^{+2}$ and $c=-2D_0$, thus, the corresponding ordinary Hamiltonian operator $\hat{H}_{ph}$ can ban be expressed as:

$$\hat{H}_{ph}=\left(\alpha p+\beta(M+S(r))\right)+V(r) \quad (9)$$

The spinor $\Psi(r,\theta,\phi)$ can be written as [6]:

$$\Psi_{nk}(r,\theta,\phi)=\begin{pmatrix}f_{nk}(\vec{r})\\ g_{nk}(\vec{r})\end{pmatrix}=\frac{1}{r}\begin{pmatrix}F_{nk}(r)Y^{l}_{jm}(\theta,\phi)\\ iG_{n\tilde{k}}(r)Y^{\tilde{l}}_{jm}(\theta,\phi)\end{pmatrix} \quad (10)$$

Where, $F_{nk}(r)$ and $G_{n\tilde{k}}(r)$ are the upper and lower components of the Dirac spinor, $Y^{l}_{jm}(\theta,\phi)$ and $Y^{\tilde{l}}_{jm}(\theta,\phi)$ are the spin and pseudo-spin spherical harmonics while k ($\tilde{k}$) is related to the total angular momentum quantum numbers for spin symmetry l and p-spin symmetry $\tilde{l}$ as [6]:

$$k=\begin{cases}-(l+1) & \text{if } -(j+1/2),(s_{1/2},p_{3/2},etc),\ j=l+\frac{1}{2},\ \text{aligned spin }(k\langle 0)\\ +l & \text{if } j=l+\frac{1}{2},(p_{1/2},d_{3/2},etc),\ j=l-\frac{1}{2},\ \text{unaligned spin }(k\rangle 0)\end{cases} \quad (11)$$

$$\tilde{k}=\begin{cases}-\tilde{l} & \text{if } -(j+1/2),(s_{1/2},p_{3/2},etc),\ j=\tilde{l}-\frac{1}{2},\ \text{aligned -spin }(k\langle 0)\\ +(\tilde{l}+1) & \text{if } j=\tilde{l}+\frac{1}{2},(p_{1/2},d_{3/2},etc),\ j=\tilde{l}+\frac{1}{2},\ \text{unaligned- spin }(k\rangle 0)\end{cases} \quad (12)$$

The radial functions ($F_{nk}(r)$, $G_{nk}(r)$) are obtained by solving the following differential equations [6]:

$$\left[\frac{d^2}{dr^2}-\frac{k(k+1)}{r^2}-\left[M+E_{nk}-\Delta(r)\right]\left[M-E_{nk}+\Sigma(r)\right]+\frac{\frac{d\Delta(r)}{dr}\left(\frac{d}{dr}+\frac{k}{r}\right)}{M+E_{nk}-\Delta(r)}\right]F_{nk}(r)=0 \quad (13)$$

$$\left[\frac{d^2}{dr^2}-\frac{k(k-1)}{r^2}-\left[M+E_{nk}-\Delta(r)\right]\left[M-E_{nk}+\Sigma(r)\right]+\frac{\frac{d\Sigma(r)}{dr}\left(\frac{d}{dr}-\frac{k}{r}\right)}{M+E_{nk}-\Sigma(r)}\right]G_{n\tilde{k}}(r)=0 \quad (14)$$

The bound state solutions of the PH potential for the spin symmetric case obtained in the exact spin symmetry $\frac{d\Delta(r)}{dr}=0$ and then the energy eigenvalues depend on n and l. According to LTA and asymptotic interaction method, which was applied in [6], the upper component $F_{nk}(r)$ of the Dirac spinor gives by:

$$F_{nk}(r)=Nr^{\nu+1}e^{-\frac{\mu r^2}{2}}\,{}_1F_1\left(-n,\nu+\frac{3}{2},r^2\right) \quad (15)$$

Where, N and ${}_1F_1(-n,2\gamma+1,2\varepsilon r)$ are the normalization constant and the confluent hyper-geometric functions, the relativistic positive energy eigenvalues with the PH potential under the spin-symmetry condition is obtained as [6]:

$$\frac{r_0}{\sqrt{D_0}}\left(2D_0+E-M\right)\sqrt{M+E-C}-\sqrt{(2k+1)^2+4D_0^2r_0^2(M+E-C)}=4(n+1/2) \quad (16)$$

For the exact pseudo-spin symmetric case, the lower $G_{nk}(r)$ component of the Dirac spinor [6]:

$$G_{nk}(r)=\tilde{N}r^{\nu+1}e^{-\frac{\mu r^2}{2}}\,{}_1F_1\left(-n,\nu+\frac{3}{2},r^2\right)\ \text{with}\ \nu(\nu+1)=k(k+1)+(M+E-C)D_0r_0^2\ \text{and}\ \mu^2=\frac{D_0}{r_0^2}(M+E-C) \quad (17)$$

Here $\tilde{N}$ denote to the normalization constant and the relativistic negative energy eigenvalues with the PH potential under the pseudo-spin spin-symmetry condition is obtained as [6]:

$$\frac{r_0}{\sqrt{D_0}}\left(2D_0+E-M\right)\sqrt{E-M-C}-\sqrt{(2k+1)^2+4D_0^2r_0^2(M-E+C)}=4(n+1/2) \quad (18)$$

It is well known that, the generalized Laguerre polynomials $L_n^{(p)}(x)$ can be expressed as a function of the confluent hyper-geometric functions as:

$$L_n^{(p)}(x) = \frac{\Gamma(n+p+1)}{n!\Gamma(p+1)}\,{}_1F_1(-n,p+1;x) \tag{19}$$

Which allow us to rewritten the upper component $F_{nk}(r)$ and the lower $G_{nk}(r)$ component of the Dirac spinor for the spin symmetric case and the pseudo-spin spin-symmetry, respectively:

$$F_{nk}(r) = Nn!\frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\Gamma\left(n+\nu+\frac{3}{2}\right)} r^{\nu+1} e^{-\frac{\mu r^2}{2}} L_n^{\left(\nu+\frac{1}{2}\right)}\left(r^2\right) \text{ and } G_{nk}(r) = \tilde{N}n!\frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\Gamma\left(n+\nu+\frac{3}{2}\right)} r^{\nu+1} e^{-\frac{\mu r^2}{2}} L_n^{\left(\nu+\frac{1}{2}\right)}\left(r^2\right) \tag{20}$$

NC Relativistic Hamiltonian Operator for MPH Potential

Overview of Bopp's shift method

In order to obtain the MDE for MPH potential $V(\hat{r})$, we replace both ordinary Hamiltonian operator $\hat{H}(p_i,x_i)$, ordinary spinor $\Psi(\vec{r})$ and ordinary energy E by NC Hamiltonian operator $\hat{H}(\hat{p}_i,\hat{x}_i)$, new spinor $\hat{\Psi}(\vec{\hat{r}})$ and new energy E_{nc-ph} and the ordinary product will be replace by new star product $*$, respectively. Allow us to writing the new MED for MPH potential as follows [22-34]:

$$\hat{H}(\hat{p}_i,\hat{x}_i) * \hat{\Psi}(\vec{\hat{r}}) = E_{nc-ph}\hat{\Psi}(\vec{\hat{r}}) \tag{21}$$

It is worth to motioning that the Bopp's shift method permutes to reduce the above equation to simplest the form:

$$H_{nc-ph}(\hat{p}_i,\hat{x}_i)\psi(\vec{r}) = E_{nc-ph}\psi(\vec{r}) \tag{22}$$

Where, $\psi(\vec{r})$ is a solution of the Dirac equation and the new operator of Hamiltonian $H_{nc-ph}(\hat{p}_i,\hat{x}_i)$ can be expressed in three general varieties: both NC space and NC phase (NC-3D: RSP), only NC space (NC-3D: RS) and only NC phase (NC: 3D-RP) as, respectively [35-44]:

$$H_{nc-ph}(\hat{p}_i,\hat{x}_i) \equiv H\left(\hat{p}_i=p_i-\frac{1}{2}\bar{\theta}_{ij}x_j;\hat{x}_i=x_i-\frac{1}{2}\theta_{ij}p_j\right) \text{ for NC-3D: RSP} \tag{23}$$

$$H_{nc-ph}(\hat{p}_i,\hat{x}_i) \equiv H\left(\hat{p}_i=p_i;\hat{x}_i=x_i-\frac{1}{2}\theta_{ij}p_j\right) \text{ for NC-3D: RS} \tag{24}$$

$$H_{nc-ph}(\hat{p}_i,\hat{x}_i) \equiv H\left(\hat{p}_i=p_i-\frac{1}{2}\bar{\theta}_{ij};x_j,\hat{x}_i=x_i\right) \text{for NC-3D: RP} \tag{25}$$

In recently work, we are interest with the above second variety and then the new modified Hamiltonian $H_{nc-ph}(\hat{p}_i,\hat{x}_i)$ defined as a function of $\hat{x}_i = x_i - \frac{1}{2}\theta_{ij}p_j$ and $\hat{p}_i = p_i$:

$$H_{nc-ph}(\hat{p}_i,\hat{x}_i) = \alpha\hat{\mathrm{P}} + \beta(M + S(\hat{r})) + V(\hat{r}) \tag{26}$$

Where the MPH potential $V(\hat{r})$ is given by:

$$V(\hat{r}) = \frac{a}{\hat{r}^2} - \frac{b}{\hat{r}} + c \tag{27}$$

The Dirac equation in the presence of above interaction $V(\hat{r})$ can be rewritten according Bopp's shift method as follows:

$$(\alpha\mathrm{P}+\beta(M+S(\hat{r})))\Psi(r,\theta,\phi) = (E-V(\hat{r}))\Psi(r,\theta,\phi) \tag{28}$$

The radial functions $(F_{nk}(r), G_{nk}(r))$ are obtained, in the absence of tensor interaction, by solving two equations:

$$\left[\frac{d}{dr}+\frac{k}{r}\right]F_{nk}(r) = \left[M+E_{nc-ph}-\Delta(\hat{r})\right]G_{nk}(r) \tag{29}$$

$$\left[\frac{d}{dr}+\frac{k}{r}\right]G_{nk}(r) = \left[M-E_{nc-ph}+\Sigma(\hat{r})\right]G_{nk}(r) \tag{30}$$

with $\Delta(\hat{r}) = V(\hat{r}) - S(\hat{r})$ and $\Sigma(\hat{r}) = V(\hat{r}) + S(\hat{r})$, eliminating $F_{nk}(r)$ and $G_{nk}(r)$ from Eqs. (29) and (30), we can obtain the following two Schrödinger-like differential equations as follows in (NC-3D: RS) symmetries:

$$\left[\frac{d^2}{dr^2}-\frac{k(k+1)}{r^2}-\left(M+E_{nc-ph}-\Delta(\hat{r})\right)\left(M-E_{nc-ph}+\Sigma(\hat{r})\right)\right]F_{nk}(r) = 0 \tag{31}$$

$$\left[\frac{d^2}{dr^2}-\frac{k(k-1)}{r^2}-\left(M+E_{nc-ph}-\Delta(\hat{r})\right)\left(M-E_{nc-ph}+\Sigma(\hat{r})\right)\right]G_{nk}(r) = 0 \tag{32}$$

After straightforward calculations one can obtains the two terms in (NC-3D: RS) spaces as follows:

$$\frac{a}{\hat{r}^2} = \frac{a}{r^2} + \frac{a}{r^4}\vec{\mathbf{L}}\vec{\Theta} + O\left(\theta^2\right) \text{ and } -\frac{b}{\hat{r}} = \frac{b}{r} - \frac{b}{2r^3}\vec{\mathbf{L}}\vec{\Theta} + O\left(\theta^2\right) \tag{33}$$

Which allow us to writing the MPH potential $V(\hat{r})$ as follows:

$$V(\hat{r}) = \frac{a}{r^2} - \frac{b}{r} + c + \begin{cases} \hat{V}_{1pert-ph}(r,\Theta,a,b) = \left(\frac{a}{r^4}-\frac{b}{2r^3}\right)\vec{\mathbf{L}}\vec{\Theta} & \text{for spin symmetric case} \\ \hat{V}_{2pert-ph}(r,\Theta,a,b) = \left(\frac{a}{r^4}-\frac{b}{2r^3}\right)\vec{\tilde{\mathbf{L}}}\vec{\Theta} & \text{for p-spin symmetric case} \end{cases} \tag{34}$$

It is clearly that the star product inducing the non-commutativity is replaced by the usual product plus non local corrections $\hat{V}_{1pert-ph}(r,\Theta,a,b)$ and $\hat{V}_{2pert-ph}(r,\Theta,a,b)$ in the scalar potential $V(\hat{r})$. This allows writing the modified Dirac equation in the non-commutative case as an equation similarly to the usual Dirac equation of the commutative type with a non local potential. Furthermore, using the unit step function (also known as the Heaviside step function or simply the theta function) we can rewrite the MPH potential to the following form:

$$V(\hat{r}) = \frac{a}{r^2} - \frac{b}{r} + c + \theta\left(E_{nc-ph}\right)\hat{V}_{1pert-ph}(r,\Theta,a,b) + \theta\left(-E_{nc-ph}\right)\hat{V}_{2pert-ph}(r,\Theta,a,b) \tag{35}$$

Where

$$\theta(x) = \begin{cases} 1 & \text{for } x\rangle 0 \\ 0 & \text{for } x\langle 0 \end{cases} \tag{36}$$

We generalized the constraint for the pseudo-spin (p-spin) symmetry ($\Delta(r) = V(r)$ and $\Sigma(r) = C_{ps}$ = constants which presented in [6] into the new form $\Delta(\hat{r}) = V(\hat{r})$ and $\Sigma(\hat{r}) = \hat{C}_{ps}$ = constants in (NC-3D: RS) and inserting the potential $V(\hat{r})$ into the two Schrödinger-like differential Eqs. (31) and (32), one obtains:

Citation: Maireche A (2018) A New Relativistic Bound States Solution for Modified Pseudoharmonic Potential in One-electron Atoms at Planck's and Nano-Scales. J Nanomed Res 7(1): 00168. DOI: 10.15406/jnmr.2018.07.00168

$$\left[\frac{d^2}{dr^2}-\frac{k(k+1)}{r^2}-\left(M+E_{nc-ph}\right)\left(M-E_{nc-ph}+\hat{C}_{ps}\right)-\left(\frac{a}{r^2}-\frac{b}{r}+c\right)\left(M-E_{nc-ph}+\hat{C}_{ps}\right)-\left(\frac{a}{r^4}-\frac{b}{2r^3}\right)\mathbf{L}\vec{\Theta}\left(M-E_{nc-ph}+\hat{C}_{ps}\right)\right]F_{nk}(r)=0 \quad (37)$$

$$\left[\frac{d^2}{dr^2}-\frac{k(k-1)}{r^2}-\left(M+E_{nc-ph}\right)\left(M-E_{nc-ph}+\hat{C}_{ps}\right)-\left(\frac{a}{r^2}-\frac{b}{r}+c\right)\left(M-E_{nc-ph}+\hat{C}_{ps}\right)-\left(\frac{a}{r^4}-\frac{b}{2r^3}\right)\mathbf{L}\vec{\Theta}\left(M-E_{nk}+\hat{C}_{ps}\right)\right]G_{nk}(r)=0 \quad (38)$$

and two similarly equations obtained by $\vec{\mathbf{L}}\rightarrow\tilde{\vec{\mathbf{L}}}$. It's clearly that, the additive new parts $\hat{V}_{1pert-ph}(r,\Theta,a,b)$ and $\hat{V}_{2pert-ph}(r,\Theta,a,b)$ are proportional with infinitesimal parameter Θ, thus, we can considered as a perturbations terms. Our aim is to derive the energy spectrum for a moving charged particle in the presence of a potential given by (35) analytically in a very simple way.

The Exact Relativistic Spin-orbital Hamiltonian and the Corresponding Spectrum for MPH Potential in (NC: 3D- RS) Symmetries for n^{th} **Excited States for One-electron Atoms**

The exact relativistic spin-orbital Hamiltonian for MPH Potential in (NC: 3D- RS) symmetries for one-electron atoms

The results (34) can be rewritten in a more accessible physical form, we replace both $\vec{\mathbf{L}}\vec{\Theta}$ and $\tilde{\vec{\mathbf{L}}}\vec{\Theta}$ by $\vec{S}\vec{L}$ and $\tilde{\vec{S}}\tilde{\vec{L}}$ and then the two perturbative terms $\hat{V}_{1pert-ph}(r,\Theta,a,b)$ and $\hat{V}_{2pert-ph}(r,\Theta,a,b)$ for the spin symmetric case and the pseudo-spin spin-symmetry, respectively can be rewritten to the equivalent new form for MPH potential:

$$\hat{V}_{1pert-ph}(r,\Theta,a,b)=\Theta\left(\frac{a}{r^4}-\frac{b}{2r^3}\right)\vec{S}\vec{L} \text{ and } \hat{V}_{2pert-ph}(r,\Theta,a,b)=\Theta\left(\frac{a}{r^4}-\frac{b}{2r^3}\right)\tilde{\vec{S}}\tilde{\vec{L}} \quad (39)$$

Furthermore, the above perturbative terms $\hat{V}_{1pert-ph}(r,\Theta,a,b)$ and $\hat{V}_{2pert-ph}(r,\Theta,a,b)$ can be rewritten to the following new equivalent form for MPH potential [35-47]:

$$\hat{V}_{1pert-ph}(r,\Theta,a,b)=\frac{1}{2}\Theta\left(\frac{a}{r^4}-\frac{b}{2r^3}\right)\left(\vec{J}^2-\vec{L}^2-\vec{S}^2\right) \text{ and } \hat{V}_{2pert-ph}(r,\Theta,a,b)=\frac{1}{2}\Theta\left(\frac{a}{r^4}-\frac{b}{2r^3}\right)\left(\vec{J}^2-\tilde{\vec{\mathbf{L}}}^2-\tilde{\vec{S}}^2\right) \quad (40)$$

To the best of our knowledge, we just replaced the coupling spin-orbital (pseudo spin-orbital) $\vec{S}\vec{L}$ and $\tilde{\vec{S}}\tilde{\vec{\mathbf{L}}}$ by the two expressions: $\frac{1}{2}\left(\vec{J}^2-\vec{L}^2-\vec{S}^2\right)$ and $\frac{1}{2}\left(\vec{J}^2-\tilde{\vec{\mathbf{L}}}^2-\tilde{\vec{S}}^2\right)$, respectively, in relativistic quantum mechanics. The set ($H_{nc-ph}(\hat{p}_i,\hat{x}_i)$, J^2, L^2, $\tilde{S}^2$ and J_z) forms a complete of conserved physics quantities and the spin–orbit quantum number $k(\tilde{k})$ is related to the quantum numbers for spin symmetry l and p-spin symmetry $\tilde{l}$ as follows [6]:

$$\tilde{k}=\begin{cases}\tilde{k}_1\equiv-\tilde{l} & \text{if } -(j+1/2),(s_{1/2},p_{3/2},etc),j=\tilde{l}-\frac{1}{2}, \text{ aligned spin } (k\langle 0)\\ \tilde{k}_2\equiv+\left(\tilde{l}+1\right) & \text{if } \left(j=\tilde{l}+\frac{1}{2}\right),(p_{1/2},d_{3/2},etc),j=\tilde{l}+\frac{1}{2}, \text{ unaligned spin } (k\rangle 0)\end{cases} \quad (41)$$

$$k=\begin{cases}k_1\equiv-(l+1) & \text{if } -(j+1/2),(s_{1/2},p_{3/2},etc),j=l+\frac{1}{2}, \text{ aligned spin } (k\langle 0)\\ k_2\equiv+l & \text{if } \left(j=l+\frac{1}{2}\right),(p_{1/2},d_{3/2},etc),j=l-\frac{1}{2}, \text{ unaligned spin } (k\rangle 0)\end{cases} \quad (42)$$

With $\tilde{k}(\tilde{k}-1)=\tilde{l}(\tilde{l}+1)$ and $k(k-1)=l(l+1)$, which allows us to form two diagonal (3×3) matrixes $\hat{H}_{so-ph}(k_1,k_2)$ and $\hat{\tilde{H}}_{so-ph}(\tilde{k}_1,\tilde{k}_2)$, for MPH potential, respectively, in (NC: 3D-RS) symmetries as:

$$\left(\hat{\tilde{H}}_{so-ph}\right)_{11}(\tilde{k}_1)=\tilde{k}_1\Theta\left(\frac{a}{r^4}-\frac{b}{2r^3}\right) \text{ if } -(j+1/2),(s_{1/2},p_{3/2},etc),j=\tilde{l}-\frac{1}{2}, \text{ aligned spin } (k\langle 0)$$
$$\left(\hat{\tilde{H}}_{so-ph}\right)_{22}(\tilde{k}_2)=\tilde{k}_2\Theta\left(\frac{a}{r^4}-\frac{b}{2r^3}\right) \text{ if } \left(j=\tilde{l}+\frac{1}{2}\right),(p_{1/2},d_{3/2},etc),j=\tilde{l}+\frac{1}{2}, \text{ unaligned spin } (k\rangle 0)$$
$$\left(\hat{\tilde{H}}_{so-ph}\right)_{33}=0 \quad (43)$$

$$\left(\hat{H}_{so-ph}\right)_{11}(k_1)=k_1\Theta\left(\frac{a}{r^4}-\frac{b}{2r^3}\right) \text{ if } -(j+1/2),(s_{1/2},p_{3/2},etc),j=l+\frac{1}{2}, \text{ aligned spin } (k\langle 0)$$
$$\left(\hat{H}_{so-ph}\right)_{22}(k_2)=k_2\Theta\left(\frac{a}{r^4}-\frac{b}{2r^3}\right) \text{ if } \left(j=l+\frac{1}{2}\right),(p_{1/2},d_{3/2},etc),j=l-\frac{1}{2}, \text{ unaligned spin } (k\rangle 0)$$
$$\left(\hat{H}_{so-ph}\right)_{33}=0 \quad (44)$$

The exact relativistic spin-orbital spectrum for MPH potential symmetries for n^{th} **excited states for one-electron atoms in (NC: 3D- RSP) symmetry**

In this subsection, we are going to study the modifications to the energy levels ($E_{nc-per:d}(\Theta,\tilde{k}_1)$, $E_{nc-per:u}(\Theta,\tilde{k}_2)$) for ($-(j+1/2)$, $(s_{1/2},p_{3/2},etc)$, $j=\tilde{l}+\frac{1}{2}$, aligned spin $k\langle 0$ and spin-down) and ($j=\tilde{l}+\frac{1}{2}$, $(p_{1/2},d_{3/2},etc)$, $j=\tilde{l}-\frac{1}{2}$, un aligned spin $k\rangle 0$ and spin up), respectively, at first order of infinitesimal parameter Θ, for n^{th} excited states, for the spin symmetric and the pseudo-spin spin-symmetry obtained by applying the standard perturbation theory, using Eqs. (20) and (35) as:

$$\int \overset{+}{\Psi}_{nk}(r,\theta,\phi)\left[\theta\left(E_{nc-ph}\right)\hat{V}_{1pert-ph}(r,\Theta,a,b)+\theta\left(-E_{nc-ph}\right)\hat{V}_{2pert-ph}\right](r,\Theta,a,b)\Psi_{nk}(r,\theta,\phi)r^2drd\Omega$$
$$=\theta\left(E_{nc-ph}\right)\int F^*_{nk}(r)\hat{V}_{1pert-ph}(r,\Theta,a,b)F_{nk}(r)dr-\theta\left(E_{nc-ph}\right)\int G^*_{n\tilde{k}}(r)\hat{V}_{2pert-ph}(r,\Theta,a,b)G_{n\tilde{k}}(r)dr \quad (45)$$

The first part represents the modifications to the energy levels for the spin symmetric while the second part represent the modifications to the energy levels ($E_{nc-per:d}(\Theta,\tilde{k}_1)$, $E_{nc-per:u}(\Theta,\tilde{k}_2)$) for the pseudo-spin spin-symmetry, then we have explicitly:

$$E_{nc-per:d}(\Theta,\tilde{k}_1)\equiv-\theta\left(E_{nc-ph}\right)\tilde{k}_1\int G^*_{n\tilde{k}}(r)\left(\frac{a}{r^4}-\frac{b}{2r^3}\right)G_{n\tilde{k}}(r)dr$$
$$E_{nc-per:u}(\Theta,\tilde{k}_2)\equiv\theta\left(-E_{nc-ph}\right)\tilde{k}_2\int G^*_{n\tilde{k}}(r)\left(\frac{a}{r^4}-\frac{b}{2r^3}\right)G_{n\tilde{k}}(r)dr \quad (46)$$

Now, we use eqs. (20), (40), (41) and (42) to obtain the explicit expressions for modified energy eigenvalues ($E_{nc-per:d}\left(\Theta,\tilde{k}_1\right)$, $E_{nc-per:u}\left(\Theta,\tilde{k}_2\right)$) for MDE with MPH potential under the pseudo spin symmetry conditions obtained as:

$$E_{nc-per:d}\left(\Theta,\tilde{k}_1\right)\equiv-\theta\left(-E_{nc-ph}\right)\tilde{k}_1\Theta\left(\tilde{N}n!\frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\Gamma\left(n+\nu+\frac{3}{2}\right)}\right)^2\int_0^{+\infty}r^{2\nu+2}e^{-\mu r^2}\left[L_n^{\left(\nu+\frac{1}{2}\right)}\left(r^2\right)\right]^2\left(\frac{a}{r^4}-\frac{b}{2r^3}\right)dr \quad (47)$$

$$E_{nc-per:u}\left(\Theta,\tilde{k}_2\right)\equiv-\theta\left(-E_{nc-ph}\right)\tilde{k}_2\Theta\left(\tilde{N}n!\frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\Gamma\left(n+\nu+\frac{3}{2}\right)}\right)^2\int_0^{+\infty}r^{2\nu+2}e^{-\mu r^2}\left[L_n^{\left(\nu+\frac{1}{2}\right)}\left(r^2\right)\right]^2\left(\frac{a}{r^4}-\frac{b}{2r^3}\right)dr \quad (48)$$

And using the transformation $X=r^2$, we have:

$$E_{nc-per:d}\left(\Theta,\tilde{k}_1\right)\equiv-\frac{1}{2}\theta\left(-E_{nc-ph}\right)\tilde{k}_1\Theta\left(\tilde{N}n!\frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\Gamma\left(n+\nu+\frac{3}{2}\right)}\right)^2\int_0^{+\infty}X^{\nu-\frac{1}{2}}e^{-\mu X}\left[L_n^{\left(\nu+\frac{1}{2}\right)}(X)\right]^2\left(\frac{a}{X^2}-\frac{b}{2X^{\frac{3}{2}}}\right)dX \quad (49)$$

$$E_{nc-per:u}\left(\Theta,\tilde{k}_2\right)\equiv-\frac{1}{2}\theta\left(-E_{nc-ph}\right)\tilde{k}_2\Theta\left(\tilde{N}n!\frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\Gamma\left(n+\nu+\frac{3}{2}\right)}\right)^2\int_0^{+\infty}X^{\nu-\frac{1}{2}}e^{-\mu X}\left[L_n^{\left(\nu+\frac{1}{2}\right)}(X)\right]^2\left(\frac{a}{X^2}-\frac{b}{2X^{\frac{3}{2}}}\right)dX \quad (50)$$

A direct simplification gives:

$$\begin{aligned}
E_{nc-per:d}\left(\Theta,\tilde{k}_1\right)&\equiv-\frac{1}{2}\theta\left(-E_{nc-ph}\right)\tilde{k}_1\Theta\left(\tilde{N}n!\frac{\Gamma\left(\nu_1+\frac{3}{2}\right)}{\Gamma\left(n+\nu_1+\frac{3}{2}\right)}\right)^2\left(T_{1-ph}(D_0,r_0,\nu_1,n)+T_{2-ph}(D_0,r_0,\nu_1,n)\right)\\
E_{nc-per:u}\left(\Theta,\tilde{k}_2\right)&\equiv-\frac{1}{2}\theta\left(-E_{nc-ph}\right)\tilde{k}_2\Theta\left(\tilde{N}n!\frac{\Gamma\left(\nu_1+\frac{3}{2}\right)}{\Gamma\left(n+\nu_1+\frac{3}{2}\right)}\right)^2\left(L_{1-ph}(D_0,r_0,\nu_2,n)+L_{2-ph}(D_0,r_0,\nu_2,n)\right)
\end{aligned} \quad (51)$$

Where, the four terms $T_{1-ph}\left(D_0,r_0,\nu_1,n\right)$, $T_{2-ph}\left(D_0,r_0,\nu_1,n\right)$, $L_{1-ph}\left(D_0,r_0,\nu_2,n\right)$ and $L_{2-ph}\left(D_0,r_0,\nu_2,n\right)$ are given by, respectively:

$$\begin{aligned}
&T_{1-ph}\left(D_0,r_0,\nu_1,n\right)=a\int_0^{+\infty}X^{\nu_1-\frac{5}{2}}e^{-\mu X}\left[L_n^{\left(\nu_1+\frac{1}{2}\right)}(X)\right]^2dX,\ T_{2-ph}\left(D_0,r_0,\nu_1,n\right)=-\frac{b}{2}\int_0^{+\infty}X^{\nu_1-2}e^{-\mu X}\left[L_n^{\left(\nu_1+\frac{1}{2}\right)}(X)\right]^2dX\\
&L_{1-ph}\left(D_0,r_0,\nu_1,n\right)=a\int_0^{+\infty}X^{\nu_2-\frac{5}{2}}e^{-\mu X}\left[L_n^{\left(\nu_2+\frac{1}{2}\right)}(X)\right]^2dX \text{ and } L_{2-ph}\left(D_0,r_0,\nu_1,n\right)=-\frac{b}{2}\int_0^{+\infty}X^{\nu_2-2}e^{-\mu X}\left[L_n^{\left(\nu_2+\frac{1}{2}\right)}(X)\right]^2dX
\end{aligned} \quad (52)$$

Now we apply the special integral [48]:

$$\int_0^{\infty}e^{-x\left(s+\frac{a_1+a_2}{2}\right)}x^{\alpha+\beta}L_k^{\alpha}\left(a_1x\right)L_k^{\alpha}\left(a_2x\right)dx=\frac{\Gamma(1+\alpha+\beta)\Gamma(1+\alpha+k)}{k!k!\Gamma(1+\alpha)}\left\{\frac{d^k}{dh^k}\left[\frac{F\left(\frac{1+\alpha+\beta}{2},1+\frac{\alpha+\beta}{2};1+\alpha;\frac{A^2}{B^2}\right)}{(1-h)^{1+\alpha}B^{1+\alpha+\beta}}\right]\right\}_{h=0} \quad (53)$$

Where $A^2=\frac{4a_1a_2h}{(1-h)^2}$, $B=s+\frac{a_1+a_2}{2}\frac{1+h}{1-h}$, $R\left(s+\frac{a_1+a_2}{2}\right)\rangle 0$, $a_1\rangle 0$, $a_2\rangle 0$ and $R(\alpha+\beta)\rangle-1$, which allow us to obtaining $T_{1-ph}\left(D_0,r_0,\nu_1,n\right)$ and $T_{2-ph}\left(D_0,r_0,\nu_1,n\right)$ as:

$$T_{1-ph}\left(D_0,r_0,\nu_1,n\right)=D_0r_0^{-2}\frac{\Gamma\left(\nu_1-\frac{3}{2}\right)\Gamma\left(\frac{3}{2}+\nu_1+n\right)}{n!^2\,\Gamma\left(\frac{3}{2}+\nu_1\right)}\left\{\frac{d^n}{dh^n}\left[\frac{F\left(\frac{\nu_1-3/2}{2}\right),1+\frac{\nu_1-3/2}{2};\nu_1+\frac{3}{2};\frac{A^2}{B^2}}{(1-h)^{\nu_1+\frac{3}{2}}B_1^{\nu_1-3/2}}\right]\right\}_{h=0} \tag{54}$$

$$T_{2-ph}\left(D_0,r_0,\nu_1,n\right)=-\frac{D_0r_0^{+2}}{2}\frac{\Gamma(\nu_1)\Gamma\left(\frac{3}{2}+\nu_1+n\right)}{n!^2\,\Gamma\left(\frac{3}{2}+\nu_1\right)}\left\{\frac{d^n}{dh^n}\left[\frac{F\left(\frac{\nu_1}{2}\right),1+\frac{\nu_1-1}{2};\nu_1+\frac{3}{2};\frac{A^2}{B^2}}{(1-h)^{\nu_1+\frac{3}{2}}B^{\nu_1}}\right]\right\}_{h=0} \tag{55}$$

Here $\nu_1\left(\nu_1+1\right)=\tilde{k}_1(\tilde{k}_1+1)+\left(M+E-C\right)D_0r_0^{\,2}$, $A^2=\frac{4h}{(1-h)^2}$ and $B=\mu-1+\frac{1+h}{1-h}$, thus, the new factors $L_{1-ph}\left(D_0,r_0,\nu_2,n\right)=T_{1-ph}\left(D_0,r_0,\nu_1\to\nu_2,n\right)$ and $L_{2-ph}\left(D_0,r_0,\nu_2,n\right)=T_{2-ph}\left(D_0,r_0,\nu_1\to\nu_2,n\right)$ are determined by the following results:

$$\begin{aligned}
L_{1-ph}\left(D_0,r_0,\nu_2,n\right)&=D_0r_0^{-2}\frac{\Gamma\left(\nu_2-\frac{3}{2}\right)\Gamma\left(\frac{3}{2}+\nu_2+n\right)}{n!^2\,\Gamma\left(\frac{3}{2}+\nu_2\right)}\left\{\frac{d^n}{dh^n}\left[\frac{F\left(\frac{\nu_2-3/2}{2}\right),1+\frac{\nu_2-3/2}{2};\nu_2+\frac{3}{2};\frac{A^2}{B^2}}{(1-h)^{\nu_2+\frac{3}{2}}B_1^{\nu_2-3/2}}\right]\right\}_{h=0}\\
L_{2-ph}\left(D_0,r_0,\nu_2,n\right)&=-\frac{D_0r_0^{+2}}{2}\frac{\Gamma(\nu_2)\Gamma\left(\frac{3}{2}+\nu_2+n\right)}{n!^2\,\Gamma\left(\frac{3}{2}+\nu_2\right)}\left\{\frac{d^n}{dh^n}\left[\frac{F\left(\frac{\nu_2}{2}\right),1+\frac{\nu_2-1}{2};\nu_2+\frac{3}{2};\frac{A^2}{B^2}}{(1-h)^{\nu_2+\frac{3}{2}}B^{\nu_2}}\right]\right\}_{h=0}
\end{aligned} \tag{56}$$

Where, $\nu_2\left(\nu_2+1\right)=\tilde{k}_2(\tilde{k}_2+1)+\left(M+E-C\right)D_0r_0^{\,2}$, substituting Eqs. (54), (55) and (56) into Eq. (51), we obtain the modifications to the energy levels ($E_{nc-per:d}\left(\Theta,\tilde{k}_1\right)$, $E_{nc-per:u}\left(\Theta,\tilde{k}_2\right)$) produced by relativistic spin-orbital effect under the pseudo-spin symmetry conditions. Now, the energy levels ($E_{nc-per:d}\left(\Theta,k_1\right)$, $E_{nc-per:u}\left(\Theta,k_2\right)$) produced by relativistic spin-orbital effect under the spin symmetry conditions, can be determined by means of same procedures as before, and to avoid repetition we just make the following steps:

$$\tilde{N}\to N,\tilde{k}_1\to k_1,\tilde{k}_2\to k_2 \text{ and } \theta\left(-E_{nc-ph}\right)\to-\theta\left(E_{nc-ph}\right) \tag{57}$$

This implies that ($E_{nc-per:d}\left(\Theta,k_1\right)$, $E_{nc-per:u}\left(\Theta,k_2\right)$) can be expressed as, respectively:

$$\begin{aligned}
E_{nc-per:d}\left(\Theta,k_1\right)&\equiv\frac{1}{2}\theta\left(E_{nc-ph}\right)k_1\Theta\left(Nn!\frac{\Gamma\left(\nu(k_2)+\frac{3}{2}\right)}{\Gamma\left(n+\nu(k_2)+\frac{3}{2}\right)}\right)^2\left(T_{1-ph}\left(D_0,r_0,\nu(k_2),n\right)+T_{2-ph}\left(D_0,r_0,\nu(k_2),\nu,n\right)\right)\\
E_{nc-per:u}\left(\Theta,k_2\right)&\equiv\frac{1}{2}\theta\left(E_{nc-ph}\right)k_2\Theta\left(Nn!\frac{\Gamma\left(\nu(k_1)+\frac{3}{2}\right)}{\Gamma\left(n+\nu(k_1)+\frac{3}{2}\right)}\right)^2\left(L_{1-ph}\left(D_0,r_0,\nu(k_1),n\right)+L_{2-ph}\left(D_0,r_0,\nu(k_1),n\right)\right)
\end{aligned} \tag{58}$$

The negative and positive signs of the coefficients $\theta\left(-E_{nc-ph}\right)$ and $\theta\left(E_{nc-ph}\right)$ are necessary to ensure that the modifications to the energy levels under the pseudo spin symmetry conditions and spin symmetry conditions are negative and positive, respectively.

The exact relativistic magnetic spectrum for MPH potential for n^{th} **excited states for one-electron atoms in (NC: 3D- R S) symmetries**

Having obtained the exact modifications to the energy levels ($E_{nc-per:d}\left(\Theta,\tilde{k}_1\right)$, $E_{nc-per:u}\left(\Theta,\tilde{k}_2\right)$) and ($E_{nc-per:d}\left(\Theta,k_1\right)$, $E_{nc-per:u}\left(\Theta,k_2\right)$) under the pseudo spin symmetry conditions and spin symmetry conditions, respectively, for n^{th} exited states, produced by NC spin-orbital Hamiltonian operator, we now consider another interested physically meaningful phenomena, which also produced from the perturbative terms of PH potential related to the influence of an external uniform magnetic field, it's sufficient to apply the following two replacements to describing these phenomena:

$$D_0\left(\frac{r_0^{-2}}{r^4}-\frac{r_0^{+2}}{2r^3}\right)\mathbf{L}\Theta \to \chi D_0\left(\frac{r_0^{-2}}{r^4}-\frac{r_0^{+2}}{2r^3}\right)\vec{B}\vec{L} \text{ or } \chi D_0\left(\frac{r_0^{-2}}{r^4}-\frac{r_0^{+2}}{2r^3}\right)\vec{B}\vec{L}$$

and $\Theta \to \chi B$ (59)

Here χ is infinitesimal real proportional's constants, and we choose the magnetic field $\vec{B} = B\vec{k}$, which allow us to introduce the modified new magnetic Hamiltonian $\hat{H}_{mag-ph}(r,a,b,\chi)$ in (NC: 3D-RS), as:

$$\hat{H}_{mag-ph}(D_0,r_0,\chi) = \chi D_0\left(\frac{r_0^{-2}}{r^4}-\frac{r_0^{+2}}{2r^3}\right)\begin{cases}\vec{B}\vec{L} & \text{for pseudo spin symmetry}\\ \vec{B}\vec{L} & \text{for spin symmetry}\end{cases} \tag{60}$$

Here $(-\vec{S}\vec{B})$ denote to the ordinary Hamiltonian of Zeeman effect. To obtain the exact NC magnetic modifications of energy $E_{\text{mag-ph}}(\chi,n,\tilde{m},D_0,r_0)$ and $E_{\text{mag-ph}}(\chi,n,m,D_0,r_0)$ for modified PH potential, under the pseudo spin symmetry conditions and spin symmetry conditions, respectively, which produced automatically by the effect of $\hat{H}_{m-ph}(r,D_0,r_0,\chi)$, we make the following two simultaneously replacements:

$$\tilde{k}_1 \to \tilde{\text{m}}\ ,\ k_1 \to \text{m} \text{ and } \Theta \to \chi\text{ B} \tag{61}$$

Then, the relativistic magnetic modification $E_{\text{mag-ph}}(\chi,n,\tilde{m},D_0,r_0)$ and $E_{\text{mag-ph}}(\chi,n,m,D_0,r_0)$ corresponding n^{th} excited states, in (NC-3D: RS) symmetries, can be determined from the following relation:

$$E_{\text{mag-ph}}(\chi,n,\tilde{m},D_0,r_0) = -\frac{1}{2}\theta(-E_{nc-ph})\chi\text{ B}\left(\tilde{N}n!\frac{\Gamma\left(\nu_1+\frac{3}{2}\right)}{\Gamma\left(n+\nu_1+\frac{3}{2}\right)}\right)(L_{1-ph}+L_{2-ph})\tilde{m}$$

$$E_{\text{mag-ph}}(\chi,n,m,D_0,r_0) = \frac{1}{2}\theta(E_{nc-ph})\chi\text{ B}\left(Nn!\frac{\Gamma\left(\nu(k_1)+\frac{3}{2}\right)}{\Gamma\left(n+\nu(k_1)+\frac{3}{2}\right)}\right)(L_{1-ph}+L_{2-ph})m \tag{62}$$

Where, $\tilde{m}$ and m denotes to the angular momentum quantum numbers $-\tilde{l} \le \tilde{m} \le +\tilde{l}$ and $-l \le m \le +l$, which allow us to fixing ($2\tilde{l}+1$) and $(2l+1)$ values, respectively.

The Exact Modified Global Spectrum for MPH Potential in (NC-3D: RS) Symmetries for One-Electron Atoms

Let us now resume the n^{th} excited states eigenenergies ($E_{nc-pd}(\Theta,\tilde{k}_1,\chi,n,\tilde{m},r_0,D_0)$, $E_{nc-p:u}(\Theta,\tilde{k}_2,\chi,n,\tilde{m},r_0,D_0)$) and ($E_{nc-d}(\Theta,k_1,\chi,n,m,r_0,D_0)$, $E_{ncr-u}(\Theta,k_2,\chi,n,m,r_0,D_0)$) of MDE corresponding the pseudo spin symmetry conditions and spin symmetry conditions, respectively, at first order of two parameters (Θ,χ) for MPH potential in (NC: 3D-RS) symmetries, on based to the obtained results (51), (58), and (62), in addition to the original results (18) and (20) of energies in commutative space, we obtain the following original results:

$$E_{nc-pd} = E_{n\tilde{k}_1} - \frac{1}{2}\theta(-E_{nc-ph})\left(\tilde{N}n!\frac{\Gamma\left(\nu_1+\frac{3}{2}\right)}{\Gamma\left(n+\nu_1+\frac{3}{2}\right)}\right)^2(T_{1-ph}+T_{2-ph})(\tilde{k}_1\Theta+\chi\text{ B}\tilde{m})$$

$$E_{nc-p:u} = E_{n\tilde{k}_2} + \frac{1}{2}\theta(-E_{nc-ph})\left(\tilde{N}n!\frac{\Gamma\left(\nu_1+\frac{3}{2}\right)}{\Gamma\left(n+\nu_1+\frac{3}{2}\right)}\right)^2(T_{1-ph}+T_{2-ph})(\tilde{k}_2\Theta+\chi\text{ B}\tilde{m}) \tag{63}$$

$$E_{nc-d} = E_{nk_1} - \frac{1}{2}\theta(E_{nc-ph})\left(Nn!\frac{\Gamma\left(\nu_1+\frac{3}{2}\right)}{\Gamma\left(n+\nu_1+\frac{3}{2}\right)}\right)^2(L_{1-ph}+L_{2-ph})(k_1\Theta+\chi\text{ B}m)$$

$$E_{ncr-u} = E_{nk_2} + \frac{1}{2}\theta(E_{nc-ph})\left(Nn!\frac{\Gamma\left(\nu_1+\frac{3}{2}\right)}{\Gamma\left(n+\nu_1+\frac{3}{2}\right)}\right)^2(L_{1-ph}+L_{2-ph})(k_2\Theta+\chi\text{ B}m) \tag{64}$$

Now, it is important to constructing the Hamiltonian operator $\hat{H}_{nc-ph}$ for MPH potential on based to previously obtained results. Naturally, to consider the first term in the modified Hamiltonian operator represents the kinetic energy and the potential energy in ordinary commutative space $\hat{H}_{ph}$ of the fermionic particle which presented by eq. (9), the second term $\hat{H}_{so-ph}(k_1,k_2)$ or $\hat{\tilde{H}}_{so-ph}(\tilde{k}_1,\tilde{k}_2)$ represents, the induced spin-orbital parts for the pseudo spin symmetry conditions and spin symmetry and the last term is the modified new magnetic Hamiltonian $\hat{H}_{mag-mt}(r,a,b,\chi)$:

$$\hat{H}_{nc-ph} = \hat{H}_{ph} + \begin{cases}\hat{\tilde{H}}_{so-ph}(\tilde{k}_1,\tilde{k}_2)+\chi D_0\left(\frac{r_0^{-2}}{r^4}-\frac{r_0^{+2}}{2r^3}\right)\vec{B}\vec{L} & \text{for pseudo spin symmetry}\\ \hat{H}_{so-ph}(k_1,k_2)+\chi D_0\left(\frac{r_0^{-2}}{r^4}-\frac{r_0^{+2}}{2r^3}\right)\vec{B}\vec{L} & \text{for spin symmetry}\end{cases} \tag{65}$$

In this way, one can obtain the complete energy spectra for MPH potential in (NC: 3D-RS) symmetries. Know the following accompanying constraint relations:

a. The original spectrum contains two possible values of energies in ordinary three dimensional spaces which presented by Eqs. (16) and (18),

As mentioned in the previous subsection, the quantum numbers $\tilde{m}$ and m satisfied the two intervals: $-\tilde{l} \le \tilde{m} \le +\tilde{l}$ and $-l \le m \le +l$, thus we have ($2\tilde{l}+1$) and $(2l+1)$ values, respectively,

We have also two values for ($j = \tilde{l} + \frac{1}{2}$ and $j = \tilde{l} - \frac{1}{2}$) and ($j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$) for pseudo spin symmetry conditions and spin symmetry. Allow us to deduce the important original results: every state in usually 3-dimensional spaces will be replacing by $2(2\tilde{l}+1)$ and $2(2l+1)$ sub-states.

Then the degenerated state can be take $2\sum_{i=0}^{n-1}(2l+1) \equiv 2n^2$ values in (NC: 3D-RS) symmetries. Finally, we resume our original results in this article, the first one is the induced pseudo-spin-orbital and spin-orbital Hamiltonian operators ($\hat{\tilde{H}}_{so-ph}\left(\tilde{k}_1,\tilde{k}_2\right)$ and $\hat{H}_{so-ph}\left(k_1,k_2\right)$) and corresponding eigenvalues ($E_{nc-per:d}\left(\Theta,\tilde{k}_1\right)$, $E_{nc-per:u}\left(\Theta,\tilde{k}_2\right)$) and ($E_{nc-per:d}\left(\Theta,k_1\right)$, $E_{nc-per:u}\left(\Theta,k_2\right)$), respectively as:

$$\hat{\tilde{H}}_{so-ph}\left(\tilde{k}_1,\tilde{k}_2\right)\Psi_{nk}\left(r,\theta,\phi\right)\Rightarrow\begin{cases}\left(\hat{\tilde{H}}_{so-ph}\right)_{11}\left(\tilde{k}_1\right)\left(i\frac{G_{n\tilde{k}}(r)}{r}Y_{jm}^{\tilde{l}}(\theta,\phi)\right)=E_{nc-per:d}\left(\Theta,\tilde{k}_1\right)\left(i\frac{G_{n\tilde{k}}(r)}{r}Y_{jm}^{\tilde{l}}(\theta,\phi)\right)\\ \left(\hat{\tilde{H}}_{so-ph}\right)_{22}\left(\tilde{k}_2\right)\left(i\frac{G_{n\tilde{k}}(r)}{r}Y_{jm}^{\tilde{l}}(\theta,\phi)\right)=E_{nc-per:u}\left(\Theta,\tilde{k}_1\right)\left(i\frac{G_{n\tilde{k}}(r)}{r}Y_{jm}^{\tilde{l}}(\theta,\phi)\right)\\ \left(\hat{\tilde{H}}_{so-ph}\right)_{33}\left(\tilde{k}_1\right)\left(i\frac{G_{n\tilde{k}}(r)}{r}Y_{jm}^{\tilde{l}}(\theta,\phi)\right)=0\end{cases}\quad(66)$$

$$\hat{H}_{so-ph}\left(k_1,k_2\right)\Psi_{nk}\left(r,\theta,\phi\right)\Rightarrow\begin{cases}\left(\hat{H}_{so-ph}\right)_{11}\left(k_1\right)\left(\frac{F_{nk}(r)}{r}Y_{jm}^{l}(\theta,\phi)\right)=E_{nc-per:d}\left(\Theta,k_1\right)\left(\frac{F_{nk}(r)}{r}Y_{jm}^{l}(\theta,\phi)\right)\\ \left(\hat{H}_{so-ph}\right)_{22}\left(k_2\right)\left(\frac{F_{nk}(r)}{r}Y_{jm}^{l}(\theta,\phi)\right)=E_{nc-per:u}\left(\Theta,k_2\right)\left(\frac{F_{nk}(r)}{r}Y_{jm}^{l}(\theta,\phi)\right)\\ \left(\hat{H}_{so-ph}\left(k_1,k_2\right)\right)_{33}\left(\frac{F_{nk}(r)}{r}Y_{jm}^{l}(\theta,\phi)\right)=0\end{cases}\quad(67)$$

The second original results are the induced the modified new magnetic Hamiltonian operator $\hat{H}_{mag-mt}\left(r,a,b,\chi\right)$ and corresponding eigenvalues $E_{\text{mag-ph}}\left(\chi,n,\tilde{m},D_0,r_0\right)$ and $E_{\text{mag-ph}}\left(\chi,n,m,D_0,r_0\right)$, respectively as:

$$\hat{H}_{mag-mt}\left(r,a,b,\chi\right)\Psi_{nk}\left(r,\theta,\phi\right)\equiv\frac{\chi D_0}{r}\left(\frac{r_0^{-2}}{r^4}-\frac{r_0^{+2}}{2r^3}\right)\begin{pmatrix}\vec{B}\vec{L} & 0\\ 0 & \vec{B}\vec{\tilde{L}}\end{pmatrix}\begin{pmatrix}F_{nk}(r)Y_{jm}^{l}(\theta,\phi)\\ iG_{n\tilde{k}}(r)Y_{jm}^{\tilde{l}}(\theta,\phi)\end{pmatrix}=\begin{pmatrix}E_{\text{mag-ph}}\left(\chi,n,m,D_0,r_0\right)\frac{F_{nk}(r)}{r}Y_{jm}^{l}(\theta,\phi)\\ E_{\text{mag-ph}}\left(\chi,n,\tilde{m},D_0,r_0\right)i\frac{G_{n\tilde{k}}(r)}{r}Y_{jm}^{\tilde{l}}(\theta,\phi)\end{pmatrix}\quad(68)$$

It is worth to mention that (in the limit $\Theta\to 0$) we obtain the commutative result of the relativistic negative energy eigenvalues and positive energy eigenvalues under pseudo spin symmetry and spin-symmetry in addition to the relativistic Hamiltonian operator for PH potential.

Concluding Remarks

In this paper we have performed the exact analytical bound state solutions: the energy spectra and the corresponding NC Hermitian Hamiltonian operator for three dimensional MDE in spherical coordinates for MPH potential by using Bopp's Shift method and standard perturbation theory. It is found that the energy eigenvalues depend on the dimensionality of the problem and new atomic quantum numbers ($j=\tilde{l}\pm 1/2$, $j=l\pm 1/2$, $\tilde{s}=\pm 1/2, l$, $\tilde{l}$, $\tilde{m}$ and m) in addition to the two infinitesimal parameters(Θ and χ), and we also showed that the obtained energy spectra degenerate and every old state will be replaced by $2\left(2\tilde{l}+1\right)$ and $2\left(2l+1\right)$ sub-states under the pseudo spin symmetry and spin symmetry conditions, respectively, for n^{th} exited states.

Acknowledgement

This work was supported with search laboratory of: Physics and Material Chemistry, in Physics department, Sciences faculty, University of M'sila, Algeria, The author would like to thank the kind's referees for the positive comments and suggestions which led to the improved standard of this article.

Conflict of Interest

None.

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Citation: Maireche A (2018) A New Relativistic Bound States Solution for Modified Pseudoharmonic Potential in One-electron Atoms at Planck's and Nano-Scales. J Nanomed Res 7(1): 00168. DOI: 10.15406/jnmr.2018.07.00168