Impendant method for determining the reduction of hydraulic resistance in large arterial vessels with permeable walls

Abstract

The article deals with the problem of stationary blood flow in vessels with permeable walls. To determine the hydraulic resistance in an arterial vessel, the blood is considered to be a Newtonian viscous fluid, and the flow is stationary. In solving problems, formulas are obtained for determining the corresponding hydrodynamic parameters, such as velocity, fluid flow, and pressure gradient. Impedance method determined the hydraulic resistance. With a steady flow, the hydraulic resistance in the permeable vessel depends significantly on the permeability coefficient: with increasing this coefficient, it decreases.

Keywords: newton’s liquid, continuity, oscillation parameter, pulsating motions

Introduction

During the contraction of the heart from the heart along the walls of the blood vessels, a wave of pressure propagates, which is called a pulse wave. As the wave moves away from the heart, this wave gradually weakens and practically fades in the capillaries. The speed of the pulse wave propagation depends on many factors, among which, for example, the elastic-viscous properties of the vessel wall, blood pressure, its density, viscosity, permeability of the vessel wall, etc. can be noted. In most works, the main attention is paid to determining the propagation of a pulse pressure wave taking into account the elasticity of the vessel wall, and its permeability is not taken into account anywhere. However, the permeability of the wall significantly affects the propagation of the pulse pressure wave and its damping. The branching of the arterial tree is modelled by introducing a “permeability” of the wall with volume outflows of transverse velocity to the walls. In fact, this velocity is a strongly discontinuous function of the coordinate, when using the model of a permeable tube this function is always smoothed by the wall surface. Apparently, the idea of modelling an artery in the form of a permeable tube was first expressed in and is developed further in. The most complete study of such a model is contained in, where the system of blood circulation of a dog is modelled. Unlike other works, to determine the hydraulic resistance in the central arterial vessel, the formulation of the problem of pulsating blood flow in the arterial channel was considered in, where the outflow of blood is mathematically modelled as a permeable wall of blood vessels. It should be noted that in addition to wave flows, it is advisable to study the steady flow of blood in a vessel with permeable walls. However, this case does not follow from the solution for wave fluxes as the limit as the frequency tends to zero, so the stationary problem should be considered in a separate formulation.

Statement of problem and methods of solution

For a steady flow of a fluid,

\[ \frac{\partial v_x}{\partial t} = \frac{\partial p}{\partial x} = 0, \frac{\partial v_x}{\partial x} = \frac{\partial v_y}{\partial t} = \frac{\partial v_y}{\partial y} = 0 \]

Taking them into account, we obtain from [1-5] a system of equations

\[ \frac{1}{\rho} \frac{\partial p}{\partial r} = \left( \frac{\partial^2 v_x}{\partial r^2} + \frac{1}{r} \frac{\partial v_x}{\partial r} + \frac{\partial^2 v_y}{\partial z^2} \right) \]

\[ \frac{1}{\rho} \frac{\partial p}{\partial r} = \left( \frac{\partial^2 v_x}{\partial r^2} + \frac{1}{r} \frac{\partial v_x}{\partial r} - \frac{1}{r^2} v_x + \frac{\partial^2 v_y}{\partial z^2} \right) \]

\[ \frac{\partial v_x}{\partial r} + \frac{1}{r} v_x + \frac{\partial v_y}{\partial z} = 0 \]

And boundary conditions

\[ v_x = \frac{R^*}{\mu} (p - p_0), v_y = 0, r = R \]

\[ \frac{\partial v_x}{\partial r} = 0, v_x = 0, r = 0 \]

\[ p = p_0; Q = Q_0, x = 0 \]

Where \( \rho \) - density of blood; \( r \) - radial; \( x \) - longitudinal coordinate; \( v_x \) and \( v_y \) - radial and axial velocity components; \( \gamma \) - cinematic viscosity of blood; \( p \) - internal pressure; \( p_0 \) - pressure in the environment; \( p_0, u, Q_0 \) - average pressure values and flow rate; \( \gamma^* \) - coefficient of permeability

From the system (1), after making some calculations, we find the equation for the pressure

\[ \frac{\partial^2 p}{\partial x^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} = 0 \]

According to which in a stationary flow the pressure distribution obeys the Laplace equation. We seek the solution of (3) in the form

\[ p = p_1(r) \exp(i\gamma^* x) + p_2(r) \exp(-i\gamma^* x) \]
This procedure follows from the general solution of the condition that, in this case, the complex argument is transformed into the real one, i.e.

\[ p = p_1(r) \exp \left( \frac{\lambda}{r} \right) + p_2(r) \exp \left( -\frac{\lambda}{r} \right) \]

Here all functions are real. Therefore, in the stationary problem, there is no need for a complex solution.

The substitution of (5) into (3) leads to the Bessel equation

\[ \frac{d^2 p_2}{dr^2} + \frac{1}{r} \frac{dp_2}{dr} + \frac{\lambda^2}{r^2} p_2 = 0 \]

His fundamental decisions will be:

\[ J_0 \left( \frac{\lambda}{r} \right) \quad \text{and} \quad Y_0 \left( \frac{\lambda}{r} \right) \]

In this case the general solution has the form

\[ p_{1,2} = c_1 J_0 \left( \frac{\lambda}{r} \right) + c_2 Y_0 \left( \frac{\lambda}{r} \right) \]

At the pressure becomes infinite, so the coefficients in front of the function \( Y_0 \left( \frac{\lambda}{r} \right) \) must vanish and in this case the solution of (6) takes the form

\[ p_{1,2} = c_1 J_0 \left( \frac{\lambda}{r} \right) \]

Or finally, we get a solution

\[ p - p_c = J_0 \left( \frac{\lambda}{r} \right) \left[ c_1 \exp \left( \frac{\lambda}{r} \right) - c_2 \exp \left( -\frac{\lambda}{r} \right) \right] \]

Where the coefficients \( c_1 \) and \( c_2 \) are determined from the boundary conditions (2). The calculation of (8) for the first equation of system (1) gives the equation

\[ \frac{d^2 v_r}{dr^2} + \frac{1}{r} \frac{dv_r}{dr} - \frac{\lambda^2}{r^2} v_r = \frac{\lambda}{r^2} J_0 \left( \frac{\lambda}{r} \right) \]

whose solution is sought in the form

\[ v_r = v_{1,2} \left( r \right) \left[ c_1 \exp \left( \frac{\lambda}{r} \right) - c_2 \exp \left( -\frac{\lambda}{r} \right) \right] \]

Then

\[ \frac{d^2 v_{1,2}}{dr^2} + \frac{1}{r} \frac{dv_{1,2}}{dr} + \frac{\lambda^2}{r^2} v_{1,2} = \frac{\lambda}{r^2} J_0 \left( \frac{\lambda}{r} \right) \]

Equation (11) is solved under the condition that the liquid adheres to the wall of the tube, i.e. \( v_r \rightarrow 0 \) at \( r = R \) and limitations \( v_r \rightarrow 0 \) on the axis of the pipe.

The fundamental solutions (11) of the Bessel function of zero order are expressed in this way

\[ J_0 \left( \frac{\lambda}{r} \right) \quad \text{and} \quad Y_0 \left( \frac{\lambda}{r} \right) \]

Equation (11) is inhomogeneous, and therefore its particular solution is found from formula

\[ v_{1,2} = x J_0 (x) f(x) \exp \left( \frac{x}{r} \right) - \frac{2}{\pi} J_0 (x) x Y_0 (x) f(x) \exp \left( \frac{x}{r} \right) \]

Where

\[ f(x) = \frac{\lambda}{r} J_0 (x), \quad x = \frac{\lambda}{r} \]

Sometimes, taking into account the relation

\[ J_0 (x) Y_0 (x) - J_0 (x) Y_0 (x) = \frac{2}{\pi x} \]

We get it

\[ v^*_{1,2} \left( \frac{\lambda}{r} \right) = \frac{1}{2\mu} J_0 \left( \frac{\lambda}{r} \right) \]

The use of the fundamental solution (12) and the limited velocity on the axis of the tube gives

\[ v_{1,2} = c_1 J_0 \left( \frac{\lambda}{r} \right) + \frac{1}{2\mu} J_0 \left( \frac{\lambda}{r} \right) \]

From condition \( v_r = 0 \) at \( r = R \), define

\[ \tau = \frac{1}{2\mu} J_0 (\lambda) \]

Substituting (17) into (16), from (10) we find

\[ v_r = R \frac{J_0 (\lambda)}{2\mu} \left[ \frac{1}{\lambda} J_1 (\lambda) + \frac{1}{\lambda^2} J_0 (\lambda) \right] \]

In the same way, using the solution (18) and the second equation of the system (1), we obtain a formula for the distribution of the transverse velocity:

\[ v_r = R \frac{J_0 (\lambda)}{2\mu} \left[ \frac{1}{\lambda} J_1 (\lambda) + \frac{1}{\lambda^2} J_0 (\lambda) \right] \]

Using the wall permeability condition (2) leads to an equation for determining the eigenvalue \( \lambda \)

\[ \tau^* = \frac{1}{2\mu} \left[ \frac{1}{\lambda} J_1 (\lambda) + \frac{1}{\lambda^2} J_0 (\lambda) \right] \]

If \( \lambda \approx i \) then, expanding the functions \( J_0 (\lambda), J_1 (\lambda) \) in convergent series

\[ J_0 (z) = 1 - \frac{z^2}{4} + \frac{z^4}{12} - \frac{z^6}{30} + \ldots,
\]

\[ J_1 (z) = z - \frac{z^3}{2} + \frac{z^5}{2} - \frac{z^7}{30} + \ldots,
\]

From (20), after some calculations, we find

\[ \lambda = 4 \sqrt{\tau} \]

Equation (20) is transcendental and has an infinite set of roots \( \lambda_n \).

Summation over eigenvalues \( \lambda_n \) gives formulas for pressure, radial and axial velocities

\[ p - p_c = \sum_{n=1}^{\infty} J_0 \left( \frac{\lambda_n}{r} \right) \left[ A_n \exp \left( \frac{\lambda_n}{r} \right) + B_n \exp \left( -\frac{\lambda_n}{r} \right) \right] \]

\[ v^*_{1,2} \left( \frac{\lambda_n}{r} \right) = \frac{1}{2\mu} J_0 \left( \frac{\lambda_n}{r} \right) \]

Where

\[ \tau^* = \frac{1}{2\mu} \left[ \frac{1}{\lambda_n} J_1 (\lambda_n) + \frac{1}{\lambda_n^2} J_0 (\lambda_n) \right] \]

[80]

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Results and discussion

Formulas (23) allow us to investigate the characteristics of the internal structure of the flow of viscous blood in a pipe with permeable walls: hydraulic resistance on a permeable wall, fluid flow along the length of the tube, and a number of other hydrodynamic parameters. Using (23), we find the distribution of pressure drop, blood flow and shear stress on the wall:

\[ \frac{-dP}{dx} = \sum_{m=1}^{\infty} \frac{A_m}{R} \exp \left( \frac{A_m x}{R} \right) \frac{\lambda}{R} B_m \exp \left( -\frac{\lambda}{R} x \right) \] (25)

\[ Q = 2\pi \int_0^R \left[ \frac{1}{\lambda} \sum_{m=1}^{\infty} \frac{A_m}{R} \exp \left( \frac{A_m x}{R} \right) \frac{\lambda}{R} B_m \exp \left( -\frac{\lambda}{R} x \right) \right] dx \] (26)

Using formulas (25) and (26), we determine the ratio of the pressure drop per unit length to the blood flow:

\[ \frac{-dP}{dx} = \frac{8\mu}{\pi R^4} \sum_{m=1}^{\infty} \frac{A_m}{R} J_0(\lambda_m) \] (28)

Or, in a dimensionless form,

\[ \frac{\pi R^4}{8\mu} \frac{-dP}{dx} = \sum_{m=1}^{\infty} \frac{A_m}{R} J_0(\lambda_m) \] (29)

Attitude \( \frac{-dP}{dx} \) is called the effective impedance, in the general case it is expressed by a function of a complex variable. Therefore, its main part determines the hydraulic resistance of the flow. It can be seen from Figure 1 that the effective impedance at a stationary flow in a permeable pipe essentially depends on the permeability coefficient: with an increase in this coefficient, it decreases. In mathematical modelling of blood flow in the arterial bed, where the outflow of blood from the main trunk is taken into account as a permeable wall of the vessel. For a quantitative analysis, we cite data from2 for a dog. With the aid of Figure 2 we calculate the sum of the transverse areas of the lateral branch, it is approximately 3.6 cm², and the surface area of the main arterial trunk is 240 cm². The coefficient of permeability of the wall is determined by the ratio of the total transverse areas of the lateral branch to the surface area of the main trunk. In this case, it is 0.015. It can be seen from Figure 1 that in the arterial vessel the hydraulic resistance decreases due to the permeability of the wall, when it is 0.015, corresponds to a decrease by 9-10 percent.

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Conflict of interest

The authors declare no conflict of interest.

References


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