The Case of Optimal Control with Exceptional Role of the Conditions of Transversality

Abstract

This research considered solving the optimal control problem of spacecraft (as solid body) when the conditions of transversality have key significance. It is shown that the assumed criterion of optimality guarantees motion of spacecraft with energy not exceeding the required value. Topicality of article is caused by fact that on concrete example, the conditions of transversality are demonstrated to be very important mathematical instrument (even the only) for finding the main properties, laws and key characteristics (parameters, constants, integrals of motion) of optimal solution of control problem.

Keywords: Optimal control; Criterion of optimality; Maximum principle; Conditions of transversality; Controlling functions

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Introduction

Its investigate motion of spacecraft (as solid body) relative to centre of mass was investigated in this research work. Spacecraft attitude is described by the quaternion Λ (it give position of the body axes relative to inertial coordinate system) and the vector ω of absolute angular velocity. Then, equation of motion [1]

$$2\dot{\Lambda} = \Lambda \circ \omega \qquad (1)$$

(It is assumed that $\|\Lambda(0)\| = 1$). In order to estimate the efficiency of control, the functional to be optimized is introduced as presented in equation 2.

$$G = \int_0^T \left(a_1\left(J_1\omega_1^2 + J_2\omega_2^2 + J_3\omega_3^2\right) + a_2 \right) dt \qquad (2)$$

Where J_i are the spacecraft central principal moment of inertia; ω_i are the components of vector $\omega\left(i=\overline{1,3}\right)$; a_1 =const>0, a_2 =const>0. Let solve following problem of control: take the spacecraft from initial attitude Λ_{in} into final attitude Λ_f obeying Eq. (1) so as to minimize integral (2) (the time T is not given). The taken criterion of optimality combines (in given proportion) the time and integral of energy to be expended for slew maneuver. Aspects of finding economical control are topical now.

For solving the formulated problem, use Pontryagin's maximum principle [2] and the universal variables [3] (because the optimized functional does not include positional coordinates). The assumed integral (2) does not include the torques M_i; the sought-for function $\omega(t)$ is piecewise continuous function of time. For our optimization problem, the Hamiltonian is

$$H = r_1\omega_1 + r_2\omega_2 + r_3\omega_3 - a_1\left(J_1\omega_1^2 + J_2\omega_2^2 + J_3\omega_3^2\right) - a_2,$$

Where r_i are universal variables (as the components of vector r) satisfying equations [3]

$$\dot{r}_1 = \omega_3 r_2 - \omega_2 r_3, \dot{r}_2 = \omega_1 r_3 - \omega_3 r_1, \dot{r}_3 = \omega_2 r_1 - \omega_1 r_2 \qquad (3)$$

The Hamiltonian H is written, ignoring the constraint $\|\Lambda\| = 1$ since $\|\Lambda(t)\| = 1$ under any $\omega(t)$ for Eq. (1) (of course $\|\Lambda_{in}\| = \|\Lambda_f\| = 1$). Optimal function r($t$) is computed by the quaternion $\Lambda(t)$ [1,3]:

$$r = \tilde{\Lambda} \circ_{CE} \circ \Lambda, \text{ where } CE = Const = \Lambda_{in} \circ r(0) \circ \tilde{\Lambda}_{in}$$

For the vector r of universal variables $|r|$ = const ≠ 0. The function H is maximal if the relations

$$\omega_i = r_i / 2a_1 J \qquad (4)$$

are satisfied. As is known, the functions r_i and ω_i should satisfy the conditions of transversality which are r(0) ≠ 0, r(T)≠0 (since left and right endpoints of the trajectory $\Lambda(t)$ are fixed) and H=0 because the maneuver end time T is not fixed and the Hamiltonian H is independent of time in explicit form. After substitution Eq.(4) in expression for H and the requirement H = 0, obtain the equation

$$\left(r_1^2/J_1 + r_2^2/J_2 + r_3^2/J_3\right)/4a_1 - a_2 = 0$$

Through which have following key properties of the controlled motion:

$$r_1^2/J_1 + r_2^2/J_2 + r_3^2/J_3 = const = 4a_1a_2,$$
$$J_1\omega_1^2 + J_2\omega_2^2 + J_3\omega_3^2 = const, \qquad (5)$$
$$J_1^2\omega_1^2 + J_2^2\omega_2^2 + J_3^2\omega_3^2 = const$$

Last property follows directly from the demands (4) (they formalize condition of maximum for H). The condition of transversality H=0 takes place at each instant of time [4].

The problem of optimal control is reduced to finding the solution to the system of differential equations (1), (3) under the condition that the control ω is chosen based on condition (4) with the simultaneous satisfaction of the condition of transversality

H=0 and the boundary conditions $\Lambda(0)=\Lambda_{in}, \Lambda(T)=\Lambda_f$ (the conditions of transversality r(0) ≠ 0 and r(T)≠0 are satisfied automatically, as it follows from first equality (5) written for optimal motion). The system of differential equations (3) for the variables r_i, together with the requirement of maximizing the Hamiltonian H and the condition H=0, provides the necessary optimality conditions. Reminding that the coefficients $a_1 \neq 0$ and $a_2 \neq 0$. If we take the ort $p = r/|r|$ then $r_0 = 2\sqrt{a_1 a_2}/C$; $E_k = a_2/2a_1$; $|L| = \sqrt{a_2/a_1}/C$ where $r_0 = |r|$; $C = \sqrt{P_{10}^2/J_1 + P_{20}^2/J_2 + P_{30}^2/J_3}$; p_{i0} are the components of the vector $p_0 = p(0)$; E_k is rotary energy; L is angular momentum;

$$P_1^2/J_1 + P_2^2/J_2 + P_3^2/J_3 = const, \text{ since } |r| = \text{const}.$$

The boundary value problem of the maximum principle is to determine such value of the vector p_0 at which the solution $\Lambda(t)$ of the motion equation (1) and differential equations (3) (with the simultaneous satisfying the equalities (4) at each instant of time) satisfies the maneuver conditions $\Lambda(0)=\Lambda_{in}$ and $\Lambda(T)=\Lambda_f$ (the quantity r_0 is calculated unambiguously by p_0 and the coefficients a_1, a_2). Optimal vector p_0 is determined only by the values Λ_{in}, Λ_f and J_1, J_2, J_3.

Punctual consecutive implementation of procedure of the maximum principle for dynamical problem of optimal slew maneuver (when $\omega(0)=\omega(T)=0$ and the control torque M is limited) show that maximal rotary energy E_k is no more the ratio $a_2/2a_1$ for any instant of time $t \in [0, T]$ (independently of duration of acceleration and braking). I.e. always, during optimal rotation from the position Λ_{in} into the position Λ_f (in the sense of minimum (2)), rotary energy of spacecraft have restriction by known upper level determined by the coefficients a_1, a_2 of the minimized functional. If assume a_1 = 0.5 and $a_2 = E_{ad}$ then optimization of motion program by criterion (2) give satisfaction of the inequality $J_1\omega_1^2 + J_2\omega_2^2 + J_3\omega_3^2 \le 2E_{ad}$ for any instant of time, where E_{ad} is admissible rotary energy. In our variation problem, find the main properties, laws and key characteristics (parameters, constants, integrals of motion) of optimal solution of control problem using the conditions of transversality as very important and unique mathematical instrument. Chosen criterion of optimality guarantees motion of solid body with rotary energy not exceeding the required value.

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Conflict of Interest

No conflict of interest exists.

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