

Peculiarity of Taylor's and wavy vortices beginning, generated by a rotating magnetic field

Abstract

Results of research of stationary instability of axisymmetric laminar flow of a conducting liquid, arising under the influence of the rotating magnetic field of any rotary symmetry, are briefly presented. It is shown, that the secondary flow arises in the form of Taylor's vortices or wavy vortices of this or that order. The correctness of linear statement of a problem of stability is established.

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Introduction

Stationary instability of axisymmetric laminar flow of a viscous conducting liquid in the infinitely long circular cylinder, arising under the influence of coaxially rotating magnetic field (RMF) of any rotary symmetry, has been investigated. The nearest analogue of the conducting liquid flow raised by the rotating magnetic field is Couette flow between two concentric cylinders from which internal one rotates and external remains motionless. While analyzing stability of such flow, Lin¹ has noticed, that the first approximation equations for small perturbations of velocity and pressure allow for periodic solution with respect to ϕ and z :

$$f=f(r)\exp(\sigma t+in\phi+iaz), \quad (1)$$

where σ is the complex constant, n is the integer number (order of sinuosity), and a is the real (dimension less wave number).

Usually one deals with a special case of rotary symmetry $n=0$. In this case a primary flow is independent of ϕ , but disturbances of velocity u_r , u_ϕ , u_z and pressure q are not zero. The series of papers in which stability was investigated in the approximation of small values of relative frequency $\omega^{2,3}$ in the approximation of big ω^4 and for intermediate values $\omega^{5,6}$ has been issued by the Donetsk group of fluid dynamicists, lead by A. B. Kapusta. Within all investigated ranges of flow parameters only one-vortical (in the radial direction) structures of Taylor's vortices have been found in the calculations.

The case $n \neq 0$ corresponds to the occurrence of the so-called wavy vortices. For the first time, such problem for Couette flow between coaxial cylinders have been considered by Davey et al.⁷ The work investigated bifurcation of Taylor vortices into wavy vortices, i.e. in such flow when waves extend along the axis of Taylor's vortices. As the number of waves which are passing on these vortices, can be only integer it is clear why in (1) n is the integer (n is the order of sinuosity). The results obtained in this work, will well be in agreement with results of experiments by Coles⁸ & Gollub et al.⁹

Current study briefly presents results of researching the case of $n \neq 0$ as applied to the flow of a conducting liquid arising in a cylindrical vessel under the influence of rotating magnetic field. The problem of stability for arbitrary symmetry of RMF has been considered in linear formulation.¹⁰ Certainly, the use of linear statement in the research of loss of stability of Taylor's vortices and transition to wavy vortices, can raise some doubts. After the loss of stability resulting flow will represent superposition of a primary azimuthal flow and system of

Taylor's vortices. Furthermore it is necessary to investigate stability of already complex flow involving occurrence of wavy vortices. The strategy of such a research is illustrated by the citation from the monograph of Richtmyer.¹¹

In recent years, the nonlinear theory developed by Davey et al.⁷ has led to an understanding of the structure and stability of the finite-amplitude Taylor vortices, the second bifurcation to the wavy vortices, and the structure and stability of the wavy vortices, as discussed below.

In a problem like this, involving a sequence of bifurcations, the theory consists ideally of a sequence of alternately linear and nonlinear investigations. After each bifurcations the structure and amplitude of the new flow is found by a nonlinear calculations. Its stability is then investigated by linear zing the equations about the new flow and studying the growth of infinitesimal disturbances, in order to find the next bifurcation, and so on.

Apparently, the MHD flow raised by weak rotating magnetic field is most closely analogous to Couette flow between two concentric cylinders. Davey et al.⁷ have theoretically established, that Taylor's critical number at which Taylor's vortices become unstable and wavy vortices appear exceeds by 8% the Taylor's number at which the primary one-dimensional flow loses the stability and for the first time Taylor's vortices appear. In the assumption, that in weak RMF and the primary flow will be weak, and nonlinear additions will be even weaker, the similar estimation for linear statement of a problem of stability has been made. As Davey et al.⁷ used Taylor's number as criterion of an estimation of the stability for the correct comparison of the results in our case, we will take advantage of analogue of Taylor's number for magnetic hydrodynamics – Taylor's magnetic number. Corresponding excess of Taylor's magnetic number for the case of one pair of poles of the RMF makes up 6.78% that close to the result obtained by Davey et al.⁷

The problem is solved for two cases: in low-frequency approach and for any value of relative frequency. With the use of Galerkin's method the curves of neutral stability corresponding to occurrence of Taylor's and wavy vortices are calculated. Already for $p=1$ interesting enough results have been obtained. It was found, that the curves of neutral stability obtained for $n=1$ cross the curves of neutral stability for $n=0$; curves for $n=2$ cross curves for $n=1$ etc. Taylor's vortices ($n=0$) arise in wide enough, but limited range of flow parameters, but upon the increase in parameters, they lose stability and convert to wavy vortices ($n>0$). It is established, that under certain conditions the loss of stability of a primary flow leads to the direct appearance of

wavy vortices of some order, by-passing the stage of Taylor's vortices. All this means that the use of linear statement in the study of stability of the flow raised by the RMF, is quite correct. On the curve of neutral stability separating area of one-dimensional azimuthal flow from the region of a three-dimensional vortical flow, points of bifurcation are defined corresponding to the transition from secondary flow in the form of Taylor's vortices to the secondary flow in the form of wavy vortices with $n=1$ and further with consecutively higher orders of a sinuosity. It is typical that such transitions are accompanied by step-wise increase in wave number, i.e. transition to smaller and smaller scale vortices. Such a cascade of bifurcations is observed both on the branch of the neutral stability corresponding to low-frequency approach, and on the branch corresponding to the case of arbitrary values of relative frequency.

As well the interesting phenomena are observed at an increase in the order of rotary symmetry of the RMF. At $p=2$ the range of parameters at which the loss of stability of a primary flow leads to occurrence of Taylor's vortices, is reduced. At $p=3$ Taylor's vortices do not arise at all, instead the wavy vortices appear with $n \geq 1$. At $p=4$ a zone of occurrence of wavy vortices with $n=1$ is reduced, and at $n \geq 2$, wavy vortices appear already only at $n \geq 2$. The increase in the values of Hartmann number, the relative frequency and in the order of rotary symmetry of the RMF, reduces the characteristic size both Taylor's and wavy vortices. The vortices centre is thus displaced towards the cylinder wall.

The completed research has allowed establishing in general terms that at the loss of stability of a primary flow, the occurrence of wavy vortices without an intermediate stage in the form of Taylor's vortices is possible. And, as it was already noted, at the increase in the order of rotary symmetry of the RMF Taylor's vortices may not appear at all. This is the basic difference between our results and the results of the classical research problem of stability of Couette flow between coaxial cylinders Davey et al.,⁷ when transition to wavy vortices necessarily is preceded by a stage of Taylor's vortices. This is because Couette flow arises thanks to forces of viscosity (liquid sticking to the rotating cylinder) while in our case of using the RMF the flow is created due to field acting on the conducting liquid at a motionless wall of the cylinder. Profiles of a primary flow essentially differ: in our case there is always at least a thin boundary layer between a kernel of a flow and a cylinder wall.⁸⁻¹¹

The completed research allows one to expand our views about the mechanisms of the rise of the instability of the conducting liquid flow generated by the RMF of any rotary symmetry in infinitely long cylindrical vessel. The results obtained thus far allow us to predict the

loss of stability of a primary flow occurrence of a secondary flow in Taylor's vortices or wavy vortices of this or that order depending on the level of the power affecting the liquid and on the order of rotary symmetry of the RMF. Apparently, these conclusions would likely be fair and for sufficiently long cylinders of finite length when it is possible to neglect the influence of end faces on the flow in the central part of the vessel.

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Conflicts of interest

Author declares that there is no conflicts of interest.

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