New Wavelength-Time Codes for High Speed Optical CDMA Networks

Abstract

One new wavelength-time code family with short code lengths is proposed for optical code-division multiple access networks. With the special structure of the proposed codes, economical optical components can be used for coder realization and the resulting tunable coders are simpler with fast tuning speeds. The performance analysis is taken the beat noise during the photo-detecting process into account, and it shows that these code words can obtain bit error rates comparable to that of previous codes with short code lengths.

Index Terms: Fiber-Bragg Grating (FBG); Optical Code-Division Multiple-Access (OCDMA); Wavelength time codes; Access network

Introduction

Optical code-division multiple access (OCDMA) is a promising solution and it provides secure and asynchronous transmission in optical networks. Since high statistical multiplexing gain can be offered even in bursty traffic, OCDMA techniques are very suitable for access networks [1-3]. To reduce the code lengths, traditional one-dimensional time-spreading codes such as optical orthogonal codes (OOCs) [4] and prime codes [5] were combined with wavelength-hopping patterns to yield two-dimensional (2-D) codes that maintain the cardinality and correlation constraints [6]. Multi-level sequences such as prime sequences [7] provided large number of wavelength-hopping patterns for 2-D codes, and these original prime sequences and their cyclic shifts were named shifted prime sequences (SPSs) in this letter. However, since the prime codes had large autocorrelation sidelobes, the prime-hop sequences (PHSs) in [6] cannot use the cyclic shifts of the prime hopping patterns in order to reserve ideal correlation constraint. 2-D codes adopting OOCs as time spreading pattern were proposed [8] and they were able to adopt all of the SPSs as hopping patterns. Though this code family had large cardinality and short code lengths at the same time, the weights of these codes were limited when the code lengths are short and this affected the signal’s extinction ratio. Other code families with short code lengths and ideal correlation properties were proposed [9-12], and the comparisons between these code families were given in [11,12]. Among these code families, one code family generated from the wavelength spreading and time hopping (WS-TH) scheme with short code lengths was proposed in [12]. When the number of wavelengths for one codeword is \(2^n\) (\(m\) is a positive integer), the corresponding code words belonging to the same code group can share the same \(2^n \times 2^n\) arrayed waveguide gratings (AWGs) for encoding/decoding. However, this advantage exists only when the transmitting/receiving codewords for all users are fixed. When this type of encoders (or decoders) is needed to be operated in a tunable fashion, each user needs one \(2^n \times 2^n\) AWG and \(2^n\) optical switches for the manipulation of wavelengths in one encoder (or decoder), which is more expensive for the construction of the encoder. Here one new code family named folded prime-hop sequences (FPHSs) is proposed. These codes have code lengths the same as that of the codes in [11,12] for the same code weights, and thus are suitable for high-speed optical transmission with high signal’s extinction ratio. When the number of wavelengths for one codeword is \(M=p^2\) (\(p\) is a prime), the corresponding tunable encoder (or decoder) only needs one \(1 \times p\) thin film filter, \(p\) fiber Bragg gratings for the manipulation of wavelengths. Therefore, it is a promising solution for OCDMA-based networks with low cost and simple hardware.

Code Construction and Implementation

Each FPHS is constructed by combining two prime sequences in a similar manner as that of traditional time spreading/wavelength hopping code families, and can be viewed as a folded version of PHSs. However, the differences between the FPHS and PHS code families are obvious. Since the time spreading patterns of PHSs use prime codes which have large autocorrelation sidelobes, cyclic shifts of the hopping patterns cannot be used in order to keep the cross-correlation of PHSs no more than one. For the FPHSs, prime sequences can be viewed as the “time-spreading” patterns. Since the cross-correlation between any two prime sequences is no more than one, SPSs can be used as the “wavelength-hopping” patterns and the cardinality is increased. For explanation, the construction of FPHSs is described similar to that of traditional time spreading/wavelength hopping codes in the following. Suppose FPHSs with code length \(L=p\), number of wavelengths \(M=p^2\) and weight \(w=p\) are needed to be constructed, prime sequences \(T_{sp}\) and \(H_{sp}\) with lengths \(p\) (\(p\) is a prime) are used for “time-spreading” and “wavelength-hopping”, respectively. Their \(i\)-th elements are obtained as
\[ T_{a,b}(i) = \begin{cases} b+1, & \text{if } a=0 \text{ and } 0 \leq b < p-1, \\ a \mod p + 1, & \text{if } 1 \leq a \leq p-1, b=0, \end{cases} \]  
\hspace{1cm} (1)

and

\[ H_{c,d}(i) = c \mod (p-1) + 1, \text{ if } 1 \leq c \leq p-1 \text{ and } 0 \leq d \leq p-1, \]  
\hspace{1cm} (2)

Respectively, where \( \mod p \) and \( \bullet \) are the modulo-\( p \) addition and multiplication, respectively. \( T_{a,b} \)'s and \( H_{c,d} \)'s for \( p=3 \) is shown in (Table 1). Note that the FPHSs in one code can be divided into two parts: \( T_{a,b} \)'s and \( H_{c,d} \)'s. The elements of one FPHS \( T_{a,b} \) belonging to the first part of the code can be obtained as follows:

\[ T_{a,b}(j,i) = \begin{cases} 1, & \text{if } p(H_{c,d}(i)-1) + T_{a,b}(i) - 1 = j, \\ 0, & \text{otherwise,} \end{cases} \]  
\hspace{1cm} (3)

where \( T_{a,b}(j,i) \) is an element of \( T_{a,b} \) at row \( #j \) and column \( #i \). Though there are \( 2p-1 \) \( T_{a,b} \)'s and \( p(p-1) \) \( H_{c,d} \)'s, only \( p^2 \) \( T_{a,b} \) \( H_{c,d} \)'s can be obtained since \( T_{a,b} \)'s only can be combined with \( H_{c,d} \)'s to yield the 2-D codewords that satisfy the correlation constraint. The elements of one FPHS \( PT_{a,b} \) (for \( 0 \leq a, b \leq p-1 \)) belonging to the second part of the code can be obtained as follows:

\[ PT_{a,b}(j,0) = 1, \text{ if } pi + T_{a,b}(i) - 1 = j. \]  
\hspace{1cm} (4)

Thus the total number of code words in the second part of the code is \( p^2 \). Note that in addition to the elements that satisfy (3), other elements of \( PT_{a,b} \) are zero and thus this part of FPHSs is somewhat like the codewords used in the spectral-amplitude-coding OCDMA networks [13]. From the discussion above, it is found that the total number of FPHSs obtaining from (2) and (3) is \( \Phi=p^2 \). From (2) and (3), it is known that FPHSs may not exist for several pairs \((a,b)\) in the range of \( 0 \leq a, b \leq p-1 \), thus the existences of FPHSs are shown in (Table 1(a)). The ticking region in (Table 1a) can be divided into two parts: the left part is for the FPHSs generated from (2) (e.g. \( T_{a,b} \)'s) and the right part is for the FPHSs generated from (3) (e.g. \( PT_{a,b} \)). For example, though \( T_{a,b} \)'s for \( 0 \leq a, b \leq p-1 \) are all shown in (Table 1[a]), only the \( T_{a,b} \)'s satisfy the ranges indicated in (2) (e.g. \( a=0, 0 \leq b \leq p-1 \) and \( 1 \leq a \leq p-1, b=0 \)) are ticked in the left part of the ticking region. However, since all of the \( PT_{a,b} \)'s exist for \( 0 \leq a, b \leq p-1 \), they are ticked in the right part of the ticking region. For the same values of \((a,b)\), the situations of the existence for \( T_{a,b} \)'s are the same even when the values of \((c,d)\) are different. For clarification, five FPHSs \( T_{a,b} \) \( T_{a-1,b} \) \( T_{a,b+1} \) \( T_{a+1,b} \) \( PT_{a,b} \) and \( PT_{a,b} \) are also shown in (Table 1[b]) for illustration. However, it should be pointed that these ticked FPHSs are the codewords that are suitable for the proposed tunable encoder/decoder described in the following and there are other FPHSs satisfying the correlation constraint. The tunable encoder that can generate all the ticked FPHSs in (Table 1) is shown in (Figure 1(a)). Here each chip of FPHSs is assumed to have spectral width \( \Delta \lambda \) and time duration \( T \). Thus the spectrum width of multiple wavelength laser (MWL) should be \( p\Delta \lambda \) and the wavelength \( \#j \lambda \) is used by the elements of FPHSs at row \( \#j \). Two kinds of optical filters are used in this encoder: One is thin film filter (TFF) with channel bandwidth \( p\Delta \lambda \) and the other is fiber Bragg grating (FBG) with channel bandwidth \( \Delta \lambda \). Due to the special code structure of FPHSs, there is only one tunable delay line (TDL) connected to each output port of TFF and there is only one FBG connected to the output port of each TDL. Therefore, the optical components needed to construct the encoder of \( p \times p \) FPHSs is one MWL, one electrical-optical modulator (EOM), one circulator, one \( 1 \times p \) TFF, \( p \) TDLs and \( p \) FBGs. It is obvious that the proposed tunable encoder is relatively low cost as compared to the tunable version of the encoder proposed in [12]. The encoding process is described as follows: the signal modulation format used here is on-off keying, which is the same as that in most time-spreading/wavelength-hopping OCDMA schemes. Thus only when the information bit is \( 1 \), the wavelengths from the MWL pass the EOM, the circulator, and enter the input port of \( 1 \times p \) TFF, \( \lambda_0, \lambda_1, \ldots, \lambda_{p-1} \) appear at the output port \( \#e(p) \) and one of these wavelengths is reflected by the FBG belonging to this TFF output port. The time delay of each reflected wavelength chip is determined by the TDL at each TFF output port. Since the needs for tuning ranges of these FBGs are only \( p\Delta \lambda \) (which is equal to \( 1/p \) of the total encoded bandwidth), no special FBG manufacturing is required and the tuning speed is faster. In addition, since the code lengths of FPHSs are short, the needs for tuning ranges of TDLs are also relaxed. This also makes the encoder more economical. The tunable decoder that can be used to decode all the ticked FPHSs in (Table 1) is shown in (Figure 1(b)). The main part of the decoder is the same as that of the encoder in (Figure 1(a)), except that the time delays of the TDLs in the decoder should be “complement” to that of the corresponding encoder. Due to the use of TFF for the de-multiplexing of wavelength chips, the problems of splitting losses occurred at the conventional encoder/decoder for 2-D codes adopting wavelength and time domains are also improved [6].

**System Performance**

Suppose that all the FPHSs of length \( L=p \) that can be applied to the tunable encoder/decoder mentioned above are indexed with one single number \( u(0<\Phi-1) \), the average probability of getting one hit between the desired FPHS \#0 and other interfering FPHSs can be obtained by using the following equation:

\[ P_{hit}(\tau) = \frac{1}{2L(\Phi-1)} \sum_{u=0}^{\Phi-1} \sum_{\tau=0}^{L-1} R(\tau), \]  
\hspace{1cm} (5)
Table 1: (a) The existence of FPHSs for \( p = 3 \) and (b) Several FPHSs.

<table>
<thead>
<tr>
<th>( T_{a}H_{c} )</th>
<th>( PT_{a} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{0})</td>
<td>( H_{0} = [111] )</td>
</tr>
<tr>
<td>( T_{1})</td>
<td>( H_{1} = [231] )</td>
</tr>
<tr>
<td>( T_{2})</td>
<td>( H_{2} = [321] )</td>
</tr>
</tbody>
</table>

\[ H_{1} = [123] \]
\[ H_{2} = [132] \]

\[ T_{0}H_{0} = [000] \]
\[ T_{0}H_{1} = [010] \]
\[ T_{0}H_{2} = [100] \]

\[ T_{1}H_{0} = [000] \]
\[ T_{1}H_{1} = [010] \]
\[ T_{1}H_{2} = [100] \]

\[ T_{2}H_{0} = [000] \]
\[ T_{2}H_{1} = [010] \]
\[ T_{2}H_{2} = [100] \]

Figure 1: The encoder and (b) decoder for FPHSs.

Figure 2: BER v.s. number of active users.

where

1) \( R(t) \) is the periodic correlation function between the desired codeword;

2) \( u \)-th codeword and

3) \( \tau \) is the relative time shift between these two codewords.

Assume that there are \( k_{y} \) pulses beating against each other at a particular wavelength \( \lambda \). Therefore, when \( k \) pulses due to the interferers fall at the photodetector, they will be distributed over \( \lambda_{y} \)'s according to the factors \( k_{y} \)'s. Since the nonzero elements are uniformly distributed among the available wavelengths in the code patterns of OOC-PS codewords in [12] and FPHSs, the components of the distribution vector \( \{ k_{0}, k_{1}, ..., k_{w-1} \} \) are modeled to obey the multinomial distribution with equal probability \( P_{i} = 1/w \). Therefore, when the beat noise is considered, the bit error rate (BER) of the FPHSs is [12,14].

\[ \text{BER} = \frac{k_{y}^{K-1}}{k_{y}^{K-i}} \cdot \frac{k^{i}}{w^{i}} \cdot \left( 1 - \frac{P_{i}}{w} \right)^{K-i} \cdot \left( 1 + \frac{P_{i}}{w} \right)^{K-i} \]

\[ \frac{1}{2} \cdot \left( \text{erf} \left( \frac{\text{SNR}_{0}}{2} \right) + \text{erf} \left( \frac{\text{SNR}_{0}}{2} \right) \right) \]

(6)

Where

\( K \) is the number of active users, and the values for \( \text{SNR}_{0} \) and \( \text{SNR}_{0} \) are given in [12].

Figure 2 shows the relationships between BER and \( K \) for OOC-PS and FPHS codes with the same code lengths, weights and number of wavelengths, and three cases for code length \( L = 5, 7, 11 \) are discussed. It can be found that when the code lengths are the same, OOC-PS and FPHS codes obtain similar BER performance. In addition, when \( L \) increases, BERs decrease for these two code families. Note that the performance for incoherent OCDMA schemes using both time and wavelength domains are always degraded by the beat noise seriously. Though the performance of these schemes can be improved by the use of forward error correction codes, the code families with short code lengths such as OOC-PS and FPHS codes can use error correction codes without sacrificing the bit rates too much.

Conclusion

One new code family and the corresponding simple encoder/decoder are proposed for high speed OCDMA networks. Due to the structures of the proposed code words, the locations of the first nonzero chips of any two code words are located nearby, and the same is true for the second nonzero chips, third nonzero chips, etc. Therefore, the tuning range of each chip is limited and the time for the encoder to tune from one codeword to another
is decreased. Since the structure for decoders is similar to the one for encoders for the proposed codes, the decoders also have the advantage of fast tuning speeds. Since the need of tuning ranges for the tunable delay lines and optical filters in encoders and decoders is alleviated, the cost of the tunable delay lines and optical filters in encoders and decoders is decreased. Therefore, the coder of the proposed codes has advantages of low cost and fast tuning speed. The associated BER performance with the consideration of beat noise is also comparable to that of other wavelength/time code proposed previously.

Conflict of Interest
None.

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None.

References