Research Article
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Zero- truncated discrete shanker distribution and its applications

Abstract

Discrete analogue of the continuous Shanker distribution, which may be called a discrete Shanker distribution, has been introduced. The probability mass function and probability generating function of the distribution have been obtained. Zero truncated form of the distribution has been investigated. Certain recurrence relations for probabilities and moments have been also derived. The parameters of Zero- truncated discrete Shanker distribution have been estimated by using Newton- Raphson method. The distributions have been fitted to eight numbers of well- known data sets, which are used by other authors. A comparative study has been made among ZTP, ZTPL and ZTDS distributions, using the same data set based on the goodness of fit test. It has been observed that in most cases ZTPL gives much closer fit than ZTP distribution. While ZTDS gives very closer fit to ZTPL and in some cases ZTDS gives better fit than ZTPL distribution.

Keywords: discrete shanker distribution, zero-truncated discrete shanker distribution, zero- truncated Poisson- lindley distribution, recurrence relations, survival function

Volume 5 Issue 6 - 2017

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Received: March 10, 2017 | **Published:** May 16, 2017

Abbreviations: DS: Discrete Shanker; ZTP: Zero–Truncated Poisson; ZTPL: Zero–truncated Poisson Lindley; ZTDS: Zero–Truncated Discrete Shanker; PDF: Probability Density Function; pmf: probability mass function; $S(x)$: survival function; $r(x)$: failure hazard rate, $r^*(x)$: reversed failure rate; $f_D(x;\theta)$: pmf of DS distribution, $f_z(x;\theta)$: pmf of ZTDS distribution; $\eta'_{[r]}$: $i_{[r]}$ factorial moment of ZTDS distribution; $\mu_{[r]}$: r^{th} raw moment of DS distribution; P_r: r^{th} Probability of DS distribution; P^z_r: r^{th} Probability of ZTDS distribution

Introduction

It is sometimes inconvenient to measure the life length of a device, on a continuous scale. In practice, we come across situation, where lifetime of a device is considered to be a discrete random variable. For example, in the case of an on off switching device, the lifetime of the switch is a discrete random variable. If the lifetimes of individuals in some populations are grouped or when lifetime refers to an integral numbers of cycles of some sort, it may be desirable to treat it as a discrete random variable. When a discrete model is used with lifetime data, it is usually a multinomial distribution. This arises because effectively the continuous data have been grouped. Such situations may demand another discrete distribution, usually over the non negative integers. Such situations are best treated individually, but generally one tries to adopt one of the standard discrete distribution. Some of those works are by Nakagawa and Osaki,[1] where the discrete Weibull distribution is obtained; Roy[2] studied discrete Rayleigh distribution; Kemp[3] derived discrete Half normal distribution. Krishna and Pundir[4] investigated the discrete Burr and the discrete Pareto distribution. Gomez-Deniz[5] derived a new generalization of the geometric distribution obtained from the generalized exponential distribution of Marshall and Olkin.[6] Borah et al.,[7,8] studied on two parameter discrete quasi- Lindley and discrete Janardan distributions respectively. Borah and Saikia[9] introduced discrete Sushila distribution. Dutta and Borah[10] studied zero- modified Poisson- Lindley distribution.

Derivation of the proposed distribution

One parameter continuous Shanker distribution introduced by Shanker[11] with parameter θ is defined by its probability density function (pdf)

$$f(x:\theta)=\frac{\theta^2}{\theta^2+1}(\theta+x)e^{-\theta x}.x>0.\theta>0. \tag{2.1}$$

Discretization of continuous distribution can be done using different methodologies. In this paper we deal with the derivation of a new discrete distribution which may be called discrete Shanker (DS) distribution. It takes values in {0, 1, 2, . . .,}. This distribution is generated by discretizing the survival function of the continuous Shanker distribution

$$S(x)=\int_x^\infty f(x:\theta)\,dx$$

$$=\frac{\theta^2+1+\theta x}{\theta^2+1}e^{-\theta x},x>0.\theta>0. \tag{2.2}$$

$$S\left(x+1\right)=\frac{\theta^2+1+\theta\left(x+1\right)}{\theta^2+1}e^{-\theta\left(x+1\right)}, x>0.\theta>0. \quad (2.3)$$

Where $f\left(x;\theta\right)$ denotes the pdf of Shanker distribution.

The pmf of discrete Shanker distribution $f_D\left(x;\theta\right)$ may be obtained as

$$f_D\left(x;\theta\right)=S\left(x\right)-S\left(x+1\right)$$

$$=\frac{(\theta^2+1+\theta x)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}{\theta^2+1}e^{-\theta x}, x=0,1,2,3.... \quad (2.4)$$

Proposition 1: The probability generating function (pgf) of DS distribution is given by

$$G_D\left(t\right)=\frac{\left(\theta^2+1\right)\left(1-e^{-\theta}\right)-\theta}{\left(\theta^2+1\right)\left(1-e^{-\theta}t\right)}+\frac{\theta\left(1-e^{-\theta}\right)}{\left(\theta^2+1\right)\left(1-e^{-\theta}t\right)^2}$$

Proposition 2: The cumulative distribution of DS distribution is given by

$$F\left(x\right)=\frac{\left(\theta^2+1\right)-\left(\theta^2+1+\theta\left(x+1\right)\right)e^{-\theta\left(x+1\right)}}{\left(\theta^2+1\right)}$$

The survival function of DS distribution has obtained as

$$S_D\left(x\right)=\frac{\left(\theta^2+1+\theta\left(x+1\right)\right)e^{-\theta\left(x+1\right)}}{\left(\theta^2+1\right)}$$

The failure hazard rate may be obtained as

$$r_D\left(x\right)=\frac{(\theta^2+1+\theta x)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}{\left(\theta^2+1+\theta x\right)}$$

The reversed failure rate

$$r^*\left(x\right)=\frac{\left[(\theta^2+1+\theta x)\left(1-e^{-\theta}\right)-\theta e^{-\theta}\right]e^{-\theta x}}{\left(\theta^2+1\right)-\left(\theta^2+1+\theta\left(x+1\right)\right)e^{-\theta\left(x+1\right)}}$$

The second rate of failure is obtained as

$$r^*\left(x\right)=\log\left[\frac{s\left(x\right)}{s\left(x+1\right)}\right]$$

$$=\log\left[\frac{\left(\theta^2+1+\theta\left(x+1\right)\right)}{e^{-\theta}\left(\theta^2+1+\theta\left(x+2\right)\right)}\right]$$

The Proportions of probabilities is given by

$$\frac{f_D\left(x+1;\theta\right)}{f_D\left(x;\theta\right)}=e^{-\theta}\left[1+\frac{\theta\left(1-e^{-\theta}\right)}{(\theta^2+1+\theta x)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}\right]$$

Probability recurrence relation:

Probability recurrence relation of DS distribution may be obtained as

$$P_{r+2}=e^{-\theta}\left(2P_{r+1}-e^{-\theta}P_r\right) \text{r}=1,2,3,\ldots \quad (2.5)$$

Where $P_0=\frac{(\theta^2+1)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}{\theta^2+1}$,

and $P_1=\frac{(\theta^2+1+\theta)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}{\theta^2+1}e^{-\theta}$ (2.6)

Here P_r denotes Pr(X= r).

Factorial moment recurrence relation

Factorial moment generating function (fmgf) may be obtained as

$$M_D\left(t\right)=\frac{\left(\theta^2+1\right)\left(1-e^{-\theta}\right)-\theta}{\left(\theta^2+1\right)\left(1-e^{-\theta}-e^{-\theta}t\right)}+\frac{\theta\left(1-e^{-\theta}\right)}{\left(\theta^{'}+1\right)\left(1-e^{-\theta}-e^{-\theta}{}_t\right)^2}. \quad (2.7)$$

First four factorial moments may be obtained as

$$\mu'_{[1]}=\frac{e^{-\theta}\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta\right]}{\left(\theta^2+1\right)\left(1-e^{-\theta}\right)^2}$$

$$\mu'_{[2]}=\frac{2e^{-2\theta}\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+2\theta\right]}{\left(\theta^2+1\right)\left(1-e^{-\theta}\right)^3}$$

$$\mu'_{[3]}=\frac{6e^{-3\theta}\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+3\theta\right]}{\left(\theta^2+1\right)\left(1-e^{-\theta}\right)^4}$$

Citation: Borah M, Saikia KR. Zero- truncated discrete shanker distribution and its applications. *Biom Biostat Int J.* 2017;5(6):232–237. DOI: 10.15406/bbij.2017.05.00152

$$\mu'_{[4]} = \frac{24e^{-4\theta}\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+4\theta\right]}{\left(\theta^2+1\right)\left(1-e^{-\theta}\right)^4}$$

Proposition 3: The general form of factorial moment may also be written as

$$\mu'_{[r]} = \frac{r!e^{-\theta r}\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta r\right]}{\left(\theta^2+1\right)\left(1-e^{-\theta}\right)^{r+1}} \quad (2.8)$$

Hence, mean and variance may be obtained as

$$\text{Mean} = \frac{e^{-\theta}\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta\right]}{\left(\theta^2+1\right)\left(1-e^{-\theta}\right)^2}, \text{ and}$$

$$Variance = \frac{e^{-\theta}\left[\left(\theta^2+1\right)^2\left(1-e^{-\theta}\right)^2+\left(\theta^2+1\right)\left(1-e^{-\theta}\right)\theta-e^{-\theta}\theta^2\right]}{\left(\theta^2+1\right)^2\left(1-e^{-\theta}\right)^4}$$

respectively.

Zero truncated discrete shanker (ZTDS) distribution:

Zero- truncated distributions are applicable for the situations when the data to be modeled originate from a generating mechanism that structurally excludes zero counts. The discrete Shanker distribution must be adjusted to count for the missing zeros. Here the zero-truncated discrete Shanker distribution has been derived.

The pmf $f_z(x;\theta)$ of Zero-truncated DS distribution has been derived as

$$f_z(x;\theta) = \frac{P_x}{1-P_0} \quad (3.0)$$

Where P_x denotes the pmf of discrete Shanker distribution.

$$Hence, f_z(x;\theta) = \frac{(\theta^2+1+\theta x)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}{(\theta^2+\theta+1)} e^{-\theta(x-1)}, x = 1,2,3,\ldots \quad (3.1)$$

Probability recurrence relation for ZTDS distribution

The pgf $G_z(t)$ of zero-truncated DS distribution may be obtained as

$$G_z(t) = \Sigma_{x=1}^{\infty} t^x f_z(x;\theta),$$

$$= \frac{t\left[\left\{\left(\theta^2+1\right)\left(1-e^{-\theta}\right)-\theta e^{-\theta}\right\}\left(1-te^{-\theta}\right)+\theta\left(1-e^{-\theta}\right)\right]}{\left((\theta^2+\theta+1\right)\left(1-te^{-\theta}\right)^2} \quad (3.2)$$

Probability recurrence relation **ZTDS** distribution may obtained as

$$P^z{}_r = e^{-\theta}\left[2P^z{}_{r-1} - e^{-\theta}P^z{}_{r-2}\right] r = 2,3,4,\ldots.$$

Where $P^z{}_1 = \dfrac{(\theta^2+1+\theta)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}{(\theta^2+\theta+1)}$ and

$$P^z{}_2 = \frac{(\theta^2+1+2\theta)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}{(\theta^2+\theta+1)} e^{-\theta}. \quad (3.3)$$

Proposition 4: The cumulative distribution of ZTDS distribution is given by

$$F_z(x) = \frac{\left(\theta^2+\theta+1\right)-\left(\theta^2+\theta+\theta x+1\right)e^{-\theta x}}{\left(\theta^2+\theta+1\right)}$$

The survival function of ZTDS distribution is given by

$$S_z(x) = \frac{\left(\theta^2+\theta+\theta x+1\right))e^{-\theta x}}{\left(\theta^2+\theta+1\right)}$$

The Failure hazared rate may be obtained as

$$r_z(x) = \frac{P\left(X=x\right)}{P\left(X \ge x-1\right)},$$

$$= \frac{(\theta^2+1+\theta x)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}{\left(\theta^2+1+\theta x\right)}.$$

The reversed failure rate

$$r_z^*(x) = \frac{P\left(X=x\right)}{P\left(X \le x\right)}$$

$$= \frac{[(\theta^2+1+\theta x)\left(1-e^{-\theta}\right)-\theta e^{-\theta}]e^{-\theta(x-1)}}{\left(\theta^2+\theta+1\right)-\left(\theta^2+\theta+1+\theta x\right))e^{-\theta x}}.$$

The second rate of failure is obtained as

$$r_z^{**}(x)(x) = \log\left[\frac{s(x)}{s(x+1)}\right],$$

$$= \log\left[\frac{\left(\theta^2+\theta+1+\theta x\right)}{e^{-\theta}\left(\theta^2+\theta+1+\theta\left(x+1\right)\right)}\right].$$

Citation: Borah M, Saikia KR. Zero- truncated discrete shanker distribution and its applications. *Biom Biostat Int J.* 2017;5(6):232–237.
DOI: 10.15406/bbij.2017.05.00152

The proportions of probabilities is given by

$$\frac{f_z(x+1;\theta)}{f_z(x;\theta)} = e^{-\theta}\left[1+\frac{\theta\left(1-e^{-\theta}\right)}{(\theta^2+1+\theta x)\left(1-e^{-\theta}\right)-\theta e^{-\theta}}\right]$$

Factorial moment recurrence relation for ZTDS distribution

Factorial moment generating function $M_z(t)$ of ZTDS distribution may be obtained as

$$M_z(t) = \frac{(1+t)\left[\left\{\left(\theta^2+1\right)\left(1-e^{-\theta}\right)-\theta e^{-\theta}\right\}\left(1-t-te^{-\theta}\right)+\theta\left(1-e^{-\theta}\right)\right]}{\left((\theta^2+\theta+1)\left(1-t-te^{-\theta}\right)^2\right)} \quad (3.4)$$

Factorial moment recurrence relation of ZTDS distribution may be obtained as

$$\eta'_{[r]} = \frac{e^{-\theta}}{\left(1-e^{-\theta}\right)^2}\left[2\left(1-e^{-\theta}\right)r - e^{-\theta}\eta'_{[r-1]} - r(r-1) - e^{-\theta}\eta'_{[r-2]}\right]$$

$, r \geq 2$ - (3.5)

where η

$$\eta'_{[1]} = \frac{\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta\right]}{\left(\theta^2+\theta+1\right)\left(1-e^{-\theta}\right)^2}$$

$$\eta'_{[2]} = \frac{2e^{-\theta}\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+2\theta\right]}{\left(\theta^2+\theta+1\right)\left(1-e^{-\theta}\right)^3} \quad (3.6)$$

Variance σ_Z^2 of ZTDS distribution may be obtained as

$$.\sigma_Z^2 = \frac{\left[\left(\theta^2+1\right)^2\left(1-e^{-\theta}\right)^2+5\left(\theta^2+1\right)\left(1-e^{-\theta}\right)e^{-\theta}\theta+4\theta^2 e^{-\theta}\right]}{\left(\theta^2+\theta+1\right)^2\left(1-e^{-\theta}\right)^4}$$

Proposition 5: The general form of factorial moment may be written as

$$\eta'_{[r]} = \frac{r!\,e^{-\theta(r-1)}\left[\left(\theta^2+1\right)\left(1-e^{-\theta}\right)+\theta r\right]}{\left(\theta^2+\theta+1\right)\left(1-e^{-\theta}\right)^{r+1}}. \quad (3.7)$$

Method of estimation

The parameter θ of ZTDS distribution has been estimated using Newton-Rapson iterative method, selecting appropriate initial guest value θ_0 for θ, where the function of θ may be written as

$$f(\theta) = 1-e^{-\theta}-\frac{\theta e^{-\theta}}{\theta^2+\theta+1}-f_o,$$

$$f'(\theta) = e^{-\theta}+\frac{e^{-\theta}\left(\theta^3+2\theta^2+\theta-1\right)}{\left(\theta^2+\theta+1\right)^2}$$ based on relative frequency f_o.

Similarly, function of θ may be written as

$$f(\theta) = 1-e^{-\theta}-\frac{\theta e^{-\theta}}{\theta^2+\theta+1}-\mu$$

$$f'(\theta) = e^{-\theta}+\frac{e^{-\theta}\left(\theta^3+2\theta^2+\theta-1\right)}{\left(\theta^2+\theta+1\right)^2}$$, based on mean μ

Newton- raphson iterative method

$$\theta_{n+1} = \theta_n - \frac{f(\theta)}{f'(\theta)}$$, n=0, 1, 2, …, where θ_0 is the initial guest value.

Replacing θ_0 by θ_1 and repeating the process till it converse. (Balagurusamy.[12])

Goodness of fit

In this section, an attempt has been made to test the suitability of ZTDS distribution. Eight data sets, which are used by Shanker et. al.,[13] have been used for a comparative study (Tables 1-8).

Table 1 Number of mothers in rural area having at least one live birth and neonatal death

No. of neonatal death	Observed no. of mothers	Expected frequency		
		ZTDS	ZTP	ZTPL
1	409	399.7	399.7	408.1
2	88	102.3	102.3	89.4
3	19	17.5	17.5	19.3
4	5	2.2	2.2	4.1
5	1	0.3	0.3	1.1
	522	522	522.2	522
	Estimate θ	1.7914	0.512047	4.199697
Total	X^2	0.181	3.464	0.145
	d.f.	2	1	2
	p- value	0.9137	0.0627	0.9301

Table 2 The number of estate area having at least one live birth and one neonatal death

No. of neonatal death	Observed no. of mothers	Expected frequency		
		ZTDS	ZTP	ZTPL
1	71	71	66.5	72.3
2	32	29.43	35.1	28.4
3	7	12.3	10.9	10.9
4	5	4.11	3.3	4.1
5	3	2.2	0.8	2.2
	118	118	118	118
	Estimate θ	1.2053	1.055102	2.049609
Total	x^2	2.289	0.696	2.274
	d.f.	3	1	2
	p- value	0.5147	0.4041	0.3208

Table 3 Number of mothers in urban area with at least two live births by the number of infant and child deaths

No. of neonatal death	Observed no. of mothers	Expected frequency		
		ZTDS	ZTP	ZTPL
1	176	176	164.3	171.6
2	44	50.13	61.2	51.3
3	16	13.35	15.2	15
4	6	3.41	2.8	4.3
5	2	1.11	0.5	1.7
	244	244	244	244
	Estimate θ	15,499	0.744522	2.209422
Total	x^2	2.852	7.301	1.882
	d.f.	2	1	2

Table 4 Number of mothers in rural area with at least two live birth by the numbers of infant and child deaths

No. of neonatal death	Observed no. of mothers	Expected frequency		
		ZTDS	ZTP	ZTPL
1	745	744.97	708.9	738.1
2	212	215.02	255.1	214.8
3	50	58.01	61.2	61.3
4	21	15	11	17.2
5	7	3.77	1.6	4.8
6	3	1.33	0.2	1.8
	1038	1,038	1038	1038
	Estimate θ	1.5376	0.719783	3.007722
Total	x^2	8.256	37.046	4.773
	d.f.	4	2	3
	p-value	0.0826	0	0.1892

Table 5 Number of literate mothers with at least one live birth by the number of infant deaths.

No. of neonatal death	Observed no. of mothers	Expected frequency		
		ZTDS	ZTP	ZTPL
1	683	683.04	659	674.4
2	145	150.81	177.4	154.1
3	29	31.36	31.8	34.6
4	11	6.27	4.3	7.7
5	5		1.22	2.2
	873	873	873	873
	Estimate θ	2	0.538402	4.00231
Total	x^2	10.022	8.718	5.31
	d.f.	3	1	2
	p- value	0.0184	0.0031	0.0703

Table 6 Number of mothers having experienced at least one child death

No. of neonatal death	Observed no. of mothers	Expected frequency		
		ZTDS	ZTP	ZTPL
1	89	89	76.8	83.4
2	25	31.26	39.9	32.3
3	11	10.2	13.8	12.2
4	6	3.18	3.6	4.5
5	3	0.96	0.7	1.6
6	1	0.4	0.2	0.9
	135	135	135	135
	Estimate θ	1.3568	1.038289	2.089084
Total	x^2	3.912	7.90	3.428
	d.f.	2	1	2
	p- value	3.912	7.9	3.428

Table 7 Number of mothers having at least one neonatal death

No. of neonatal death	Observed no. of mothers	Expected frequency		
		ZTDS	ZTP	ZTPL
1	567	567.04	545.8	561.4
2	135	138.37	162.5	139.7
3	28	31.71	32.3	34.2
4	11	6.98	4.8	8.2
5	5	1.9	0.6	2.6
	746	746	746	746
	Estimate θ	2	0.595415	3.625737
Total		6.227	26.855	3.839
	d.f.	3	2	2
	p-value	0.1012	0	0.1467

Citation: Borah M, Saikia KR. Zero- truncated discrete shanker distribution and its applications. *Biom Biostat Int J.* 2017;5(6):232–237.
DOI: 10.15406/bbij.2017.05.00152

Table 8 Number of european red mites on apple leaves, reported by german

No. of neonatal death	Observed no. of mothers	Expected frequency		
		ZTDS	ZTP	ZTPL
1	38	38	28.7	36.1
2	17	21.32	25.7	20.5
3	10	10.9	15.3	11.2
4	9	5.28	6.9	3.1
5	3	2.47	2.5	1.6
6	2	1.13	0.7	0.8
7	1	1	0.2	0.8
8	0	0.39	0.1	
	80	80	80	80
	Estimate θ	0.9316	1.791615	1.185582
Total	x^2	3.753	9.827	2.467
	d.f.	3	2	3
	p- value	3	2	3

Conclusion

The discrete Shanker distribution has been introduced by discretizing the continuous Shanker distribution. Zero- truncated discrete Shanker (ZTDS) distribution have also been investigated. The parameter of the distribution has been estimated using Newton – Raphson iterative method. The application of ZTDS distribution to eight sets of data covering demography, biological sciences and social sciences have been studied. A comparative study has been made with ZTP and ZTPL distributions of Shanker et al.,13 It is observed that in most cases ZTPL gives much closer fits than ZTP distribution. It is also observed that ZTDS gives very closer fit to ZTPL and in some cases ZTDS gives better fit than ZTPL distribution.[14-18]

Acknowledgement

None.

Conflict of interest

None.

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