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On the curvature of reflected diffracted shock wave interacted by an yawed wedge (Subsonic case)

Abstract

Lighthill considered the diffraction of normal shock wave past a small bend of angle δ. Srivastava and Srivastava and Chopra extended the work of Lighthill to the diffraction of oblique shock wave (consisting of incident and reflected shock wave). Chopra and Srivastava further carried forward their work when the diffraction of oblique shock wave takes place with yawed wedges. In the present investigation curvature of the reflected diffracted is obtained when the interaction takes place with yawed wedge and the relative outflow behind reflected shock wave before diffraction is subsonic.

Keywords: curvature, diffraction reflection, yawed wedge, subsonic.

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Introduction

Lighthill[1] considered the diffraction of a normal shock wave past a small bend of small angle δ. The analogous problem of a plane shock wave hitting the wall obliquely together with the reflected shock has been solved by Srivastava[2] and Srivastava & Chopra.[3] Srivastava[2] solved the problem when the relative outflow behind reflected shock before diffraction is sonic and subsonic. Srivastava & Chopra[3] solved the problem when the relative outflow behind the reflected shock wave before diffraction is supersonic. Srivastava[4] gave the results concerning curvature of the reflected diffracted shock when the relative outflow behind the reflected shock before diffraction is sonic; subsequently he solved the curvature of reflected diffracted shock[5] when the relative outflow before diffraction is subsonic. Chester[6] considered the problem of reflection and diffraction of normal shock wave interacted by yawed wedges which was the extension of Lighthill's[1] problem of diffraction of a normal shock wave past a small bend. The results of Srivastava[2] and Srivastava & Chopra[3] have been extended results to the case of yawed wedges Chopra & Srivastava.[7] More specifically the attempt is concerned with the interaction of an oblique shock configuration (consisting of incident and reflected shock) with an yawed wedge i.e. the shock line (line of intersection of incident and reflected shock) makes some non zero angle with the leading edge of the wedge. In the present case curvature results have been obtained for the yawed case when the relative outflow behind reflected shock before diffraction is subsonic.

Mathematical formulation

Let the velocity pressure, density and sound speed ahead of the shock wave be denoted by o, p_0, ρ_0, a_0 in the intermediate region by q_1, p_1, ρ_1, a_1 and behind the reflected shock by q_2, p_2, ρ_2, a_2. Let U denote the velocity of intersection of the incident and reflected shock, δ the angle of the bend, α_0 is the angle of incidence and α_2 is the angle of reflection. The Rankine-Hugoniot equation across incident and reflected shock for $\gamma = 1.4$ (γ being the ratio of specific heats are gives as follows Srivastava.[8,9] Across the incident shock (Figure 1).

$$q_1 = \frac{5}{6} U \sin\alpha_0 \left(1 - \frac{a_0^2}{U^2 \sin^2\alpha_0}\right) \tag{1}$$

$$p_1 = \frac{5}{6}\rho_0 \left(U^2 \sin^2\alpha_0 - \frac{a_0^2}{7}\right) \tag{2}$$

$$\rho = \frac{6\rho_0}{\left(1 + \frac{5a_0^2}{U^2 \sin^2\alpha_0}\right)}$$

$$a_0 = \sqrt{\frac{\gamma p_0}{\rho_0}} \tag{3}$$

Across the reflected shock

$$\bar{q}_2 = \bar{q}_1 + \frac{5}{6}\left(U^* - \bar{q}_1\right)\left(1 - \frac{a_1^2}{\left(U^* - \bar{q}_1\right)^2}\right) \tag{4}$$

$$p_2 = \frac{5}{6}\rho_1 \left\{\left(U^* - \bar{q}_1\right)^2 - \frac{a_1^2}{7}\right\} \tag{5}$$

$$\rho_2 = \frac{6\rho_1}{1 + \frac{5a_1^2}{\left(U^* - \bar{q}_1\right)^2}} \tag{6}$$

Where $\bar{q}_2 = q_2 \sin\alpha_2$, $\bar{q}_1 = -q_1 \cos\left(\alpha_0 + \alpha_2\right)$

$$U^* = U \sin\alpha_2, \; a_1 = \sqrt{\frac{\gamma p_1}{\rho_1}}$$

Also we have

$$q_1 \cos\theta' = q_2 \cos\alpha_2, \quad \theta' = \alpha_0 + \alpha_2 - \frac{\pi}{2}$$

As the oblique shock configuration advances over a yawed wedge (shock line making some non zero angle with the leading edge of the wedge) the velocity of the point of intersection of the leading edge and shock line moves with velocity $U\operatorname{cosec}\chi$, U being the velocity of shock line and χ being the angle of yaw (Figure 2).

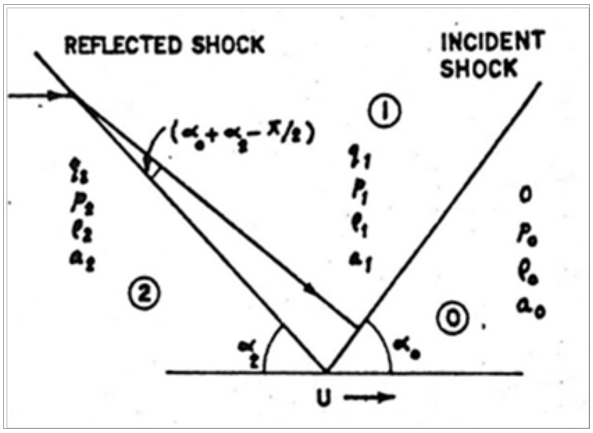

Figure 1 Oblique shock configuration.

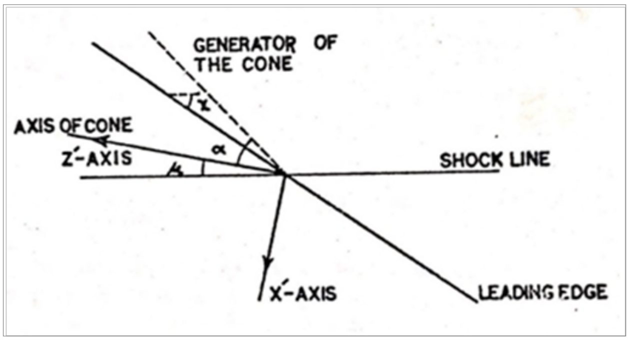

Figure 2 Configuration after interaction in the x′-z′ plane.

By superimposing a velocity on the whole field in a direction opposite to the direction of motion of the point of intersection of the shock line and the leading edge, the shock configuration becomes stationary and the resulting velocity behind the reflected shock for stationary configuration say V_2 is given by

$$V_2^2 = U^2 \operatorname{cosec}^2\chi + q_2^2 - 2Uq_2 \tag{7}$$

For conical field flow to occur behind the reflected shock V_2/a_2 should be greater than 1 which from (7) gives the condition that we should have

$$\sin^2\chi < \frac{U^2}{a_2^2 + 2Uq_2 - q_2^2} \tag{8}$$

We have also the relation

$$\tan\mu = \frac{U - q_2}{U\operatorname{cosec}\chi} \tag{9}$$

Further the semi angle of the Mach cone is given by

$$\sin\alpha = \frac{a_2}{V_2} \tag{10}$$

Let the disturbed flow variable behind the reflected diffracted shock referred to $Ox'y'z'$ axes be denoted by

$$\vec{V}_2 = \left(u_2, v_2, V_2 + w_2\right), p_2', \rho_2', S_2' \tag{11}$$

where u_2, v_2, w_2 are small perturbation in the velocity along $0x', 0y'$ and $0z'$ respectively, p_2' is the pressure, ρ_2' is the density and S_2' is the entropy. Using conservation laws we obtain the flow equations as

$$\overrightarrow{V'}_2 \nabla p_2' + \rho_2' \nabla \overrightarrow{V'}_2 = 0 \tag{12}$$

$$\left(\overrightarrow{V'}_2 \nabla\right)\overrightarrow{V'}_2 + \frac{1}{\rho_2'}\nabla p_2' = 0 \tag{13}$$

$$\overrightarrow{V'}_2 \nabla S_2' = 0 \tag{14}$$

We introduce the following transformations

$$\begin{aligned} x &= \frac{x'}{z'\tan\alpha} \\ y &= \frac{y'}{z'\tan\alpha} \\ p &= \frac{p_2' - p_2}{a_2\rho_2 q_2} \\ \rho &= \frac{a_2\left(\rho_2' - \rho_2\right)}{\rho_2 q_2} \\ u &= \frac{u_2}{q_2\cos\alpha} \\ v &= \frac{V_2}{q_2\cos\alpha} \\ w &= -\frac{W_2}{q_2\sin\alpha} \end{aligned} \tag{15}$$

On the assumption that the flow variable in the perturbed region differ by small quantities from their values in the uniform region and using the transformation given by equation (15) we obtain a single second order differential in p, namely

$$\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1\right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y}\right) = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \tag{16}$$

The characteristics of equation (16) are tangents to the unit circle $x^2 + y^2 = 1$ which in $0x'y'z'$ axes becomes the cone $x'^2 + y'^2 = z'^2\tan^2\alpha$. The region of disturbance will therefore be bounded by cone of disturbance, the shock front and the wall of the wedge (Figure 3).

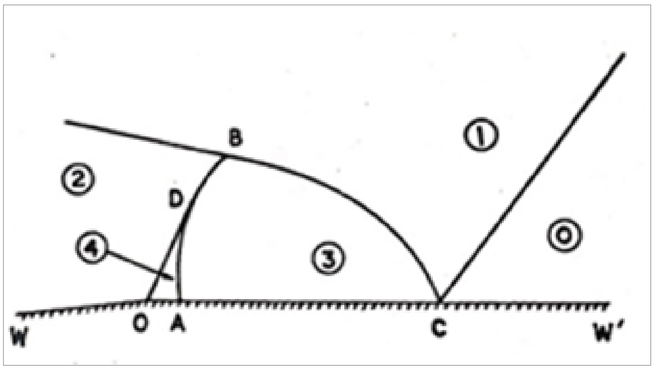

Figure 3 Flow picture in a plane perpendicular to the axis of the cone of disturbance.

The position of the shock line referred to (x, y) system is $\left(\frac{\tan\mu}{\tan\alpha}, 0\right)$ and it will lie inside on the cone of disturbance and outside the cone of disturbance according as

$$\frac{\tan\mu}{\tan\alpha} \le 1 \tag{17}$$

and

$$\frac{\tan\mu}{\tan\alpha} > 1 \tag{18}$$

Following Chopra & Srivastava,[7] the undisturbed part of the reflected shock lies in the plane

$$x = k - y\cot\alpha_2 \sec\mu\ ,\ k = \frac{\tan\mu}{\tan\alpha} \tag{19}$$

The equation of the reflected diffracted shock may therefore be written as

$$x\tan\alpha = \tan\mu - y\cot\alpha_2 \sec\mu\tan\alpha + f(y)\sec\mu \tag{20}$$

where $f(y)$ is small

The radius of curvature κ is given by

$$\frac{1}{\kappa} = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \tag{21}$$

Using equation (20), we obtain

$$\kappa = \frac{f''(y)\cot\alpha_2 \cot\alpha\sec^2\mu}{\left(1+\cot^2\alpha_2\sec^2\mu\right)^{3/2}} \tag{22}$$

Following Srivastava,[7] we have

$$v = A_1 + B_1\left(f - yf'\right) + C_1 f''(y) \tag{23}$$

Equation (23) gives

$$f''(y) = \frac{\partial v}{\partial y}\cdot\frac{1}{\left(C_1 - B_1 y\right)} \tag{24}$$

Combining (22) and (24) we have

$$\kappa = \frac{\partial v}{\partial y}\cdot\frac{1}{\left(C_1 - B_1 y\right)}\times\frac{\sec^2\mu\cot\alpha\cot\alpha_2}{\left(1+\cot^2\alpha_2\sec^2\mu\right)^{3/2}} \tag{25}$$

Following Srivastava[2] and Srivastava & Chopra[3] we have

$$\frac{\partial v}{\partial y} = \frac{C_1 - B_1 y}{C_3 - B_3 y}\cdot\frac{\partial p}{\partial y} \tag{26}$$

From (25) and (26) we have

$$\kappa = \frac{1}{C_3 - B_3 y}\frac{\partial p}{\partial y}\cdot\frac{\sec^2\mu\cot\alpha\cot\alpha_2}{\left(1+\cot^2\alpha_2\sec^2\mu\right)^{3/2}} \tag{27}$$

We have the relation

$$y = \kappa\left\{\cos\phi + \sin\phi\tan\theta\right\}\ ,\ \kappa = \frac{U - q_2}{a_2}\cdot\sin\alpha_2 \tag{28}$$

and $\cot\phi = \cot\alpha_2 \cdot \sec\mu$

$$\tan\theta = \frac{\kappa'}{\kappa}\frac{\left(Z^2 - 1\right)}{\left(Z^2 + 1\right)}\ ,\ \kappa' = \sqrt{1-\kappa^2} \tag{29}$$

Following Chopra[10] and Srivastava[11] the relation between Z and z_1 is given by

$$z_1 = \frac{1}{2}\left[\left(\frac{bz+1}{bz-1}\right)^{\pi/\lambda} + \left(\frac{bz+1}{bz-1}\right)^{-\pi/\lambda}\right] \tag{30}$$

where $b = \left(\frac{\kappa'\sin\phi + \kappa\cos\phi}{\kappa'\sin\phi - \kappa\cos\phi}\right)^{1/2}$

and $\lambda = \cot^{-1}\left(\frac{\cot\phi}{\left(\sin^2\phi - \kappa^2\right)^{1/2}}\right)$

From (30) we obtain

$$z = -\frac{1}{b}\frac{\left\{1+\left(z_1 + \sqrt{z_1^2 - 1}\right)^{\lambda/\pi}\right\}}{\left\{1-\left(z_1 + \sqrt{z_1^2 - 1}\right)^{\lambda/\pi}\right\}} \tag{31}$$

In (28) z is substituted in terms of z_1 actually in terms of x_1 as on the real axis $y_1 = 0$, z_1 being equal to $x_1 + iy_1$, we will then obtain $\frac{dy}{dx_1}$.

The numerical values for the calculation are

$$\frac{p_0}{p_1} = 0, \quad \alpha_0 = 39.97^0, \quad \chi = 40^0$$

These data provide $\frac{U - q_2}{a_2} = 0.94699$ (subsonic)

The solution of the problem is obtained by the introduction of the complex function

$$\omega(z_1) = \frac{\partial p}{\partial x_1} - i\frac{\partial p}{\partial y_1} \quad (32)$$

$\omega(z_1)$ is given by Chopra[10]

$$\omega(z_1) = \frac{G\delta\left[H(z_1 - x_0) - 1\right]\cos\chi \sec\alpha}{(z_1 - x_0)(z_1^2 - 1)^{1/2}}(z_1 - 1)^{\beta/\pi} e^{\phi + i\beta} \quad (33)$$

where

$$\phi = \frac{z_1}{12\pi}\left[\frac{1.51716 - \beta}{1} + \frac{4(0.00505 - \beta)}{(1 - 0.25z_1)} + \frac{2(-0.10311 - \beta)}{(1 - 0.50z_1)} + \frac{4(-0.22845 - \beta)}{(1 - 0.75z_1)} + \frac{(-1.57080 - \beta)}{(1 - z_1)}\right] \quad (34)$$

$$\beta = \tan^{-1}\left\{-\left(\frac{\partial p}{\partial y_1}\right) \Big/ \left(\frac{\partial p}{\partial x_1}\right)\right\}_{x_1 = t = \frac{1}{x} = z_1} \quad (35)$$

$$\left(\frac{\partial p}{\partial y_1}\right) \Big/ \left(\frac{\partial p}{\partial x_1}\right) = \frac{0.16931 - 0.09429\tan\theta - 0.05812\tan^2\theta + 0.02859\tan^3\theta}{(0.75607 - 0.24393\tan^2\theta)^{1/2} \times (0.36323 + 0.11275\tan\theta - 0.06569\tan^2\theta)} \quad (36)$$

The curvature κ from (27) can be put in the form

$$\kappa = -\frac{1}{C_3 - B_3 y}\cdot\frac{\partial p}{\partial x_1}\cdot\frac{\partial x_1}{\partial y}\cdot\frac{\sec^2\mu\cot\alpha\cot\alpha_2}{(1 + \cot^2\alpha_2\sec^2\mu)^{3/2}} \quad (37)$$

On the shock from $z_1 = x_1 + iy_1 = x_1$ (y_1 being zero) and varies from $x_1 = 1$ to $x_1 = \infty$.

The real part on the right hand side of 33 with z_1 replaced by x_1 gives the value of $\frac{\partial p}{\partial x_1}$.

As mentioned earlier $\frac{dy}{dx_1}$ is obtained from (28). Now that $\frac{\partial p}{\partial x_1}$ is known and $\frac{dy}{dx_1}$ is known then from (37) κ is known. We therefore have obtained final expression for κ.

We have the relation $\tan\theta = \frac{\kappa'}{\kappa}\frac{(z^2 - 1)}{(z^2 + 1)}$ when $z \to \infty$, $\tan\theta = \frac{\kappa'}{\kappa}$.

So from the relation (28)

$$y = \kappa\left(\cos\phi + \sin\phi\frac{\kappa'}{\kappa}\right) = \kappa\cos\phi + \kappa'\sin\phi$$

So

$$\frac{y}{(\kappa\cos\phi + \sin\phi\,\kappa')} = 1$$

when $z \to \frac{1}{b}$, then we have

$$\tan\theta = \frac{\kappa'}{\kappa}\frac{(1 - b^2)}{(1 + b^2)} = -\cot\phi$$

We have then $y = \kappa\{\cos\phi + \sin\phi(-\cot\phi)\} = 0$

or

$$\frac{y}{(\kappa\cos\phi + \sin\phi\,\kappa')} = 0$$

So in the final analysis $z \to \infty$ ($z_1 \to 1$ i.e. $x_1 \to 1$)

$$\frac{y}{(\kappa\cos\phi + \kappa'\sin\phi)} = 1$$

and

$$z_1 \to \frac{1}{b}(z_1 \to \infty,\ x_1 \to \infty)$$

$$\frac{y}{(\kappa\cos\alpha + \kappa'\sin\phi)} = 0$$

Numerical analysis

Taking $\frac{\partial p}{\partial x_1}$ into consideration from equation (33) it could be seen that κ is zero at $x_1 = 1$ (see equation 38) i.e. at the point of intersection of shock and Mach Cone. This is physically consistent. Also at $x_1 \to \infty, \kappa$ tends to ∞ i.e. at the point of intersection of wall surface of the wedge and shock front intersection.

Referring to equation (33) we see that the point of inflexion over the curvature of the reflected diffracted shock is given by when

$$H\left(x_1 - x_0\right) - 1 = 0 \quad (38)$$

i.e. when $$x_1 = x_0 + \frac{1}{H} \quad (39)$$

From the calculation we have (Chopra[10])

$x_0 = 0.75595$ and $H = 0.51062$

with these values of x_0 and H we obtain from (39)

$x_1 = 2.71935$

This indicates that at $x_1 = 2.71935$, we find that there is a point of inflexion over the reflected diffracted shock. The curvature has infinite value, then it passes through point of inflexion and finally it becomes zero. This is the qualitative estimate of the curvature.

Conclusion

The results obtained here give more general results as intersection is considered with yawed wedges. The results when there is no yaw in the wedge will reduce to the results of paper (2). The results are general and could be used in aeronautics depending on the situations that arise.

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Conflict of interest

The authors declare that there is no conflict of interest.

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